Tri-copter Drone Modeling with PID Control Tuned by PSO Algorithm

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ABSTRACT

The tri-rotors are more recent kinds of drones, as compared with the mostly used quad-rotors because of the numerous special characteristics over the other types of multi-copters. Many technical features specialize tri-copters like small volume that is useful in slender places, light weight, extended battery existence, and agility in translation and turns. In this paper, (single tri-rotor) design is theorized and the nonlinear mathematical model is derived completely by Newton-Euler formula then the Proportional -Integral and Derivative (PID) controller is utilized to control the rotational and translational equations , six PID controllers are used for six Degrees of Freedom (DOF) equations of the model with the associated parameters are tuned by Particle Swarm Optimization (PSO) method to minimize the whole Integral Time Absolute Errors (ITAEs) for the tri-rotor model and the effects are gained by Simulink in MATLAB . The results were satisfactory with the stability of the system and with little delay.

Keywords
Tri-rotor, Particle Swarm Optimization algorithm, Integral Time Absolute Error, Proportional-Integral-Derivative control, Cost Function

1. INTRODUCTION
Generally, Unmanned Aerial Vehicles (UAVs) can be flied autonomously or remotely and it can transfer payloads depending on the power plant of the vehicle. One of the UAV types is the Vertical Take Off and Landing (VTOL) which is capable to do some different movements such as hovering, vertical take-off and landing from dynamic or static place, lateral movement. Multi-rotors are VTOL vehicles under the UAV classification that have lately developed. Such multi-rotors make use of multiple rotors to operate [1]. Recently, multi-copters such as tri-copters, quad-copters, and penta-copters have become very famous Unmanned Aerial Vehicles (UAVs) that is known commercially as drones and they are attracting much interest among amateurs, and engineers. Multi-copter is a vehicle with further than two motors without an aircrew on board [2]. The Tri-Rotor drone is a modern multi-rotor UAV type used in many applications such as target acquisition, reconnaissance, battle damage assessment, monitoring, imaging of forest fires, transportation, and many dangerous tasks [3]. There are some motivations that Tri-rotor is chosen over quad-rotor such as [4]:

I. A clear alignment in the long distances of the tri-rotor contrast to the quad-rotor type, which is dissipated at long distances.

II. These tri-rotors are flying at a professional speed and similarly to fixed-wing UAVs.

Tri-rotor UAV is assumed to be nonlinear system, rigid body that can be defined by body frame reference and global (earth) frame reference and this UAV type has three motors such that each motor is placed on each arm which has an equal length to the center of gravity [5]. In addition, a servo motor is fitted on the tilt arm to rectify the generated reaction torque, this design made the tri-rotor movement excellent especially in back turns. Tri-rotor has a huge studies place as it has large variations to different other multi-copters in model and control attitude and position due to its unique yaw motion trouble caused by the unpaired rotor in the tail [3]. To overcome for this problem, more than one design solution has been offered to control the yawing.

The first design is called (single Tri-rotor), in this design three rotors are placed on the three arms and a servo rotor is placed with the tail motor in order to incline it by a specific angle to minimize the reaction torque, this design is characterized by the ease of movement of the aircraft, especially in the turns, this design is used in this work. The second design is called (coaxial tri-rotor), in this design six rotors has been used such that every two counter rotating motors are positioned on one arm to terminate the generated reaction moments, this design is characterized by high stability [6]. In this paper, the mostly used linear PID controller is used with Particle Swarm Optimization (PSO) tuning method. PSO is utilized to optimize the PID controllers' gains.

The PSO algorithm was firstly offered by Kennedy and Eberhart in 1995. These theory concepts depend on the behavior of groups such as bird, ants or bees. Its technique is based totally at the concept of looking the nearest route to food in the cluster, if a specific particle discovers the best route, the alternative particles also will observe the same route to the meals even though their localities are some distance away from the meals [7]. In [5], tri-copter model is illustrated with using a PID control for angular velocity, altitude, and Euler angles. In [6], the two single and coaxial tri-rotor designs are introduced with a PID control to improve altitude and attitude stability, the simulations have been done in MATLAB for both designs with good obtained stability results.

In [3], three different configurations of the rotor directions are explained and the best one with good control performance has been chosen in the work, then a PID control was applied but the results suffered from overshoot and oscillation then a back-stepping control was applied with fine satisfactory results. In [8], PID control with complimentary Filters was used in the work to control rotational actions and the results was implemented on a real tri-rotor vehicle with acceptable results. This paper is organized as follows, section two reviews the dynamics and concepts of tri-rotor vehicle, section three shows the model with its derived equations, section four provides the PID controllers tuned through PSO

set of rules and applied to the tri-rotor, finally sections five and six survey the simulation results and deduction.

2. TRI-ROTOR VEHICLE'S FLIGHT CONCEPTS

Tri-rotor has been made up of a triangular frame with three rotors, the front rotors rotates oppositely while the tail rotor rotates as the same as the front left rotor as shown in Figure 1 [6]
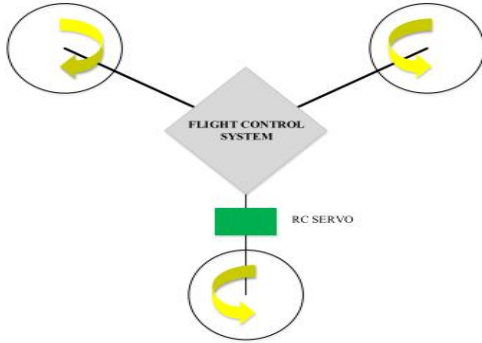

Fig 1: Tri-rotor principles [5]

As any UAV, the tri-rotor has six degrees of freedom and four control actions can be controlled directly, first control action is the altitude which is created by increasing or decreasing the angular velocities for all brushless motors equally as shown in Figure 2-a. Second control is the roll moment which is accomplished by varying the angular velocities of the front rotors with numbers 2 and 3 while rotor number 1 is fixed as shown in Figure 2-b. Third control is generated by varying the angular velocity of rotor1 and fixing the front rotor angular velocities to give the pitch movement as shown in Figure 2-c. Fourth control is the yaw moment and it is obtained by orientating the servo motor in the tail by a small angle as shown in Figure 2-d. The tri-rotor can fly ahead by means of reducing speed of the front rotors and increasing speed of the tail-rotor so to incline the frame of the plane forwards, and it will be flown to right side by speed up the left rotor and slow down the right rotor and vice versa [9].
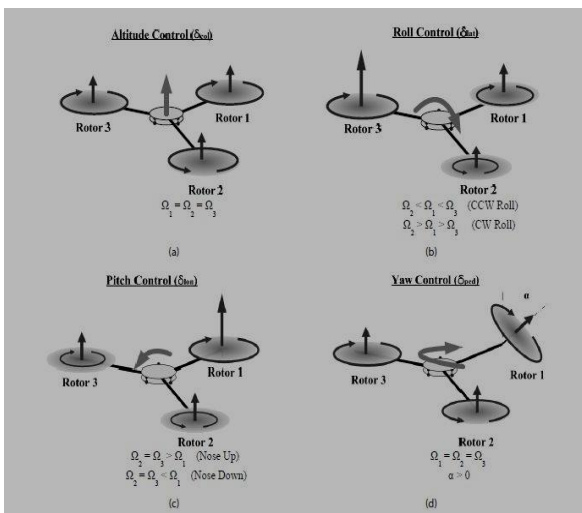

Fig 2: Control actions of Tri-rotor UAV [6]

3. TRI-ROTOR MATHEMATICAL MODEL DERIVATION

Y shape tri-rotor is presented in this work with three arms separated by an angle 120°. Any flying multi-copter is defined as rigid body and it is designated by motions with six degrees of freedom (DOF). This tri-rotor model is derived via Newton's second rule. The UAV system defined by three translational motion of body (local) and earth (global) coordinate systems equations and three rotating motion equations. Any point must be described with the aid of the coordinate system that is comprised as will be illustrated in Figure 3. The body coordinate expresses the placement of the tri-rotor and the earth coordinate expresses its place in the area. In order to translate between the two systems, a rotation matrix is needed which is described by this equation:

$$Q_{xyz}^{\emptyset\theta\Psi} =$$
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\emptyset & \sin\emptyset \\ 0 & -\sin\emptyset & \cos\emptyset \end{bmatrix} \cdot \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \cdot \begin{bmatrix} \cos\Psi & \sin\Psi & 0 \\ -\sin\Psi & \cos\Psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
$$=$$
$$\begin{bmatrix} \cos\theta\cos\Psi & \cos\theta\sin\Psi & -\sin\theta \\ \sin\emptyset\sin\theta\cos\Psi - \cos\emptyset\sin\Psi & \sin\emptyset\sin\theta\sin\Psi + \cos\emptyset\cos\Psi & \sin\emptyset\cos\theta \\ \cos\emptyset\sin\theta\cos\Psi + \sin\emptyset\sin\Psi & \cos\emptyset\sin\theta\sin\Psi - \sin\emptyset\cos\Psi & \cos\emptyset\cos\theta \end{bmatrix}$$
(1)

Where:

$\emptyset, \theta$ and Ψ --- are "Euler angles.

$Q_x^{\emptyset}$ --- signifies rotation of x-axis via an angle $\emptyset$.

Q_y^{θ} --- signifies rotation of y-axis via an angle θ.

Q_z^{Ψ} --- signifies rotation of z-axis via an angle Ψ.
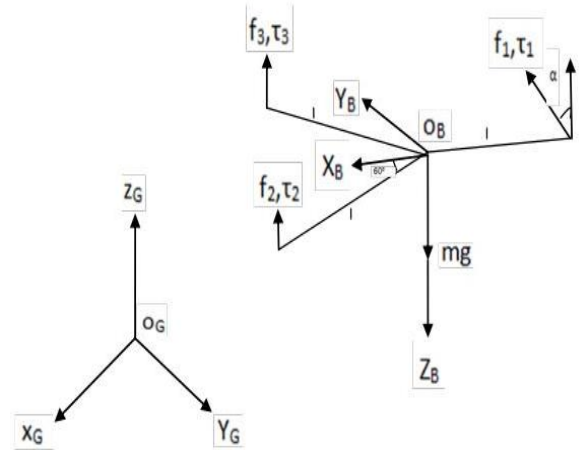

Fig 3: Body (B) and global (G) coordinate systems of a UAV

Here are three external separated forces acted on tri-rotor body (F_x^B, F_y^B, F_z^B) and three induced separated moments (M_x^B, M_y^B, M_z^B) which can be described by these equations:

$$F_{ext}^B = \begin{bmatrix} F_x^B \\ F_y^B \\ F_z^B \end{bmatrix} = \begin{bmatrix} 0 \\ f_1.\sin\alpha \\ f_1.\cos\alpha + f_2 + f_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ U_1 \end{bmatrix}$$
(2)

Where:

$F_{ext}^B \rightarrow$ Denotes the external force vector performing on the body.

$F_x^B, F_y^B \ F_z^B \rightarrow$ Denotes the external forces performing on the body in the $x, y,$ and z axes respectively.

$f_1, f_2, f_3 \rightarrow$ Denotes the forces produced from three motors.

$\alpha \rightarrow$ Denotes the incline "angle of the servo motor.

$U_1 \rightarrow$ Denotes the vertical control action.

$$M_{ext}^B = \begin{bmatrix} \frac{\sqrt{3}}{2} * l * (f_2 - f_3) \\ \frac{1}{2} * l * (f_2 + f_3) - l * f_1 * \cos \alpha + \tau_1 * \sin \alpha \\ -l * f_1 \ \sin \alpha - \tau_1 \cos \alpha - \tau_2 - \tau_3 \end{bmatrix} = \begin{bmatrix} U_2 \\ U_3 \\ U_4 \end{bmatrix} \quad (3)$$

$$f_i = b * \omega_i^2 \tag{4}$$

$$\tau_i = d * \omega_i^2 \text{ for } i = 1, 2, 3. \tag{5}$$

Where:

$M_{ext}^B \rightarrow$ denotes the external moment vector performing on the tri-copter body.

$l \rightarrow$ Denotes the length from the motor to the center of gravity.

$\tau_1, \tau_2, \tau_3 \rightarrow$ Denote torques caused from the motors.

$U_2, U_3, U_4 \rightarrow$ Denote attitude control actions.

$\omega_i \rightarrow$ Denotes the rotational velocity of rotor i.

$b \rightarrow$ Denotes the thrust constant.

$d \rightarrow$ Denotes the torque constant.

Newton second's law states that the mass multiplied by time derivative of the translational velocity vector equals to the force vector as illustrated by the following equations:

$$m.\dot{V} = \sum F \tag{6}$$

$$m.\dot{V} = Q_{xyz}^{\phi\theta\Psi} * F_{ext}^B + \begin{bmatrix} 0 \\ 0 \\ -m.g \end{bmatrix} \tag{7}$$

$$M_{ext}^B = I.\dot{\Omega} + \Omega \times I \Omega \tag{8}$$

Where :
m → Denotes the mass of Tri-rotor.
V → Denotes the translational speed vector.
g → Denotes the gravitational force.
$\Omega \rightarrow$ Denotes the rotational speed vector.
I → Denotes the inertia matrix.

$$I = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix} \tag{9}$$

$$\dot{V} = [\ddot{x} \quad \ddot{y} \quad \ddot{z}]^T \tag{10}$$

$$\dot{\Omega} = [\ddot{\phi} \quad \ddot{\theta} \quad \ddot{\Psi}]^T \tag{11}$$

It is considered that Euler angles, and body angular rates are in hovering position, so the translational equations are obtained as follows :

$$\ddot{x} = -\frac{\sin \theta}{m} U_1 \tag{12}$$

$$\ddot{Y} = \frac{\sin \phi \cos \theta}{m} U_1 \tag{13}$$

$$\ddot{Z} = \frac{\cos \phi \cos \theta}{m} U_1 - g \tag{14}$$

And the rotational equations are illustrated as follows:

$$\ddot{\phi} = \frac{Iyy - Izz}{Ixx} \dot{\Psi}\dot{\theta} + \frac{U_2}{Ixx} \tag{15}$$

$$\ddot{\theta} = \frac{Izz - Ixx}{Iyy} \dot{\Psi}\dot{\phi} + \frac{U_3}{Iyy} \tag{16}$$

$$\ddot{\Psi} = \frac{Ixx - Iyy}{Izz} \dot{\phi}\dot{\theta} + \frac{U_4}{Izz} \tag{17}$$

4. PID CONTROLLERS TUNED BY PARTICLE SWARM OPTIMIZATION METHOD

The PID control is commonly used controller in many industrial uses due to its easiness and good performance for several structures. Six PID controllers are projected for the Tri-rotor UAV. The common PID control equation is represented as follows:

$$U(t) = K_p \ e(t) + K_i \int_0^t e(\tau) \ d\tau + K_d \ \frac{d}{dt} e(t) \tag{18}$$

The proportional factor (K_p) influences the system by reducing the rise time , the integral factor (K_i) decreases the steady state error but, it will decelerate the transient response, and finally the derivative factor (K_d) drops the overshoot and improves the transient response that will stabilize the system . These three constants of the PID control K_p ,K_i,K_d are tuned by the optimization method PSO to find the suitable values which will be applied to the tri-rotor to achieve stability and good flight [8]. Better performance and good stability can be obtained for nonlinear systems when the PID parameters are tuned by PSO. PSO technique is an optimization method which it is based in searching the optimum on social distribution of a swarm such as birds flocking or fish schooling. Each particle signifies a bird and the swarm define particles in the space [10, 11]. The particles in the swarm communicate through modification of velocity and position as the following equations:

$$V_{i,j}^{k+1} = \omega \times V_{i,j}^k + C_1 \times r_1 (Pbest_{i,j}^k - X_{i,j}^k) + C_2 \times r_2 (Gbest_j^k - X_{i,j}^k) \tag{19}$$

$$X_{i,j}^{k+1} = X_{i,j}^k + V_{i,j}^{k+1} \tag{20}$$

$$\omega = \omega_{max} - K(\omega_{max} - \omega_{min})/\text{Maxiter} \tag{21}$$

Where:

$i = 1$ - swarm size(S)

$j = 1$ - number of factors in a particle (D)

K = the iteration number.

$V_{i,j}^k$ = defines the velocity of jth parameter in i^{th} particle at k^{th} iteration.

ω = defines the inertia weight.

$C_1, C_2 = 2, r_1, r_2$ = random numbers in the range (0, 1) = defines acceleration factors.

$Pbest_{i,j}^k$ = the best j^{th} parameter1 in i^{th} particle till k^{th} iteration.

$Gbest_j^k$ = the best j^{th} parameter until k^{th} iteration.

$X_{i,j}^k$ = the position of jth parameter in i^{th} particle at k^{th} iteration.

$\omega_{max} = 0.9, \omega_{min} = 0.4$

Maxiter = the maximum number of executed iterations .The steps of PSO technique are given as follows:

1) At first step, the initialization of position and velocity for the particles in random is defined.
2) Initialize P_{best} and G_{best}.
3) Find the cost value for each particle and compare it with the previous P_{best} and update with the better value.
4) Compare the cost value of each particle with the previous G_{best} and update with the better value.
5) For each particle, update position and velocity.
6) Return to step (3) and repeat the procedure till stopping condition is encountered.

The stopping criteria can be selected either when the maximum iteration's number is reached or when there is no enhancement observed after a number of iterations. In this work, after 120 iterations no improvement in the objective function has been observed so the tuning process had been terminated. Six different PID controllers are proposed to the system to control both translational and rotational equations as demonstrated in Figure 4.
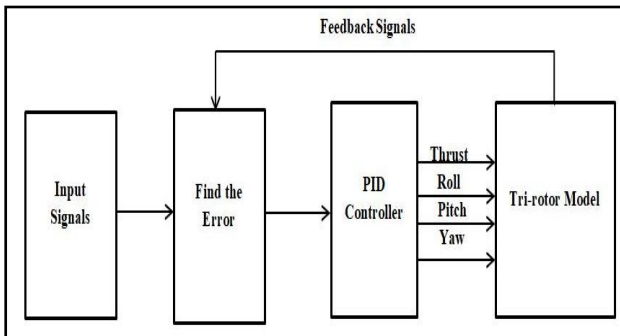

Fig 4: Tri-copter model with tuned PID controllers

Figure 5 shows the flowchart for the PSO principle working, and the mechanism of PSO process in penetrating the multidimensional search area has been presented in Figure 6.
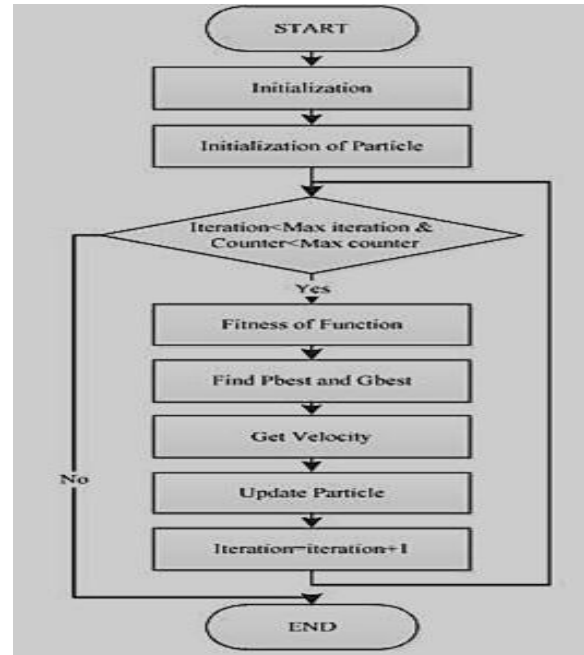

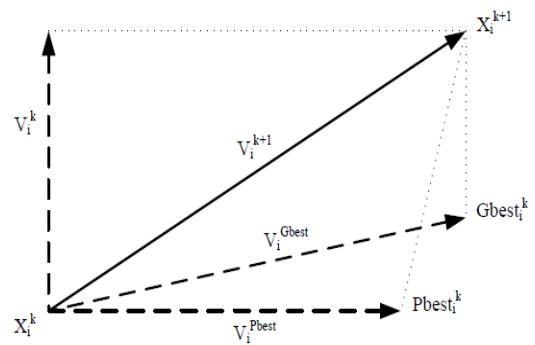
Fig 5: PSO Procedure Flowchart

Fig 6: PSO Mechanism in Searching Multidimensional Area

5. SIMULATION RESULTS

The parameters used for the simulation illustrated in Table 1. The tri-rotor model is executed by means of Simulink in MATLAB environment, and the PSO procedure is performed by using m.file to achieve reiteration in calculations. ITAE for the sum of errors has been used as the objective function to be minimized because it produces less overshoot and oscillation than other criterions.

$$\text{ITAE} = \int_0^t \tau |e| \, d\tau \qquad (22)$$

Where t =10 seconds (simulation time) and the error is expressed as:

e = desired value – actual value   (23)

Euler angles are approximated near the hovering position and as follows:

$\sin \theta = \theta$, $\cos \theta = 1$, and $\sin \emptyset = \emptyset$, $\cos \emptyset = 1$ (24)

Table 1: The Parameters for Tri-Rotor Vehicle and the PSO Needed values

Parameter	Mean	Value	Unit
Maxiter	Maximum number of iterations	142	---
S	Swarm size	30	---
D	Dimension of particles (PID gains)	18	---
m	Mass of Tri-rotor	0.814	Kg
I_x	Roll inertia	0.043	$Kg.m^2$
I_y	Pitch inertia	0.048	$Kg.m^2$
I_z	Yaw inertia	0.077	$Kg.m^2$
g	Gravitational force	9.8	m/sec^2
l	Arm length	0.15	m

Figure 7 shows the objective function values obtained along the iterations. Clearly, there is no significant change after 120 iterations so the tuning had been stopped on the 142 iterations with minimum overall error. The acquired gains for PID controllers by using PSO optimization are specified in Table 2.
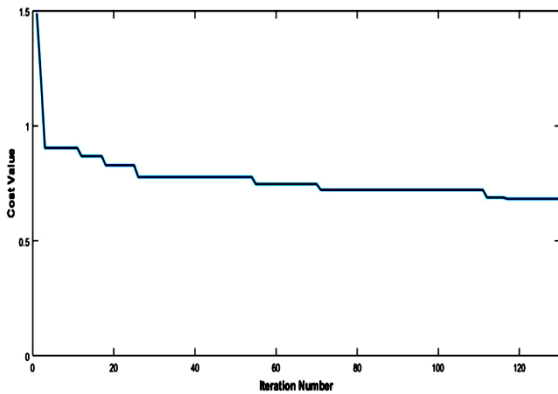

Fig 7: The objective function with iteration number.

Table 2. The tuned gains for PID controllers

Gains	X	Y	Z	Ø	θ	ψ
K_p	30.84	36.24	41.53	46.03	28.87	12.46
K_i	0.001	0.002	0.006	0.004	0.007	0.0002
K_d	6.736	7.72	10.53	15.8	19.56	7.128

The desired inputs to x, y, z, psi axes were step input to check stability ,whereas Ø and θ angles wanted values were acquired from x and y controllers since the system of the drone is under-actuated The obtained results are illustrated in Figures 8-13:
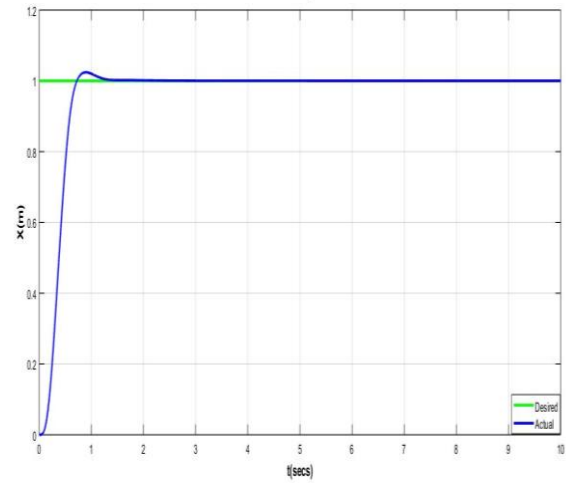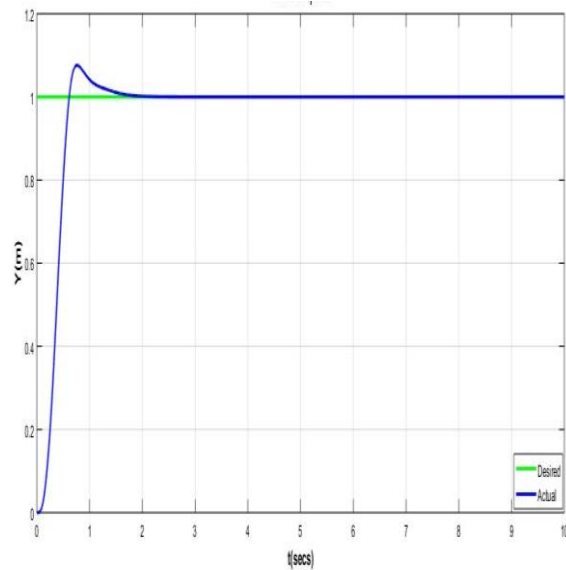

Fig 8: Step response for x-axes with PID tuned by PSO
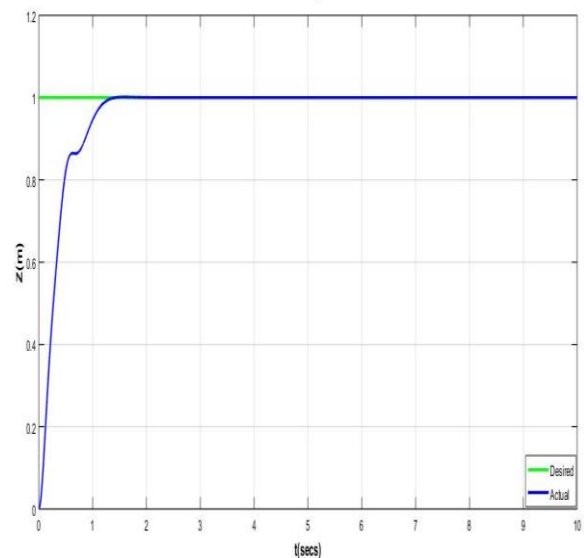

Fig 9: Step response for y-axes with PID tuned by PSO
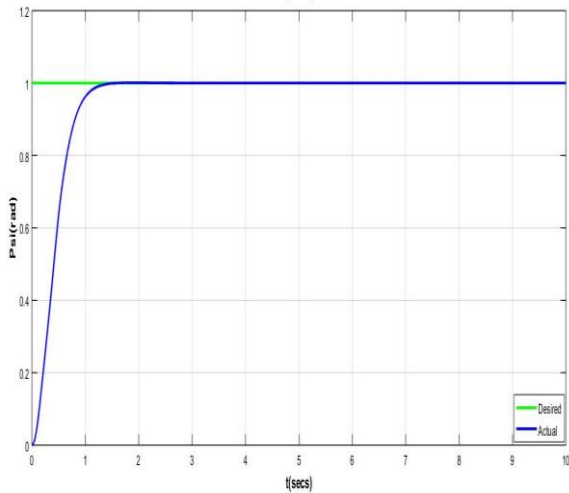

Fig 10: Step response for z-axes with PID tuned by PSO

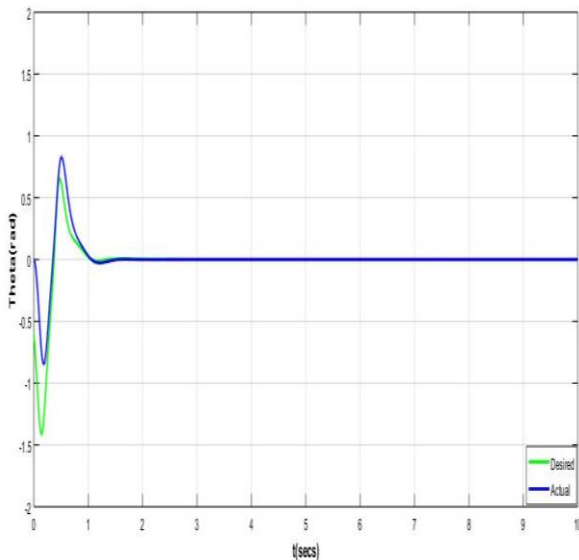**Fig 11: Step response for PSI-angle with PID tuned by PSO**

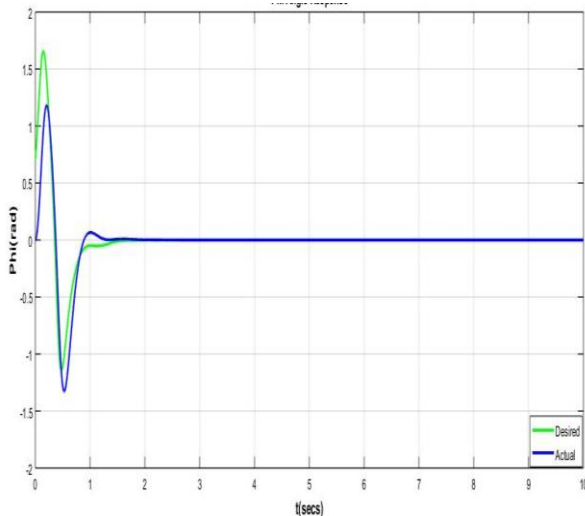**Fig 12: Response for desired theta-angle with PID tuned by PSO**

Fig 13: Response for desired phi-angle with PID tuned by PSO

After applying the obtained gains from tuning, the ITAE values have been measured and the values were very small as the following:

ITAE for x-axes = 0.0994819

ITAE for y-axes = 0.1157421

ITAE for z-axes = 0.1056444

ITAE for phi-angle = 0.1397974

ITAE for theta-angle = 0.0852669

ITAE for psi-angle = 0.1365153

6. CONCLUSION
The novel model, tri-rotor had been modeled in this paper with six DOF equations then after, the PID controllers were applied to the model in order to attain stability. The PSO technique was used to adjust the PID constants such that minimize the whole ITAE values for all the six errors. The results that were gained indicated less overshoot and small delay times so the tri-rotor will be in a stable manner when flying in hovering position and applying different tracks.

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