Regional Decomposition of Images using Three Parameter Logistic Type Mixture Model with K-Means

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ABSTRACT
For image analysis image decomposition or segmenting the images is a basic requirement. For decomposing the images probability models play a vital role. This paper addresses image decomposition using three parameter logistic type mixture distribution. Here it is assumed that the pixel intensities of image region follow a three parameter logistic type probability distribution. The estimation of parameters is carried utilizing Expectation and Maximization algorithm. The initialization of the parameters is done with K-means algorithm and moment method of estimation the number of image regions is obtained counting the peaks of the histogram drawn for the pixel intensities of the whole image. The decomposition algorithm (segmentation) is developed under maximum component likelihood function with Bayesian considerations. The efficiency of the proposed algorithm is studied by computing the metrics for segmentation such as GCE, VOI, PRI. The experimentation is conducted with five randomly chosen images taken from Berkeley image database revealed that the proposed algorithm is superior to the other model based segmentation algorithms for some images, which are having laptykurtic image regions.A comparative study with that of segmentation algorithm based on GMM is also presented.

Keywords
Image decomposition,Three parameter logistic type mixture distribution, Expectation and Maximization algorithm, Metrics of segmentation,K-means algorithm.

1. INTRODUCTION
The image decomposition is the first step in image analysis which consider dividing the image into various image regions based on the features. The features of the grey images can be characterized by pixel intensities. There are several image segmentation(image decomposition) methods available for different types of images (K SrinivasaRao et al(2007),M seshashayee et al(2011),Chandra sekhar et al(2014)).In image decomposition usually it is considered that the pixel intensities of the images are modeled as Gaussian or Gaussian mixture model.(Yunjie Chen(2014),Celia A (2014),shanazAman et al(2015),Vamsikrishna M (2015), RajkumarG.V.S et al(2017)).All these authors assumed that the pixel intensities in each image region of the whole image are measokurtic,and hence Gaussian mixture model serve the purpose.

But in some images the distribution of the pixel intensities may not be measokurtic.Hence for effective image decomposition one has to consider alternative of Gaussian mixture model which accommodate laptykurtic and platy kurtic distributed image regions. RecentlySeshashayee et al(2014) and SrinivasaRao K et al(2014) have developed and analyzed image segmentation algorithm based on new symmetric mixture distribution and generalized new symmetric mixture distributions.Jyothermayee et al ((2015),(2016),(2017)) have developed image segmentation methods using generalized laplace mixture models.The generalized laplacemixture model or the generalized new symmetric distribution includes various types of platykurticdistributions. However, in some images the pixel intensities may not be distributed aslaptykurtic. For this type of images the Gaussian mixture models or generalized new symmetric distribution or the generalized Laplace mixture models may not serve the purpose. Hence, in this paper we develop and analyze an image decomposition (segmentation) algorithm using three parameter logistic type mixture distribution. Here it is assumed that the pixel intensities of each image region follow three parameter logistic type distribution. The three parameter logistic type distribution includes several leptokurtic probability distributions.

The rest of the paper is organized as follows: section-2 deals with three parameter logistic type distribution and its mixture model. The silent features of the mixture distribution are discussed. In section -3 the estimation of the parameters using Expectation and Maximization algorithm is discussed by deriving updated equations for the model parameters. In section-4the procedure for obtaining the initial values of the parameter through K-means algorithm and moment method of estimation is presented. In section-5 the image decomposition algorithm with component maximum likelihood under Bayesian considerations is developed. In section-6, the performance of the proposed algorithm is evaluated by computing the metrics of segmentation such as, PRI, VOC and GCE. In section-7 a comparative study of the proposed algorithm with that of GMM is given. Section-8 deals with conclusions and scope for further work in this direction of research.

2. THREE PARAMETER LOGISTIC TYPE DISTRIBUTION
This section deals with the three parameter logistic type distribution and its mixture model. The pixel intensities of the image regions are considered as features of the image. Here the three parameter logistic type distribution is assumed for modeling the pixel intensities of the image regions. As a result of it the whole image is characterized as a three parameter logistic type mixture model.The probability density function (P.D.F)of the pixel intensity of the image region is of the form

$$f(x,\mu,\sigma^2)=\frac{\left[\frac{3}{(3p+\pi^2)}\right]\left[p+\left(\frac{x-\mu}{\sigma}\right)^2\right]e^{-\left(\frac{x-\mu}{\sigma}\right)}}{\sigma\left[1+e^{-\left(\frac{x-\mu}{\sigma}\right)}\right]^2} \quad (1)$$

$$Where -\infty < x < \infty, -\infty < \mu < \infty, p \geq 4$$

For various values of the parameters it generates the different shapes of probability curves associated with the three parameter logistic type distribution .The different shapes of frequency curves are shown inFigure1.

Each value of the shape parameter p (4,5,6,7….) gives a bell shaped distribution.

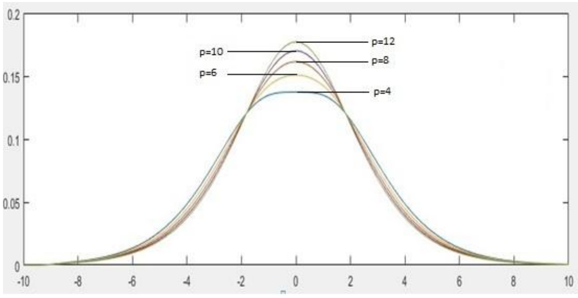

Figure .1 Frequency curve of three parameter logistic type Distribution

This distribution is symmetric about μ and the distribution function is

$$F(X)=\frac{\left[\frac{3}{3p+\pi^2}\right]\left[2\left(\frac{x-\mu}{\sigma}\right)\left[1+e^{-\left(\frac{x-\mu}{\sigma}\right)}\right]+\left[p+\left(\frac{x-\mu}{\sigma}\right)^2\right]\left[e^{-\left(\frac{x-\mu}{\sigma}\right)}-1\right]\right]}{\sigma^2 e^{\left(\frac{x-\mu}{\sigma}\right)}\left[1+e^{-\left(\frac{x-\mu}{\sigma}\right)}\right]^3}$$

The entire image is the collection of regions, hence it is assumed that the pixel intensities of the entire image follows a k-component mixture of three parameter logistic type distribution and its probability density function is of the form.

$$p(x)=\sum_{i=1}^{k}\alpha_i f_i(x,\mu,\sigma^2) \quad (2)$$

where k is the number of regions $0 \leq \alpha_i \leq 1$are weights such that $\sum \alpha_i = 1$ and $f_i(x,\mu,\sigma^2)$ is given in equation (1). α_i is the weight associated with i^{th}region in the whole image.

The mean pixel intensity of the whole image is

$$E(X)=\sum_{i=1}^{K}\alpha_i\mu_i$$

Even though the neighboring pixel intensities are correlated. The correlation can be made insignificant by considering spatial sampling proposed by Sewehand w. and Lei T (1992) and spatial averaging proposed by Kelley P.A. et al(1998).After reducing the correlations the pixel intensities are to be considered as independent .

3. UPDATED EQUATIONS OF EM-ALGORITHM FOR PARAMETER ESTIMATION:

This section deals with the estimation of the model parameters. The Expectation and Maximization algorithm can be utilized for obtaining the estimates of the parameters involved in the model. The major consideration for EM algorithm is expectation of the likelihood function and then maximization of it with respect to the parameters. Following the heuristic arguments given by Jeff A. Bilmes(1997) the updated equations of the model parameters are obtained.

This distribution of the pixel intensities of the image regions are having the three parameters namely μ, σ^2, p the shape parameters is to be first established before utilizing the EM algorithm. The p can be estimated by equating the sample kurtosis with the population kurtosis. Let the sample kurtosis is m_3 then

$$m_3=\frac{5\left(3p+\pi^2\right)\left(49p+155\pi^2\right)}{7\left(5p+7\pi^2\right)}$$

On simplification, we get

$$\left(735-175m_3\right)p^2+\left(2570-490m_3\right)\pi^2 p+\left(775-343m_3\right)\pi^4=0 \quad (10)$$

Solving equation (10) for p ,we get

$$p=\frac{2570-490m_3 \pm \sqrt{4326400-967680m_3}}{70\left(5m_3-21\right)\left(343m_3-775\right)\pi^2}$$

If $m_3 \leq 4.47089947$, Then we get two real roots and find out the value of p which is positive. After estimating the parameter p the likelihood of the function of the sample observations $x_1, x_2, x_3 ....x_N$ drawn from the image is

$$L(\theta)=\prod_{S=1}^{N}p(x_s,\theta^{(l)}) \quad (3)$$

$$L(\theta)=\prod_{s=1}^{N}\left(\sum_{i=1}^{k}\alpha_i f_i(x_s,\theta^{(l)}\right) \quad (4)$$

This implies

$$\log L(\theta)=\sum_{S=1}^{N}\log\left(\sum_{i=1}^{k}\alpha_i f_i(x_s,\theta^{(l)})\right) \quad (5)$$

Where , $\theta=\left(\mu_i,\sigma_i^2,\alpha_i\right)$ where $i=1,2,3,...n$

$$\log L(\theta)=\sum_{S=1}^{N}\log\left[\sum_{i=1}^{m}\alpha_i\frac{\left[\frac{3}{(3p+\pi^2)}\right]\left[p+\left(\frac{x_s-\mu_i}{\sigma_i}\right)^2\right]e^{-\left(\frac{x_s-\mu_i}{\sigma_i}\right)}}{\sigma_i\left[1+e^{-\left(\frac{x_s-\mu_i}{\sigma_i}\right)}\right]^2}\right] \quad (6)$$

E-STEP

In the Expectation (E) step, the expectation value of log $L(\theta)$ with respect to the initial parameter vector $\theta^{(0)}$ is

$$Q(\theta,\theta^{(0)}) = E_{\theta^{(0)}}\left[\log L(\theta)/\overline{x}\right] \quad (7)$$

Given the initial parameters $\theta^{(0)}$. One can compute the probability density function of pixel intensity X as

$$P(x_s,\theta^{(l)}) = \sum_{i=1}^{k}\alpha_i f_i(x_s,\theta^{(l)}) \quad (8)$$

$$L(\theta) = \prod_{S=1}^{N} p(x_s,\theta^{(l)}) \quad (9)$$

$$\log L(\theta) = \sum_{S=1}^{N}\log\left(\sum_{i=1}^{k}\alpha^{(l)}{}_i f_i(x_s,\theta^{(l)})\right) \quad (10)$$

The conditional probability of any observations x_s, belongs to any region K is

$$P_k(x_s,\theta^{(l)}) = \left[\frac{\alpha_k{}^{(l)} f_k(x_{s,}\theta^{(l)})}{p_i(x_s,\theta^{(l)})}\right] \quad (11)$$

$$p_k(x_s,\theta^{(l)}) = \left[\frac{\alpha_k{}^{(l)} f_k(x_{s,}\theta^{(l)})}{\sum_{i=1}^{k}\alpha_i{}^{(l)} f_i(x_s,\theta^{(l)})}\right] \quad (12)$$

The Expectation of the log likelihood function of the sample is

$$Q(\theta,\theta^{(l)}) = E_{\theta^{(l)}}\left[\log L(\theta)/\overline{x}\right]$$

But we have

$$f_i(x_s,\theta^{(l)}) = \frac{\left[\frac{3}{(3p+\pi^2)}\right]\left[p+\left(\frac{x_s-\mu_i^{(l)}}{\sigma^{(l)}}\right)^2\right]e^{-\left(\frac{x_s-\mu_i^{(l)}}{\sigma_i^{(l)}}\right)}}{\sigma_i{}^{(l)}\left[1+e^{-\left(\frac{x_s-\mu_i^{(l)}}{\sigma_i^{(l)}}\right)}\right]^2} \quad (13)$$

This implies

$$Q(\theta,\theta^{(l)}) = \sum_{i=1}^{k}\sum_{s=1}^{N}\left(P_i(x_s,\theta^{(l)})(\log f_i(x_s,\theta^{(l)}) + \log\alpha_i{}^{(l)})\right) \quad (14)$$

M-STEP:

For obtaining the estimation of model parameters one has to maximize $Q(\theta,\theta^{(l)})$ such that $\sum\alpha_i = 1$. This can be solved by applying the standard solution method for constrained maximum by constructing the first order Lagrange type function

$$F = \left[E\left(\log L(\theta^{(l)})\right) + \lambda\left(1-\sum_{i=1}^{k}\alpha_i^{(l)}\right)\right] \quad (15)$$

where, λ is Lagrangian multiplier combining the constraint with the log likelihood functions to be maximized.

The above two steps are repeated as necessary, each iteration is guaranteed to increase the likelyhood and the algorithm is guaranteed to converge to a local maximum of the likelihood function

The updated equations of α_i for $(l+1)^{th}$ iteration is

$$\alpha_i{}^{(l+1)} = \frac{1}{N}\sum_{s=1}^{N}P_i(x_s,\theta^{(l)})$$

$$\alpha_l{}^{(l+1)} = \frac{1}{N}\sum_{s=1}^{N}\left[\frac{\alpha_l{}^{(l)} f_l(x_{s,}\theta^{(l)})}{\sum_{i=1}^{k}\alpha_i{}^{(l)} f_i(x_s,\theta^{(l)})}\right] \quad (16)$$

For updating the parameter $\mu_i, i = 1,2,3..........k$ we consider the derivatives of $Q(\theta,\theta^{(l)})$ with respect to μ_i and equal to zero

We have $Q(\theta,\theta^{(l)}) = E\left[\log L(\theta,\theta^{(l)})\right]$

There fore

$$\frac{\partial}{\partial\mu_i}(Q(\theta,\theta^{(l)})) = 0$$

Implies

$$E\left[\frac{\partial}{\partial\mu_i}(\log L(\theta,\theta^{(l)}))\right] = 0$$

Taking the partial derivative with respect to μ_i ,we have

$$\frac{\partial}{\partial\mu_i}\left[\sum_{s=1}^{N}\sum_{i=1}^{K}P_i(x_{s.},\theta^l)\log\left[\alpha_i\frac{\left[\frac{3}{3p+\pi^2}\right]\left[p+\left(\frac{x_s-\mu_i}{\sigma_i}\right)^2\right]e^{-\left(\frac{x_s-\mu_i}{\sigma_i}\right)}}{\sigma_i\left[1+e^{-\left(\frac{x_s-\mu_i}{\sigma_i}\right)}\right]^2}\right]\right] = 0 \quad (17)$$

$$\mu_i^{(l+1)} = \frac{\sum_{s=1}^{n}\frac{P_i(x_{s.},\theta^{(l)})(2x_s)}{(\sigma_i^2)^{(l)}\left[p+\left(\frac{x_s-\mu_i^{(l)}}{\sigma_i^{(l)}}\right)^2\right]} - \sum_{s=1}^{n}\frac{P_i(x_{s.},\theta^{(l)})}{\sigma_i^{(l)}} + \sum_{s=1}^{n}\frac{2P_i(x_{s.},\theta^{(l)})}{\sigma_i^{(l)}\left[1+e^{\left(\frac{x_s-\mu_i^{(l)}}{\sigma_i^{(l)}}\right)}\right]}}{2\sum_{s=1}^{n}\frac{P_i(x_{s.},\theta^{(l)})}{(\sigma_i^2)^{(l)}\left[p+\left(\frac{x_s-\mu_i^{(l)}}{\sigma_i^{(l)}}\right)^2\right]}} \quad (18)$$

For updating $\sigma_i{}^2$ we differentiate $Q(\theta,\theta^{(l)})$ with respect to $\sigma_i{}^2$ and equate it to zero

That is $\frac{\partial}{\partial \sigma^2}(Q(\theta,\theta^{(l)}))=0$

This

implies $E\left[\frac{\partial}{\partial \sigma^2}(\log L(\theta,\theta^{(l)}))\right]=0$

Taking the partial derivative with respect to σ_i^2

$$\frac{\partial}{\partial \sigma_i^2}\left[\sum_{s=1}^{N}\sum_{i=1}^{K}P_i(x_s,\theta^l)\log\alpha_i\frac{\left[\frac{3}{3p+\pi^2}\right]\left[p+\left(\frac{x_s-\mu_i}{\sigma_i}\right)^2\right]e^{-\left(\frac{x_s-\mu_i}{\sigma_i}\right)}}{\sigma_i\left[1+e^{-\left(\frac{x_s-\mu_i}{\sigma_i}\right)}\right]^2}\right]=0 \quad (19)$$

$$\sigma_i^{2^{(l+1)}}=\frac{\sum_{s=1}^{N}\frac{P_i(x_s,\theta^{(l)})\left(x_s-\mu_i^{(l+1)}\right)}{2\sigma_i^{3^{(l)}}}-\sum_{s=1}^{N}\frac{P_i(x_s,\theta^{(l)})\left(x_s-\mu_i^{(l+1)}\right)}{\sigma_i^{3^{(l)}}\left[1+e^{\frac{(x_s-\mu_i)}{\sigma_i}}\right]}-\sum_{s=1}^{N}\frac{P_i(x_s,\theta^{(l)})}{2\sigma_i^{2^{(l)}}}}{\sum_{s=1}^{N}\frac{P_i(x_s,\theta^{(l)})\left(x_s-\mu_i^{(l+1)}\right)^2}{\sigma_i^{4^{(l)}}\left[p\sigma_i^{2^{(l)}}+\left(x_s-\mu_i^{(l+1)}\right)^2\right]}} \quad (20)$$

Where,

$$p_i(x_s,\theta^{(l)})=\left[\frac{\alpha_i^{(l+1)}f_i(x_s,\mu_i^{(l+1)},\sigma_i^{2^{(l)}})}{\sum_{i=1}^{k}\alpha_i^{(l+1)}f_i(x_s,\mu_i^{(l+1)},\sigma_i^{(l)})}\right] \quad (21)$$

4. INITIALIZATION OF THE PARAMETERS BY K-MEANS:

The efficiency of the EM algorithm in estimating the parameters is heavily dependent on the number of regions in the image .The number of image regions are obtained, by plotting the histogram of the pixel intensities of the whole image, and the number of peaks in the histogram are taken as the number of regions say ‘k’

A commonly used method in initializing parameters is by drawing a random sample from the entire image (McLachan G. AND Peel D.(2000). This method performs well, if the sample size is small. To overcome this problem we use the K-means algorithm to divide the whole image into homogeneous regions.

We obtain the initial estimates of μ_i,σ_i^2 and α_i for the i^th^ region with the moment method for three parameter logistic distributionThe initial estimates of the parameters are $\alpha_i=\frac{1}{k}$, where i=1,2,3…k.

$\hat{\mu}_i=\bar{X}$, and $\sigma_i^2=\frac{4n_i}{3(n_i-1)}S^2$, where S^2 is sample variance, n_i is the number of observations in the i^{th} segmentation.

5. IMAGE DECOMPOSITION ALGORITHEM:

The image segmentation algorithm is proposed in this section. The model parameters are estimated as discussed in section 2 and 3. To segment we allocate the pixels to the respective image regions. The major steps in image segmentation are as follows:

Step 1:- K-means algorithm is utilized for dividing pixel intensities of the whole image into k-image regions. Where k is number of image regions.

Step 2:-compute the initial estimates for the parameters of the model using moment method of estimation for each image region as discussed in section-3.

Step3:- The Expectation and Maximization algorithm with the updated equations of parameters given in section-2 is utilized for computing the final parameters of model.

Step-4:-The allocation of each pixel in the whole image into its corresponding j^{th} image region is done by computing the component maximum likelihood of the each image region as follows:

i.e., x_s is assigned to the j^{th} region for which L_j is maximum.

Where

$$L_j=MAX\left[\frac{\left[\frac{3}{(3p+\pi^2)}\right]\left[p+\left(\frac{x_s-\mu_i}{\sigma_i}\right)^2\right]e^{-\left(\frac{x_s-\mu_i}{\sigma_i}\right)}}{\sigma_i\left[1+e^{-\left(\frac{x_s-\mu_i}{\sigma_i}\right)}\right]^2}\right],$$

$$-\infty<\mu_i<\infty,\ -\infty<\sigma_i^2<\infty,\ \ p\geq 4$$

6. EXPERIMENTATION AND RESULTS

This section deals with the experimentation of the suggested image decomposition algorithm. The experiment is carried with five randomly taken images namely, OSTRICH,WOMAN, HILL,OCEAN and EAGLE from Berkeley image data base(http://www.eees.berkeley.edu/Research/Projects/CS/Vision/bsds/BSDS300/html).The feature of the images are obtained by considering pixel intensities. The pixel intensities are obtaining by using MATLAB. Assuming that pixel intensities of image regions follow a mixture of three parameter logistic type distribution, the model characterizing the whole image is developed. With the help of K-means algorithm, the number of image regions ‘k’ for each image is obtained and presented in Table.1

Table 1.1: Refined values of K (K-means Algorithm)

IMAGE	OSTRICH	WOMAN	OCEAN	HILLS	EAGLE
Estimate of K	2	3	3	4	2

With the pixel intensities of each image region the starting values for the model parameters μ_i,σ_i^2 and α_i where i=1, 2, 3……k are computed and presented in the Tables 1.2, 1.3, 1.4, 1.5 and 1.6 for different images. With these initial values of the parameters and the Expectation and Maximization algorithm, the refined estimates of parameters are obtained and shown in Tables:2,3,4,5 and 6

Table:1.2 ML Estimates for Ostrich data for(K=2)

Parameters	Initial Parameters		Refined estimates	
	Image Region		Image Region	
	1	2	1	2
α_i	0.500	0.500	0.2591	0.7409
μ_i	40.5146	113.260	81.456	248.14
σ_i^2	64.09627	141.798	458.4785	1214.7420

For each image region the parameter p is first estimated as $\hat{p} = 4$

Histogram, k=2 Gray Image Segmented Image

Table:1.3 Estimates of the parameters for WOMAN Image

Parameters	Initial Parameters			Refined estimates		
	Image Region			Image Region		
	1	2	3	1	2	3
α_i	0.333	0.333	0.333	0.1205	0.6177	0.2618
μ_i	219.6327	115.5619	71.7264	54.25	48.14	160.23
σ^2	920.5615	406.2072	5738.391	355.458	498.258	1958.21

Histogram, k=3 Gray Image Segmented Image

Table:1.4 Estimate of the parameters for OCEAN Image

Parameters	Initial Parameters			Refined estimates		
	Image Region			Image Region		
	1	2	3	1	2	3
α_i	0.333	0.333	0.333	0.2402	0.57	0.18

					11	87
μ_i	73.1978	125.4605	189.7868	72.99	461.91	781.25
σ_i^2	287.4086	166.946	135.257	240.2154	571.45	188.27

Histogram, k=3 Gray Image Segmented Image

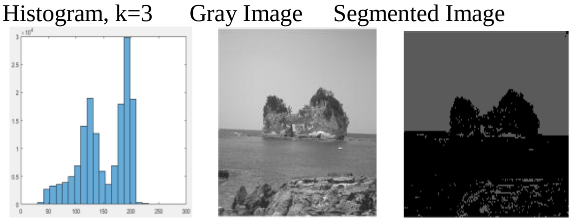

Table:1.5 Estimates of the parameters for HILLS Image

Parameters	Initial values of the Parameters				Refined Estimates			
	Image Region				Image Region			
	1	2	3	4	1	2	3	4
α_i	0.25	0.25	0.25	0.25	0.1815	0.4176	0.2029	0.1980
μ_i	189.4223	58.9282	107.5300	154.6640	249.251	649.472	440.615	372.725
σ_i^2	238.8511	364.34	152.8167	130.2383	298.4574	425.8795	114.5287	181.2541

Histogram, k=4 Gray Image Segmented Image

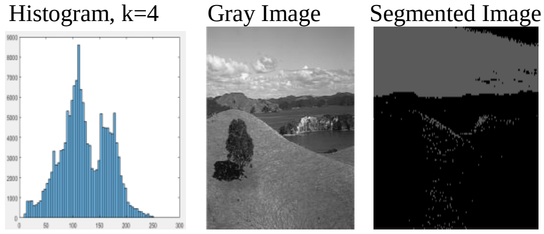

Table::1.6 TPLTM ML Estimates for EAGLE data for (K=2)

Parameters	Initial Parameters		Refined estimates	
	Image Region		Image Region	
	1	2	1	2
α_i	0.500	0.500	0.0635	0.9365
μ_i	40.5146	113.2603	23.11	99.33
σ_i^2	64.09627	141.798	63.254	181.257

Histogram, k=2 Gray Image Segmented Image

The fitted P.D.F of OSTRICH is

$$f(x_{(s)},\theta^{(l)}) = (0.2591)\frac{\left[\frac{3}{(3p+\pi^2)}\right]\left[p+\left(\frac{x_{(s)}-(81.456)}{(21.412112)}\right)^2\right]e^{-\left(\frac{x_{(s)}-(81.456)}{(21.412112)}\right)}}{(21.412112)\left[1+e^{-\left(\frac{x_{(s)}-(81.456)}{(21.412112)}\right)}\right]^2} + (0.7409)\frac{\left[\frac{3}{(3p+\pi^2)}\right]\left[p+\left(\frac{x_{(s)}-(248.14)}{(34.853191)}\right)^2\right]e^{-\left(\frac{x_{(s)}-(248.14)}{(34.853191)}\right)}}{(34.853191)\left[1+e^{-\left(\frac{x_{(s)}-(248.14)}{(34.853191)}\right)}\right]^2}$$

The fitted P.D.F of EAGLE is

$$f(x_{(s)},\theta^{(l)}) = (0.0635)\frac{\left[\frac{3}{(3p+\pi^2)}\right]\left[p+\left(\frac{x_{(s)}-(23.11)}{(7.953239)}\right)^2\right]e^{-\left(\frac{x_{(s)}-(23.11)}{(7.953239)}\right)}}{(7.953239)\left[1+e^{-\left(\frac{x_{(s)}-(23.11)}{(7.953239)}\right)}\right]^2} + (0.9365)\frac{\left[\frac{3}{(3p+\pi^2)}\right]\left[p+\left(\frac{x_{(s)}-(99.33)}{(13.463172)}\right)^2\right]e^{-\left(\frac{x_{(s)}-(99.33)}{(13.463172)}\right)}}{(13.463172)\left[1+e^{-\left(\frac{x_{(s)}-(99.33)}{(13.463172)}\right)}\right]^2}$$

The fitted P.D.F of OCEAN is

$$f(x_{(s)},\theta^{(l)}) = (0.1205)\frac{\left[\frac{3}{(3p+\pi^2)}\right]\left[p+\left(\frac{x_{(s)}-(54.25)}{(18.853594)}\right)^2\right]e^{-\left(\frac{x_{(s)}-(54.25)}{(18.853594)}\right)}}{(18.853594)\left[1+e^{-\left(\frac{x_{(s)}-(54.25)}{(18.853594)}\right)}\right]^2} + (0.6177)\frac{\left[\frac{3}{(3p+\pi^2)}\right]\left[p+\left(\frac{x_{(s)}-(48.14)}{(22.321694)}\right)^2\right]e^{-\left(\frac{x_{(s)}-(48.14)}{(22.321694)}\right)}}{(22.321694)\left[1+e^{-\left(\frac{x_{(s)}-(48.14)}{(22.321694)}\right)}\right]^2} + (0.2618)\frac{\left[\frac{3}{(3p+\pi^2)}\right]\left[p+\left(\frac{x_{(s)}-(160.23)}{(44.251667)}\right)^2\right]e^{-\left(\frac{x_{(s)}-(160.23)}{(44.251667)}\right)}}{(44.251667)\left[1+e^{-\left(\frac{x_{(s)}-(160.23)}{(44.251667)}\right)}\right]^2}$$

The fitted P.D.F of WOMAN is

$$f(x_{(s)},\theta^{(l)}) = (0.2402)\frac{\left[\frac{3}{(3p+\pi^2)}\right]\left[p+\left(\frac{x_{(s)}-(72.99)}{(15.49884)}\right)^2\right]e^{-\left(\frac{x_{(s)}-(72.99)}{(15.49884)}\right)}}{(15.49884)\left[1+e^{-\left(\frac{x_{(s)}-(72.99)}{(15.49884)}\right)}\right]^2} + (0.5711)\frac{\left[\frac{3}{(3p+\pi^2)}\right]\left[p+\left(\frac{x_{(s)}-(461.91)}{(23.905021)}\right)^2\right]e^{-\left(\frac{x_{(s)}-(461.91)}{(23.905021)}\right)}}{(23.905021)\left[1+e^{-\left(\frac{x_{(s)}-(461.91)}{(23.905021)}\right)}\right]^2} + (0.1887)\frac{\left[\frac{3}{(3p+\pi^2)}\right]\left[p+\left(\frac{x_{(s)}-(781.25)}{(13.721156)}\right)^2\right]e^{-\left(\frac{x_{(s)}-(781.25)}{(13.721156)}\right)}}{(13.721156)\left[1+e^{-\left(\frac{x_{(s)}-(781.25)}{(13.721156)}\right)}\right]^2}$$

The fitted P.D.F of HILLS is

$$f(x_{(s)},\theta^{(l)}) = (0.1815)\frac{\left[\frac{3}{(3p+\pi^2)}\right]\left[p+\left(\frac{x_{(s)}-(249.251)}{(17.275920)}\right)^2\right]e^{-\left(\frac{x_{(s)}-(249.251)}{(17.275920)}\right)}}{(17.275920)\left[1+e^{-\left(\frac{x_{(s)}-(249.251)}{(17.275920)}\right)}\right]^2} + (0.4176)\frac{\left[\frac{3}{(3p+\pi^2)}\right]\left[p+\left(\frac{x_{(s)}-(649.472)}{(20.636849)}\right)^2\right]e^{-\left(\frac{x_{(s)}-(649.472)}{(20.636849)}\right)}}{(20.636849)\left[1+e^{-\left(\frac{x_{(s)}-(649.472)}{(20.636849)}\right)}\right]^2} + (0.2029)\frac{\left[\frac{3}{(3p+\pi^2)}\right]\left[p+\left(\frac{x_{(s)}-(440.615)}{(10.701809)}\right)^2\right]e^{-\left(\frac{x_{(s)}-(440.615)}{(10.701809)}\right)}}{(10.701809)\left[1+e^{-\left(\frac{x_{(s)}-(440.615)}{(10.701809)}\right)}\right]^2} + (0.1980)\frac{\left[\frac{3}{(3p+\pi^2)}\right]\left[p+\left(\frac{x_{(s)}-(372.725)}{(13.463065)}\right)^2\right]e^{-\left(\frac{x_{(s)}-(372.725)}{(13.463065)}\right)}}{(13.463065)\left[1+e^{-\left(\frac{x_{(s)}-(372.725)}{(13.463065)}\right)}\right]^2}$$

7. COMPARITIVE STUDY OF THE ALGORITHM:-

This section deals with the performance of the proposed algorithm for image decomposition. The image segmentation quality metrics such as probabilistic rand index (PRI), global consistency error (GCE), and variation of information (VOI) are utilized. Table 7 provides the comparative image segmentation metrics obtained for the five images under experimentation with respective for the proposed algorithm and that of algorithm with GMM

Table:7 SEGMENTATION PERFORMANCE MEASURES

IMAGES	METHOD	PERFORMANCE MEASURES		
		PRI	GCE	VOI
OSTRICH	GMM	0.9147	0.2785	0.4273
	3 parameter -K Means	0.9251	0.1992	0.1258

WOMAN	GMM	0.8876	0.0232	0.1417
	3 parameter -K Means	0.9104	0.0199	0.1441
OCEAN	GMM	0.8852	0.0339	0.1927
	3 parameter -K Means	0.9145	0.0142	0.1214
HILLS	GMM	0.8688	0.2572	0.3357
	3 parameter -K Means	0.9154	0.1541	0.2347
EAGLE	GMM	0.9987	0.0023	0.0126
	3 parameter -K Means	0.9991	0.0014	0.0049

The Table.7 reveals that the proposed segmentation algorithm is much superior to that of segmentation algorithm with GMM with respect to the image segmentation quality metrics PRI,GCE, and VOI. For the images OSTRICH,WOMAN, HILL,OCEAN and EAGLE. Further the efficiency of the proposed segmentation algorithm is also studied by obtaining image quality metrics such as Average Difference, Maximum Distance, Image Fidelity, Mean Square Error, Signal to Noise Ratio, Image Quality Index. Table 6.2 presents the image quality metrics for the five images with respect to the proposed algorithm and the segmentation algorithm with GMM.

Table.8 presents the quality metrics of image segmentation with three parameter logistic type mixture model and K-means algorithm

Table.8 Quality Metrics for Comparison

IMAGES	Quality Metrics	GMM	Proposed 3parameter-K-means
OSTRICH	Average Difference	0.5315	0.4865
	Maximum Distance	0.4763	0.5715
	Image Fidelity	0.8124	0.8978
	Mean Square Error	0.0770	0.0592
	Signal to Noise Ratio	14.080	24.215
	Image Quality Index	0.8460	0.9021
WOMAN	Average Difference	0.4860	0.5845
	Maximum Distance	0.9435	0.9814
	Image Fidelity	0.4620	0.4928
	Mean Square Error	0.0803	0.0548
	Signal to Noise Ratio	4.7261	5.1878
	Image Quality Index	0.9782	0.9914
OCEAN	Average Difference	0.3211	0.1854
	Maximum Distance	0.6810	0.7514
	Image Fidelity	0.6885	0.8214
	Mean Square Error	0.0645	0.0324
	Signal to Noise Ratio	4.0802	5.879
	Image Quality Index	0.7763	0.8947
HILLS	Average Difference	0.2664	0.0958
	Maximum Distance	0.7664	0.8914
	Image Fidelity	0.9348	0.9856
	Mean Square Error	0.0138	0.0111
	Signal to Noise Ratio	0.9383	2.1987
	Image Quality Index	0.5710	0.6347
EAGLE	Average Difference	0.2350	0.3502
	Maximum Distance	0.5925	0.7817
	Image Fidelity	0.9882	0.9978
	Mean Square Error	0.0038	0.0011
	Signal to Noise Ratio	11.1494	19.245
	Image Quality Index	0.9869	0.9916

The Table.8 provides evidences for superiority of image segmentation algorithm with mixture of three component logistic probability distribution and K-means algorithm than the other algorithms under study. The quality metrics of proposed algorithm for the experimental images are very close to the standard values of the metrics.

8. CONCLUSIONS

This paper addresses a new probabilistic model in decomposing image regions. Here it is assumed that the pixel intensities are representing by image regions and they follow a three parameter logistic type distribution. The three parameter logistic type distribution is capable of portraying the image regions which are having leptokurtic distributed pixel intensities. The image decomposition algorithm is developed with component maximum likelihood by considering Bayesian frame work. The experimentation conducted with five images randomly taken from Berkeley image database revealed that the proposed image decomposition method outperforms, the existing image segmentation method for the grey images having leptokurtic pixel intensities in image regions. It is also observed that through segmentation quality metrics the proposed algorithm is superior than the segmentation algorithm based on GMM. The proposed algorithm is useful in decomposing the images at medical diagnostics, security and surveillances, remote sensing. The image decomposition using this algorithm further extended to color images by considering multivariate feature vector with multivariate three parameter logistic type mixture model, which will be taken up elsewhere.

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