

Sliding Mode Control of Inverted Pendulum with Decoupling Algorithm

Ajit Kumar Sharma
Electrical Engineering Department
Delhi Technological University, Delhi

Bharat Bhushan
Electrical Engineering Department
Delhi Technological University, Delhi

ABSTRACT

This paper presents a decoupling algorithm of sliding mode control on inverted pendulum. The decoupled method provides a simple way to achieve asymptotic stability for a nth-order nonlinear systems. The system dynamics of SMC and inverted pendulum systems are encapsulated in the algorithm form and analysed by MATLAB Simulations. The convergence of the proposed sliding mode control is verified by Lyapunov function to prove the stability of system. Numerical simulations of designed SMC control strategy for inverted pendulum demonstrate faster convergence, reduced disturbance in control input and overall robust performance.

Keywords

Sliding Mode Control (SMC), Inverted Pendulum, Decoupling Algorithm.

1. INTRODUCTION

In the past decade one may observe augmented attention on under-actuated systems in research field. The under-actuated systems have fewer actuators than the degrees of freedom to be controlled. These systems have applications in free-flying Space robots, Underwater robots, Surface vessels, Manipulators with structural flexibility, etc. The inverted pendulum has been considered as benchmark example in nonlinear control studies as a under-actuated system[1]. Dynamics of the inverted pendulum are quintessential for the balance in wheeled Mobile robots, Robo walk and Robo thrusters [2-6]. With single input and two outputs, inverted pendulum has remained an exacting control problem owing to its characteristics like instability, non-linearity. It has two equilibrium points; one being stable while other is unstable [7]. Various control strategies can be found in literature to stabilize the pendulum around unstable equilibrium point [8] Researchers have analysed PID based control [9], neural network control [11], fuzzy control [12], optimisation tools like linear quadratic regulator [10, 13]. Sliding mode control has emerged as promising method for control of inverted pendulum. [15-17]

The sliding mode is a special type of variable structure control technique [18]. Its distinct robustness and accuracy negates both intrinsic and extrinsic disturbances. The controller operates by bringing the states of system to a desired sliding

surface. Hence, design of sliding mode controller involves two stages[18]. In first stage, a sliding surface is chosen while in second stage an appropriate control law is designed which can bring the system states to desired states on sliding surface. Sliding mode control enforces reduction in order of system, thereby increasing the ability of system to minimise impact of disturbances and uncertainties [19].

In this paper, a SMC decoupling model has been designed and implemented on inverted pendulum in MATLAB environment. A decoupled sliding mode controller is designed to stabilize the pendulum at upright position point while moving the cart to a desired position. The whole system design is divided in two stages. At first stage, sliding mode surface which satisfies the desired specification of inverted pendulum is designed. During second stage, a control function is designed which handles the task of bringing inverted pendulum in equilibrium state in the presence of uncertainties and external disturbance.

The paper is organized as follows. Section I introduces the objectives of the study. In Section II analysis of inverted pendulum and its decoupling algorithm is presented. Decoupled sliding mode control is explained in Section III. In Section IV, the simulation results obtained from the study are presented. Section V embodies conclusion followed by references.

2. INVERTED PENDULUM

A single link inverted pendulum is shown in Figure 1. which consists of a moving cart and a rod hinged on it. Table 1 gives the different parameters of inverted pendulum.

Let the pendulum is at its equilibrium point, so its linear equation is given as-

$$\left. \begin{aligned} \ddot{\theta} &= \frac{m(m+M)gl}{(M+m)l+Mml^2} \theta - \frac{ml}{(M+m)l+Mml^2} (u + d(t)) \\ \ddot{x} &= -\frac{m^2 g l^2}{(M+m)l+Mml^2} \theta + \frac{l+ml^2}{(M+m)l+Mml^2} (u + d(t)) \end{aligned} \right\} \quad (1)$$

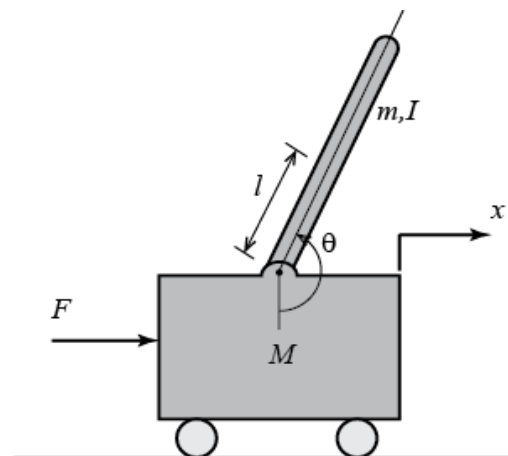


Figure 1. Single link inverted pendulum

Table 1. Parameter of Inverted Pendulum

Parameter	Meaning	Unit
θ	Pendulum angle	rad
x	Displacement of cart	m
m	Mass of pendulum	kg
M	Mass of cart	kg
l	Length of pendulum	m
g	Acceleration of gravity	m/s^2

To design sliding mode controller model should be decoupled.

Hence, defining $t_1 = \theta$, $t_2 = x$ then $w_1 = \dot{\theta}$,

$w_2 = \dot{x}$. Define $f_1(t, w) = \frac{m(m+M)gl}{(M+m)l+Mml^2} t_1$

$$g_1(w_2) = \frac{ml}{(M+m)l+Mml^2}$$

$$f_2(t, w) = -\frac{m^2gl^2}{(M+m)l+Mml^2} t_1, \text{ and}$$

$$g_2(w_2) = \frac{l+ml^2}{(M+m)l+Mml^2}$$

$$\text{Then, } \frac{g_1(w_2)}{g_2(w_2)} = \frac{\frac{ml}{(M+m)l+Mml^2}}{\frac{l+ml^2}{(M+m)l+Mml^2}} = -\frac{ml}{l+ml^2}$$

By using decoupling algorithm,

$$\left. \begin{aligned} \gamma_1 &= t_1 - \int_0^{t_2} \frac{g_1(s)}{g_2(s)} ds = t_1 + \frac{ml}{l+ml^2} t_2 \\ \gamma_2 &= t_1 - \frac{g_1(t_1)}{g_1(t_2)} w_2 = w_1 + \frac{ml}{l+ml^2} t_2 \\ \delta_1 &= t_2 \\ \delta_2 &= w_2 \end{aligned} \right\} \quad (2)$$

The system under stable point when both pendulum angle and cart position both comes at rest position, means $\theta = 0, x = 0, \dot{\theta} = 0, \dot{x} = 0$

Then $\gamma_1 = 0, \gamma_2 = 0, \delta_1 = 0, \delta_2 = 0$

By using decoupling algorithm, we get

$$\left. \begin{aligned} \dot{\gamma}_1 &= \gamma_2 \\ \dot{\gamma}_2 &= \left(\frac{m(m+M)gl}{(M+m)l+Mml^2} - \frac{ml}{l+ml^2} \frac{m^2gl^2}{(M+m)l+Mml^2} \right) t_1 \\ \dot{\delta}_1 &= \delta_2 \\ \dot{\delta}_2 &= -\frac{m^2gl^2}{(M+m)l+Mml^2} w_1 + \frac{l+ml^2}{(M+m)l+Mml^2} (u + d(t)) \end{aligned} \right\} \quad (3)$$

Let

$$T_1 = \left(\frac{m(m+M)gl}{(M+m)l+Mml^2} - \frac{ml}{l+ml^2} \frac{m^2gl^2}{(M+m)l+Mml^2} \right)$$

$$T_2 = -\frac{m^2gl^2}{(M+m)l+Mml^2}$$

$$\&, T_3 = \frac{l+ml^2}{(M+m)l+Mml^2}$$

Then Equation (3) becomes

$$\left. \begin{aligned} \dot{\gamma}_1 &= \gamma_2 \\ \dot{\gamma}_2 &= T_1 t_1 \\ \dot{\delta}_1 &= \delta_2 \\ \dot{\delta}_2 &= -T_2 w_1 + T_3 (u + d(t)) \end{aligned} \right\} \quad (4)$$

3. CONTROLLER DESIGN

Let $\mu_1 = \delta_2, \mu_2 = [\gamma_1 \ \gamma_2 \ \delta_1]^T$

Design the sliding mode controller as,

$$\sigma = \mu_1 - C\mu_2$$

where $C = [c_1 \ c_2 \ c_3]$.

Then we have

$$\dot{\sigma} = \dot{\mu}_1 - C\dot{\mu}_2 = T_2\gamma_1 + T_2T_4\delta_1 + T_3(u + d(t)) - C\dot{\mu}_2 \quad (5)$$

Design the sliding mode controller as

$$u = \frac{1}{T_3} (-T_2\gamma_1 - T_2T_4\delta_1 + C\dot{\mu}_2 - h\text{sgn}(\sigma)) \quad (6)$$

where $h \geq |d(t)|$.

Design the Lyapunov function as

$$V = \frac{1}{2} \sigma^2,$$

Then

$$\sigma \dot{\sigma} = \sigma (-h\text{sgn}(\sigma) + d(t)) \leq 0 \quad (7)$$

From equation (6) and (7) it is clear that, system can reach and thereafter stay on the manifold $\sigma = 0$ in finite time t_s .

On the manifold, we have

$$\sigma = \mu_1 - C\mu_2 = 0, \text{ i.e. } \mu_1 = C\mu_2.$$

When $t \geq t_s$, we have

$$\dot{\mu}_2 = \begin{bmatrix} \gamma_2 \\ T_1\gamma_2 + T_1T_4\delta_1 \end{bmatrix} = A_1\mu_1 + A_2\mu_2 = (A_1C + A_2)\mu_2$$

$$\text{where } A_1 = [0 \ 0 \ 1]^T, A_2 = \begin{bmatrix} 0 & 1 & 0 \\ T_1 & 0 & T_1T_4 \\ 0 & 0 & 0 \end{bmatrix}.$$

As $\mu_1 = \delta_2, \mu_2 =$

$$A_1C + A_2 = [0 \ 0 \ 1]^T [c_1 \ c_2 \ c_3] + \begin{bmatrix} 0 & 1 & 0 \\ T_1 & 0 & T_1T_4 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ c_1 & c_2 & c_3 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 0 \\ T_1 & 0 & T_1T_4 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ T_1 & 0 & T_1T_4 \\ c_1 & c_2 & c_3 \end{bmatrix}$$

The poles of $A_1C + A_2$ can be designed by the following:

$$|sI - (A_1C + A_2)| = \begin{vmatrix} s & -1 & 0 \\ -T_1 & s & -T_1T_4 \\ -c_1 & -c_2 & s - c_3 \end{vmatrix} = s^3 - c_3s^2 - T_1T_4c_1 - c_2T_1T_4s - T_1(s - c_3) = s^3 - c_3s^2 - (c_2T_1T_4 + T_1)s - T_1T_4c_1 + T_1c_3 = 0 \quad (8)$$

By comparing above equation with $(s + k)^3$

$$\begin{cases} -c_3 = 3k \\ -c_2T_1T_4 - T_1 = 3k^2 \\ -T_1T_4c_1 + T_1c_3 = k^3 \end{cases}$$

To make $A_1C + A_2$ Hurwitz, the sliding mode parameters are designed as follows:

$$\begin{cases} c_3 = -3k \\ c_2 = \frac{-3k^2 + T_1}{T_1T_4} \\ c_1 = -\frac{k^3 - T_1c_3}{T_1T_4} \end{cases}$$

4. IMPLEMENTATION OF ALGORITHM IN MATLAB

Simulation is done on MATLAB/Simulink environment to evaluate the results under various conditions (i.e. $k = 10, 50, 100$). Simulation results give a brief about the algorithm need to be implemented in the system.

The following parameters were chosen for simulation: $M=1$; $m=.010$; $L=0.50$.

The initial states are $\theta(0) = 0, \dot{\theta}(0) = 0, x(0) = 0.20, \dot{x}(0) = 0$ and ideal position signal is set as $\theta_d = \sin t$

Fig. 2 shows the simulation model of controller and pendulum.

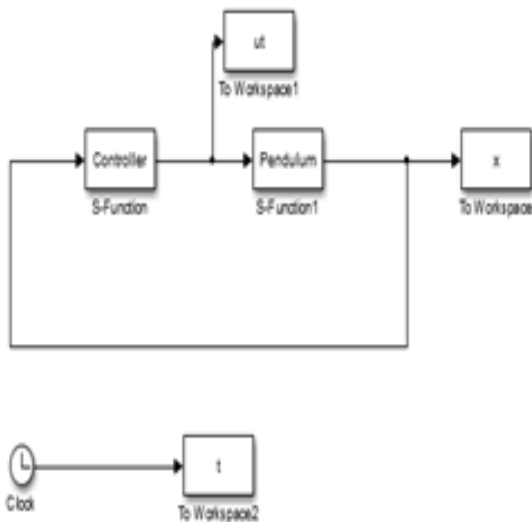


Figure 2. Simulink model of system

Case (a): For $k=10$, location of pole is closer to imaginary axis which increases the damping coefficient, hence increase the settling time and slow the system response as shown in fig (3). Slower response make system less stable, as we can see in fig (4) and fig (5).

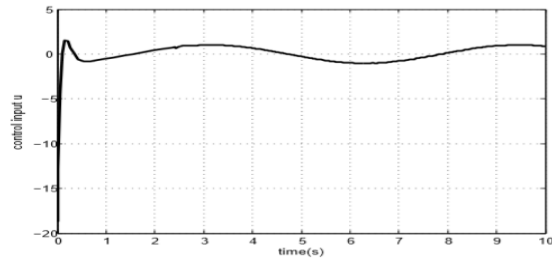


Figure 3. The response of control input (k=10)

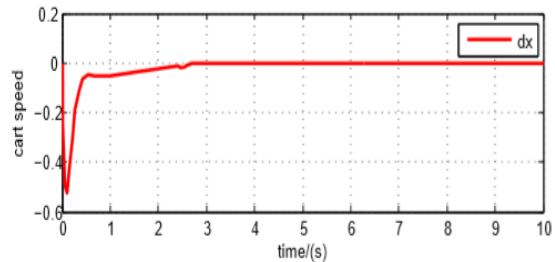
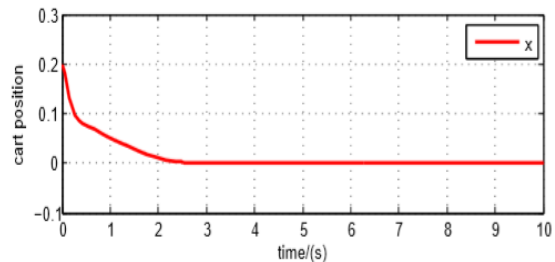


Figure 4. Cart position & Speed

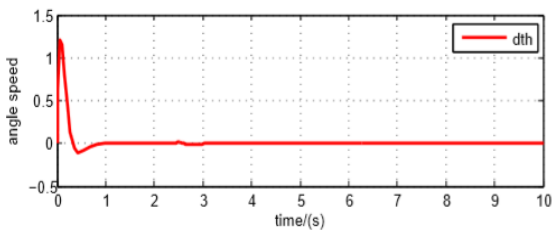
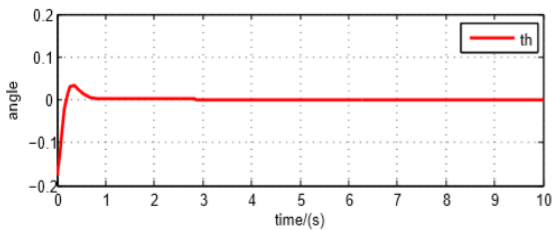


Figure 5. Pendulum angle & speed

Various cases are discussed below:

Case (b): For $k=50$, pole location moves away from imaginary axis, hence the control input has been improved which is shown in fig (6). Improved control input makes cart position in fig (7) and pendulum angle in fig (8) at rest position. Hence system become more stable.

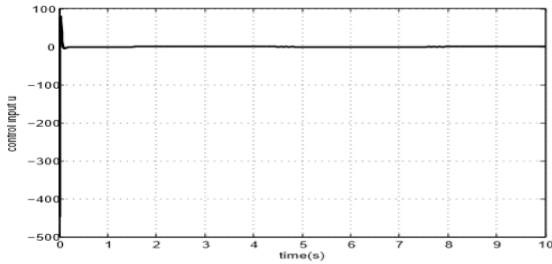


Figure 6. The response of control input ($k=50$)

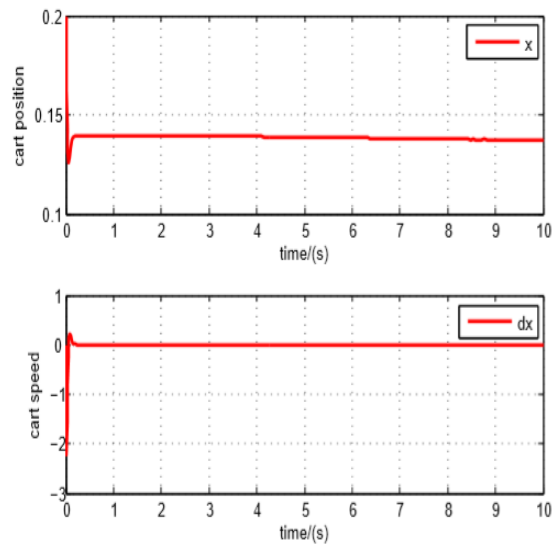


Figure 7. Cart position & Speed

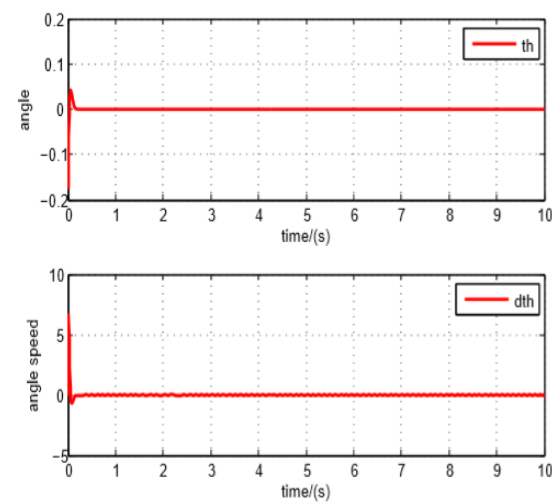


Figure 8. Pendulum angle & speed

Case (c): For $k=100$, poles are located in dominant pole region, which zero the damping coefficient and makes zero pole the transient response time as shown in fig (9). In dominant pole region cart position, fig (10) and pendulum position, fig (11) comes exactly at zero position.

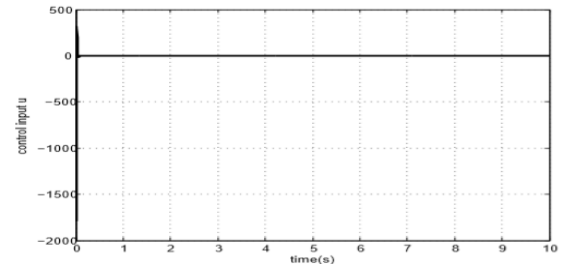


Figure 6. The response of control input ($k=100$)

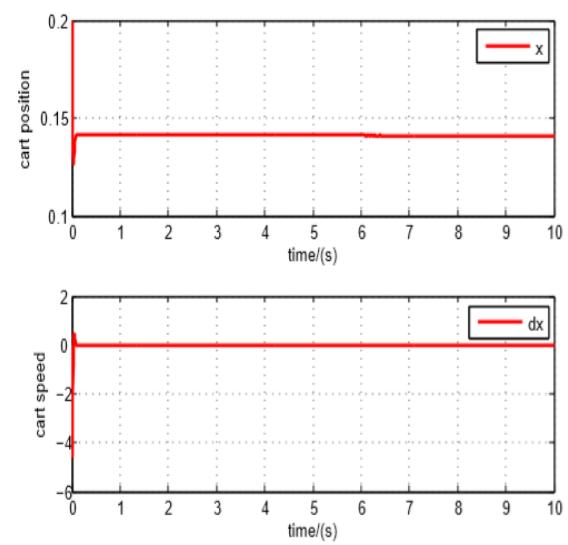


Figure 7. Cart position & Speed

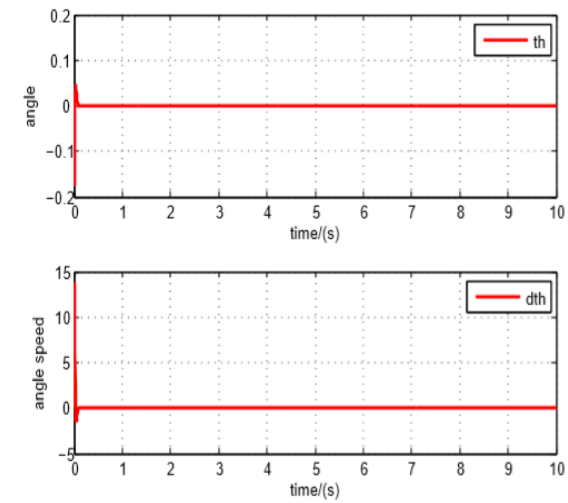


Figure 8. Pendulum angle & speed

From above figure it is clear that, as poles are moving away from imaginary axis, control input becomes more stable, hence pendulum and cart position both comes at rest position.

Table 2. Differences between All Cases

ANALYSIS	K=5	K=50	K=100
Stability	Less	More	More
Peak	More	Moderate	Less
Disturbance Rejection	Less	Less	More
Error	More	Medium	Less

5. CONCLUSION

The proposed structure gives a more stable control input for Inverted Pendulum system. There is reduction in first peak of control input response in Fig. 3 & Fig. 6. As compared to the control input in the Fig 5. Thus, implying reduced disturbance in control input. As evident from Table II, disturbance rejection, error improves as value of gain K is increased. Hence, there is overall improvement in system performance.

This study has investigated performance of decoupled algorithm with varying gain on single link inverted pendulum. It would be interesting to analyse performance of decoupled algorithm on multiple link inverted pendulum. Further, fractional order sliding mode controller can be tested for its performance on both single and multiple link inverted pendulum.

6. REFERENCES

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