Free Convective Visco-Elastic MHD Flow and Heat Transfer with Radiation

Sajal Kumar Das
Department of Mathematics
Bajali College, Pathsala, Barpeta Assam, India

ABSTRACT
An analysis of the steady free convective flow and heat transfer of a visco-elastic fluid confined between a long vertical wavy wall and a uniformly moving parallel flat wall has been presented. The x axis is taken along the length of the walls while the walls are given by y=εcos(Kx) and y=d. A uniform magnetic field is assumed to be applied normal to the flat wall. The equations governing the fluid flow and heat transfer have been solved by perturbation technique subject to the relevant boundary conditions. It is assumed that the solution consists of two parts, a mean part and a perturbed part. The long wave approximation has been used to obtain the solution of the perturbed part and to solve the mean part the well known approximation used by Ostrach [1] has been utilized. The perturbed part of the solution is the contribution from the waviness of the wall. Expressions for the zeroth-order and first order velocity, temperature, non-dimensional skin friction at the walls and pressure drop are obtained. The first order velocity, pressure drop, and skin friction coefficient have been presented graphically to observe the visco-elastic effects in combination of other flow parameters involved in the solution.

General Terms
Visco-elastic.

Keywords
Free convective, visco-elastic, perturbation technique, Prandtl number, skin friction, radiation.

1. INTRODUCTION
Analysis of fluid over a wavy wall is widely studied because of its application in different areas such as transpiration, cooling of re-entry vehicles and rocket boosters, cross hatching on ablative surfaces and film vaporization in combustion chambers. Radiation is a process of heat transfer through electromagnetic waves. The radiation effects play an important role when the surrounding temperature of a fluid is high, and this situation occurs in space technology. In such cases the investigators have to consider the effects of radiation and free convection. These types of flows are encountered in many industrial and environmental processes such as fossil fuel combustion energy processes, evaporation from large open water reservoirs, heating and cooling chambers, astrophysical flows, solar power technology and space vehicle re-entry. An important role is played by radiative heat and mass transfer in manufacturing industries for the design of reliable equipment like nuclear power plants, gas turbines and various propulsion devices for aircraft, missiles, satellites and space vehicles etc. If the temperature of the surrounding fluid is rather high, radiation effects play an important role in space related technology. Such studies were presented by Cess [2], Arpaci [3], Cheng and Ozisik [4], Hasegawa *et al*. [5], Hossain and Takhar, [6, 7], Hossain *et al.* [8], Tak and Kumar [9] and Mohamed *et al*. [10] in case of steady flows. In case of unsteady flows Raptis and Perdikis [11] have studied the flow past an accelerated plate by solving the governing equations numerically. Ganeshan *et al*. [12] have analyzed the effects of radiation and free convection using Rosseland approximation defined in Brewster [13] for an impulsively started infinite vertical isothermal plate. A linear analysis of compressible boundary layer flows over a wavy wallhas been presented by Lekoudis *et al*. [14]. Shankar and Sinha [15] have studied the Reyleigh problem for a wavy wall in detail and found that the importance of the waviness of the wall ceases quickly as the liquid is dragged along the wall at low Reynolds numbers, while the effects of viscosity are confined to a thin layer in a neighbourhood of the wall at large Reynolds numbers. Bordner [16] has presented the non-linear analysis of laminar boundary layer flow over a periodic wavy surface applying suitable orthogonal transformations to transform the wavy surface to flat one. Bordner has found that some non-linear terms in the disturbance boundary layer equations are of first order if the wave amplitude and disturbance sub-layer thickness are comparable in magnitude. He has also found the non-linear effects to be confined to the thin sub-layer adjacent to the wavy surface. The effects of small amplitude wall waviness upon the stability of the laminar boundary layer have been studied by Lesson and Gangwani [17]. A modified slip boundary conditions to represent the effect of small roughness-like (slightly wavy) perturbations to an otherwise plane fixed wall which is acting as a boundary to steady laminar flow of a viscous fluid have been obtained by Tuck and Kouzoubov [18]. In all these cases the authors have taken the wavy walls to be horizontal. The free convective flow of a viscous incompressible fluid in porous medium between two long vertical wavy walls has been studied by Patidar and Purohit [19]. Vajravelu and Sastri [20] and Das and Ahmed [21] have studied the problem of free convective flow of a viscous incompressible fluid with heat transfer confined between a vertical wavy wall and a flat wall. Mahdy . [22] have studied the problem of natural convection from a vertical wavy plate embedded in porous media for power law fluids in presence of magnetic field. Mahdy [23] has studied the mixed convection heat and mass transfer on a vertical wavy plate embedded in a saturated porous media. Ozisik [24] has analysed the radiation transfer and interactions with conduction and convection. Tak and Kumar [25] have analyzed MHD free convection flow with viscous dissipation in a vertical wavy channel. Kumar [26] has studied the effect of heat transfer and radiation on a MHD free convective flow confined between two vertical wavy walls. Choudhury and Das [27] have analysed visco-elastic effects on free convective flow confined between a long vertical wavy wall and a parallel flat wall of equal transpiration. Choudhury and Das [28] have studied visco-elastic mhd free convective flow through porous media in presence of radiation and chemical reaction with heat and mass transfer. Choudhury and Dey [29], Choudhury and Mahanta [30], Choudhury and Das [31], Sandeep *et al.*[32], Subhash *et al.* [33], and Suneetha *et al.* [34] have analyzed some problems of physical interest in this field. Cogley *et*

al. [35] have investigated the differential approximation for relative heat transfer in a grey gas near equilibrium. Choudhury and Das [36, 37, 38, 39, 40, 41] and Das [42] have analyzed some problems of physical interest in this field. Ahmed *et al* [43] have investigated the free convective MHD flow between a long wavy wall and a uniformly moving parallel flat wall with radiation, Soret and Dufour effects.

The constitutive equation for Walters liquid (Model B′)

$$\sigma^{ik} = -pg^{ik} + 2\eta_0 e^{ik} - 2k_0 e'^{ik} \qquad (1)$$

Where σik is the stress tensor, p is isotropic pressure, gik is the metric tensor of a fixed co-ordinate system xi, vi is the velocity vector, the contravariant form of e′ik is given by

$$e'^{ik} = \frac{\partial e^{ik}}{\partial t} + v^m e^{ik}{}_{,m} - v^i{}_{,m}\, e^{im} - v^i{}_{,m}\, e^{mk} \qquad (2)$$

It is the convected derivative of the deformation rate tensor eik defined by

2eik = vi,k +vk,i (3)

Here η0 is the limiting viscosity at the small rate of shear which is given by

$$\eta_0 = \int_0^\infty N(\tau)d\tau \ \text{ and } \ k_0 = \int_0^\infty \tau N(\tau)d\tau \qquad (4)$$

N(τ) being the relaxation spectrum as introduced by Walter [44, 45]. This idealized model is a valid approximation of Walters liquid (Model B′) taking very short memories into account so that terms involving

$$\int_0^\infty \tau^n N(\tau)d\tau, \quad n \geq 2 \qquad (5)$$

have been neglected.

2. MATHEMATICAL FORMULATION

The mhd flow of an electrically conducting visco-elastic fluid characterized by Walters liquid (Model B') confined between a long vertical wavy wall and a uniformly moving parallel flat wall in presence of a transverse magnetic field, radiation and temperature dependent heat source is considered. The x-axis is taken parallel to the flat wall and y-axis is perpendicular to it. The wavy and the flat walls are represented by $\bar{y} = \bar{\varepsilon}\cos k\bar{x}$ and y=d respectively, $\bar{T}_w$ and $\bar{T}_1$ being their constant temperatures, where $\bar{\varepsilon} \ll 1$.

The investigation is restricted to the following assumptions:

1. All the fluid properties, except the density in the buoyancy force term, are constants.
2. The viscous and magnetic dissipation of energy are negligible.
3. The volumetric heat source/sink term in the energy equation is constant.
4. The magnetic Reynolds number is small enough to neglect the induced magnetic field.
5. The wave length of the wavy wall, which is proportional to $\frac{1}{k}$, is large.

With the above assumptions, the equations governing the flow field are as follows.

Equation of continuity:

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0 \qquad (6)$$

Momentum equations:

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = -\frac{1}{\rho}\frac{\partial \bar{P}}{\partial \bar{x}} + \nu\left(\frac{\partial^2 \bar{u}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{u}}{\partial \bar{y}^2}\right) - \frac{K_0}{\rho}\left(\bar{u}\frac{\partial^3 \bar{u}}{\partial \bar{x}^3} + \bar{u}\frac{\partial^3 \bar{u}}{\partial \bar{x}\,\partial \bar{y}^2} + \bar{v}\frac{\partial^3 \bar{u}}{\partial \bar{y}\,\partial \bar{x}^2} + \bar{v}\frac{\partial^3 \bar{u}}{\partial \bar{y}^3} - 3\frac{\partial \bar{u}}{\partial \bar{x}}\frac{\partial^2 \bar{u}}{\partial \bar{x}^2} - \frac{\partial \bar{v}}{\partial \bar{y}}\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} - \frac{\partial \bar{u}}{\partial \bar{y}}\frac{\partial^2 \bar{v}}{\partial \bar{x}^2} - \frac{\partial \bar{u}}{\partial \bar{y}}\frac{\partial^2 \bar{u}}{\partial \bar{x}\,\partial \bar{y}} - 2\frac{\partial \bar{v}}{\partial \bar{x}}\frac{\partial^2 \bar{u}}{\partial \bar{x}\,\partial \bar{y}}\right) - g - \frac{\sigma \bar{B}^2}{\rho}\bar{u} \qquad (7)$$

$$\bar{u}\frac{\partial \bar{v}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{v}}{\partial \bar{y}} = -\frac{1}{\rho}\frac{\partial \bar{P}}{\partial \bar{y}} + \nu\left(\frac{\partial^2 \bar{v}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{v}}{\partial \bar{y}^2}\right) - \frac{K_0}{\rho}\left(\bar{u}\frac{\partial^3 \bar{v}}{\partial \bar{x}^3} + \bar{u}\frac{\partial^3 \bar{v}}{\partial \bar{x}\,\partial \bar{y}^2} + \bar{v}\frac{\partial^3 \bar{v}}{\partial \bar{y}\,\partial \bar{x}^2} + \bar{v}\frac{\partial^3 \bar{v}}{\partial \bar{y}^3} - \frac{\partial \bar{u}}{\partial \bar{x}}\frac{\partial^2 \bar{v}}{\partial \bar{x}^2} - \frac{\partial \bar{v}}{\partial \bar{x}}\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} - \frac{\partial \bar{v}}{\partial \bar{x}}\frac{\partial^2 \bar{v}}{\partial \bar{x}\,\partial \bar{y}} - 3\frac{\partial \bar{v}}{\partial \bar{y}}\frac{\partial^2 \bar{v}}{\partial \bar{y}^2} - 2\frac{\partial \bar{u}}{\partial \bar{y}}\frac{\partial^2 \bar{v}}{\partial \bar{x}\,\partial \bar{y}}\right) \qquad (8)$$

Energy equation:

$$\rho C_p\left(\bar{u}\frac{\partial \bar{T}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{T}}{\partial \bar{y}}\right) = \kappa\left(\frac{\partial^2 \bar{T}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{T}}{\partial \bar{y}^2}\right) - \frac{\partial \bar{q}_r}{\partial \bar{y}} + Q \qquad (9)$$

where $= \frac{\eta_0}{\rho}$.

The radiative heat flux $\bar{q}_r$ for an optically thin fluid (Cogley et al. [34]) is given by

$$\frac{\partial \bar{q}_r}{\partial \bar{y}} = 4I(\bar{T} - \bar{T}_s)$$

Where

$$I = \int_0^\infty (K_\lambda)_N \left(\frac{\partial e_{b\lambda}}{\partial \bar{T}}\right) d\lambda \qquad (10)$$

In static condition, (7) takes the form

$$\frac{\partial \bar{P}_s}{\partial \bar{x}} + \rho_s g = 0 \qquad (11)$$

On using (11), (7) becomes

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = -\frac{1}{\rho}\frac{\partial(\bar{P}-\bar{P}_s)}{\partial \bar{x}} + \nu\left(\frac{\partial^2 \bar{u}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{u}}{\partial \bar{y}^2}\right) - \frac{K_0}{\rho}\left(\bar{u}\frac{\partial^3 \bar{u}}{\partial \bar{x}^3} + \bar{u}\frac{\partial^3 \bar{u}}{\partial \bar{x}\,\partial \bar{y}^2} + \bar{v}\frac{\partial^3 \bar{u}}{\partial \bar{y}\,\partial \bar{x}^2} + \bar{v}\frac{\partial^3 \bar{u}}{\partial \bar{y}^3} - 3\frac{\partial \bar{u}}{\partial \bar{x}}\frac{\partial^2 \bar{u}}{\partial \bar{x}^2} - \frac{\partial \bar{v}}{\partial \bar{y}}\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} - \frac{\partial \bar{u}}{\partial \bar{y}}\frac{\partial^2 \bar{v}}{\partial \bar{x}^2} - \frac{\partial \bar{u}}{\partial \bar{y}}\frac{\partial^2 \bar{u}}{\partial \bar{x}\,\partial \bar{y}} - 2\frac{\partial \bar{v}}{\partial \bar{x}}\frac{\partial^2 \bar{u}}{\partial \bar{x}\,\partial \bar{y}}\right) + g(\rho - \rho_s) - \frac{\sigma \bar{B}^2}{\rho}\bar{u} \qquad (12)$$

The equation of state is

$$\rho_s = \rho\{1 - \beta(\bar{T} - \bar{T}_s)\} \qquad (13)$$

(12) and (13) yield

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = -\frac{1}{\rho}\frac{\partial(\bar{P}-\bar{P}_s)}{\partial \bar{x}} + \nu\left(\frac{\partial^2 \bar{u}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{u}}{\partial \bar{y}^2}\right) - \frac{K_0}{\rho}\left(\bar{u}\frac{\partial^3 \bar{u}}{\partial \bar{x}^3} + \bar{u}\frac{\partial^3 \bar{u}}{\partial \bar{x}\,\partial \bar{y}^2} + \bar{v}\frac{\partial^3 \bar{u}}{\partial \bar{y}\,\partial \bar{x}^2} + \bar{v}\frac{\partial^3 \bar{u}}{\partial \bar{y}^3} - 3\frac{\partial \bar{u}}{\partial \bar{x}}\frac{\partial^2 \bar{u}}{\partial \bar{x}^2} - \frac{\partial \bar{v}}{\partial \bar{y}}\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} - \frac{\partial \bar{u}}{\partial \bar{y}}\frac{\partial^2 \bar{v}}{\partial \bar{x}^2} - \frac{\partial \bar{u}}{\partial \bar{y}}\frac{\partial^2 \bar{u}}{\partial \bar{x}\,\partial \bar{y}} - 2\frac{\partial \bar{v}}{\partial \bar{x}}\frac{\partial^2 \bar{u}}{\partial \bar{x}\,\partial \bar{y}}\right) + g\beta(\bar{T} - \bar{T}_s) - \frac{\sigma \bar{B}^2}{\rho}\bar{u} \qquad (14)$$

The boundary conditions are

$$y = \bar{\varepsilon}\cos\bar{k}\bar{x}: \ \bar{u} = 0, \bar{v} = 0, \bar{T} = \bar{T}_w.$$

$$y = d: \quad \bar{u} = 0, \bar{v} = 0, \ \frac{\partial \bar{T}}{\partial \bar{y}} = 0. \qquad (15)$$

Now the following non-dimensional quantities are introduced:

$G_r = \frac{g\beta d^3(\bar{T}_w - \bar{T}_s)}{\nu^2}$ (Grashof Number), $M = \frac{\sigma \bar{B}^2 d^2}{\rho\nu}$ (Hartmann Number), $P_r = \frac{\eta_0 C_p}{\kappa}$ (Prandtl Number), $x = \frac{\bar{x}}{d}, y = \frac{\bar{y}}{d}, u =$

$\frac{d\bar{u}}{\nu}, v = \frac{d\bar{v}}{\nu}, P = \frac{\bar{P}d^2}{\rho\nu^2}, T = \frac{\bar{T}-\bar{T}_s}{\bar{T}_w-\bar{T}_s}, m = \frac{\bar{T}_1-\bar{T}_s}{\bar{T}_w-\bar{T}_s},\ N = \frac{4Id^2}{\rho\nu C_p}$,
$U = \frac{\bar{U}d}{\nu}$,

$\alpha = \frac{Qd^2}{\kappa(\bar{T}_w-\bar{T}_s)}$ (heat source/sink parameter), $\lambda = \bar{k}d$, $\varepsilon = \frac{\bar{\varepsilon}}{d}$ (non-dimensional amplitude ratio).

The non-dimensional form of equations (6), (14), (8) and (9) are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{16}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\partial(P-P_s)}{\partial x} + \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} - K_1\left(u\frac{\partial^3 u}{\partial x^3} + u\frac{\partial^3 u}{\partial x\partial y^2} + v\frac{\partial^3 u}{\partial y^3} + v\frac{\partial^3 u}{\partial x^2\partial y} - 3\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial x^2} - \frac{\partial v}{\partial y}\frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial x^2} - \frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y} - 2\frac{\partial v}{\partial x}\frac{\partial^2 u}{\partial x\partial y}\right) + G_rT - Mu \tag{17}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{\partial P}{\partial y} + \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} - K_1\left(u\frac{\partial^3 v}{\partial x^3} + u\frac{\partial^3 v}{\partial x\partial y^2} + v\frac{\partial^3 v}{\partial x^2\partial y} + v\frac{\partial^3 v}{\partial y^3} - 2\frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial x\partial y} - \frac{\partial v}{\partial x}\frac{\partial^2 v}{\partial x\partial y} - \frac{\partial v}{\partial x}\frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial x}\frac{\partial^2 v}{\partial x^2} - 3\frac{\partial v}{\partial y}\frac{\partial^2 v}{\partial y^2}\right) \tag{18}$$

$$P_r\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) - P_rNT + \alpha \tag{19}$$

subject to the boundary conditions

$$y = \varepsilon\cos\lambda x : u = 0, v = 0, T = 1$$
$$y = 1: u = U, v = 0, T = m \tag{20}$$

where $K_1 = \frac{K_0}{\rho d^2}$ is the visco-elastic parameter.

3. METHOD OF SOLUTION

Assuming that the solution consists of a mean part and a perturbed part, the perturbation scheme

$$u(x,y) = u_0(y) + \varepsilon u_1(x,y)$$
$$v(x,y) = \varepsilon v_1(x,y)$$
$$T(x,y) = \theta_0(y) + \varepsilon\theta_1(x,y)$$
$$P(x,y) = P_0(x) + \varepsilon P_1(x,y) \tag{21}$$

are applied to equations (16) to (19), where the perturbed quantities u_1, v_1, θ_1, P_1 are small compared with the mean quantities.

Comparing the coefficients of various powers of ε and neglecting those of second and higher powers of ε, the following equations are obtained.

Zeroth order equations:

$$\frac{d^2u_0}{dy^2} - Mu_0 = -G_r\theta_0 \tag{22}$$

$$\frac{d^2\theta_0}{dy^2} - P_rN\theta_0 = -\alpha \tag{23}$$

First order equations:

$$u_0\frac{\partial u_1}{\partial x} + v_1\frac{du_0}{dy} = -\frac{\partial P_1}{\partial x} + \frac{\partial^2 u_1}{\partial x^2} + \frac{\partial^2 u_1}{\partial y^2} - K_1\left(u_0\frac{\partial^3 u_1}{\partial x^3} + u_0\frac{\partial^3 u_1}{\partial x\partial y^2} + v_1\frac{d^3u_0}{dy^3} - \frac{\partial v_1}{\partial y}\frac{d^2u_0}{dy^2} - \frac{du_0}{dy}\frac{\partial^2 v_1}{\partial x^2} - \frac{du_0}{dy}\frac{\partial^2 u_1}{\partial x\partial y}\right) + G_r\theta_1 - Mu_1 \tag{24}$$

$$u_0\frac{\partial v_1}{\partial x} = -\frac{\partial P_1}{\partial y} + \frac{\partial^2 v_1}{\partial x^2} + \frac{\partial^2 v_1}{\partial y^2} - K_1\left(u_0\frac{\partial^3 v_1}{\partial x^3} + u_0\frac{\partial^3 v_1}{\partial x\partial y^2} - 2\frac{du_0}{dy}\frac{\partial^2 v_1}{\partial x\partial y} - \frac{\partial v_1}{\partial x}\frac{d^2u_0}{dy^2}\right) \tag{25}$$

$$\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} = 0 \tag{26}$$

$$P_r\left(u_0\frac{\partial\theta_1}{\partial x} + v_1\frac{d\theta_0}{dy}\right) = \frac{\partial^2\theta_1}{\partial x^2} + \frac{\partial^2\theta_1}{\partial y^2} - P_rN\theta_1 \tag{27}$$

The corresponding boundary conditions are

$$y = 0: u_0 = 0, \theta_0 = 1$$
$$y = 1: u_0 = 0,\ \theta_0 = m \tag{28}$$

$$y = 0:\ u_1 = -\mathrm{Re}(u_0{}'e^{i\lambda x}), v_1 = 0, \theta_1 = -\mathrm{Re}(\theta_0{}'e^{i\lambda x})$$
$$y = 1: u_1 = 0, v_1 = 0, \theta_1 = 0 \tag{29}$$

where dashes denote differentiation with respect to y.

Solutions of equations (22) and (23) are obtained as

$$u_0(y) = C_3e^{\sqrt{M}y} + C_4e^{-\sqrt{M}y} + d_1e^{C_1y} + d_2e^{C_2y} + d_3 \tag{30}$$

$$\theta_0 = C_1e^{a_1y} + C_2e^{-a_1y} + a_2 \tag{31}$$

To solve equations (24), (25) and (27), it is assumed that

$$u_1 = -\frac{\partial\psi}{\partial y}, \qquad v_1 = \frac{\partial\psi}{\partial x} \tag{32}$$

On eliminating P_1 from equations (24) and (25) and on keeping in view the equation of continuity (26), equations (24), (25) and (27) yield

$$u_0\psi_{xxx} + u_0\psi_{xyy} - u_0{}''\psi_x = \psi_{xxxx} + \psi_{yyyy} + 2\psi_{xxyy} - K_1\left(u_0\psi_{xxxxx} + 2u_0\psi_{xxxyy} + u_0\psi_{xyyyy} - u_0{}^{iv}\psi_x\right) - G_r\theta_{1y} - M\psi_{yy} \tag{33}$$

$$P_r\left(u_0\theta_{1x} + \psi_x\theta_0{}'\right) = \theta_{1xx} + \theta_{1yy} - P_rN\theta_1 \tag{34}$$

Keeping in view (32), general solutions for ψ and θ_1 are assumed as follows:

$$\psi(x,y) = e^{i\lambda x}\bar{\psi}(y) \tag{35}$$

$$\theta_1(x,y) = e^{i\lambda x}\theta(y) \tag{36}$$

And

$$\bar{\psi}(y) = \textstyle\sum_{r=0}(\psi_r(y)\lambda^r),\ \theta(y) = \sum_{r=0}(t_r(y)\lambda^r) \tag{37}$$

Using (35), (36) and (37) in equations (33) and (34) and equating coefficients of various powers of λ and neglecting those of second and higher powers of λ, the following sets of equations are obtained:

Zeroth order equations

$$\psi_0{}^{iv} - M\psi_0{}'' = G_rt_0{}' \tag{38}$$

$$t_0{}'' - a_1{}^2t_0 = 0 \tag{39}$$

First order equations

$$\psi_1{}^{iv} - M\psi_1{}'' = iK_1\left(u_0\psi_0{}^{iv} - u_0{}^{iv}\psi_0\right) + i\left(u_0\psi_0{}'' - u_0{}''\psi_0\right) + G_rt_1{}' \tag{40}$$

$$t_1{}'' - a_1{}^2t_1 = iP_r\left(u_0t_0 + \theta_0{}'\psi_0\right) \tag{41}$$

The corresponding boundary conditions are

$$y = 0: \psi_0(y) = 0, \psi_0{}'(y) = u_0{}'(y), t_0(y) = -\theta_0{}'(y)$$

$$y = 1: \psi_0(y) = 0, \psi_0{'}(y) = 0,\ t_0(y) = 0 \quad (42)$$

$$y = 0: \psi_1(y) = 0, \psi_1{'}(y) = 0, t_1(y) = 0$$

$$y = 1: \psi_1(y) = 0, \psi_1{'}(y) = 0,\ t_1(y) = 0 \quad (43)$$

The solutions of equations (38) to (41) are

$$\psi_0 = C_7 + C_8 y + C_9 e^{\sqrt{M}y} + C_{10} e^{-\sqrt{M}y} + d_9 e^{a_1 y} + d_{10} e^{-a_1 y} \quad (44)$$

$$t_0 = C_5 e^{a_1 y} + C_6 e^{-a_1 y} \quad (45)$$

$$\psi_1 = C_{13} + C_{14} y + C_{15} e^{\sqrt{M}y} + C_{16} e^{-\sqrt{M}y} + d_{37} e^{(a_1+\sqrt{M})y} + d_{38} e^{-(a_1+\sqrt{M})y} + d_{39} e^{(\sqrt{M}-a_1)y} + d_{40} e^{(a_1-\sqrt{M})y} + d_{41} y e^{\sqrt{M}y} + d_{42} y e^{-\sqrt{M}y} + d_{43} e^{2a_1 y} + d_{44} e^{-2a_1 y} + d_{45} e^{a_1 y} + d_{46} e^{-a_1 y} + d_{47} y e^{a_1 y} + d_{48} y e^{-a_1 y} + d_{49} y^2 e^{a_1 y} + d_{50} y^2 e^{-a_1 y} \quad (46$$

$$t_1 = C_{11} e^{a_1 y} + C_{12} e^{-a_1 y} + d_{24} e^{(a_1+\sqrt{M})y} + d_{25} e^{(a_1-\sqrt{M})y} + d_{26} e^{(\sqrt{M}-a_1)y} + d_{27} e^{-(a_1+\sqrt{M})y} + d_{28} e^{2a_1 y} + d_{29} e^{-2a_1 y} + d_{30} y e^{a_1 y} + d_{31} y e^{-a_1 y} + d_{32} e^{a_1 y}\left(y^2 - \frac{y}{a_1}\right) + d_{33} e^{-a_1 y}\left(y^2 + \frac{y}{a_1}\right) + d_{34} \quad (47)$$

4. RESULT AND DISCUSSION

From (32), (35), (36) and (44) to (47) and retaining up to the first power of λ in (37), the expressions for u_1, v_1 and θ_1 are obtained as follows:

$$u_1 = -\psi_{0r}{'} \cos\lambda x + \lambda \psi_{1i}{'} \sin\lambda x \quad (48)$$

$$v_1 = -\lambda \psi_{0r} \sin\lambda x - \lambda^2 \psi_{1i} \cos\lambda x \quad (49)$$

$$\theta_1 = t_{0r} \cos\lambda x - \lambda t_{1i} \sin\lambda x \quad (50)$$

where ψ_{0r}=Real ψ_0, ψ_{1i}=Imag ψ_1, t_{0r}=Real t_0, t_{1i}=Imag t_1.

The non-dimensional form of skin-friction coefficient σ_ω and σ_1 on the wavy wall y=εcos λx and the flat wall y=1 for small shear rate are given by $\sigma_\omega = (\sigma_{xy})_{y=0}$ and $\sigma_1 = (\sigma_{xy})_{y=1}$ respectively, where

$$\sigma_{xy} = u_0{'} + \mathrm{Re}\langle \varepsilon e^{i\lambda x}\left[u_0'' - \left(\psi_0'' + \lambda\psi_1''\right) - \lambda^2(\psi_0 + \lambda\psi_1) + K_1\left\{i\lambda u_0\left(\psi_0'' + \lambda\psi_1''\right) + i\lambda^3 u_0(\psi_0 + \lambda\psi_1) - i\lambda u_0''(\psi_0 + \lambda\psi_1) + 2u_0{'}(\psi_0{'} + \lambda\psi_1{'})\right\}\right]\rangle \quad (51)$$

Using equations (21), (24) and (35) to (37), the fluid pressure at any point (x,y) is found to be

$$\widehat{P}(x, y) = -K'x + Re\left[\epsilon i \frac{e^{i\lambda x}}{\lambda} Z(y)\right] + L \quad (52)$$

where L is arbitrary constant, $K' = -\frac{\partial P_0(x)}{\partial x}$ is constant and

$$Z(y) = \left(\psi_0{'''} + \lambda\psi'''_1\right) - \lambda^2(\psi_0{'} + \lambda\psi_1{'}) - i\lambda\{u_0(\psi_0' + \lambda\psi_1{'}) - u_0{'}(\psi_0 + \lambda\psi_1)\} - G_r(t_0 + \lambda t_1) - M(\psi_0 + \lambda\psi_1) + i\lambda K_1\{u_0{'''}(\psi_0 + \lambda\psi_1) - u_0(\psi_0{'''} + \lambda\psi_1{'''}) + u_0{'}(\psi_0{''} + \lambda\psi_1{''}) - u_0{''}(\psi_0{'} + \lambda\psi_1{'}) + \lambda^2 u_0(\psi_0{'} + \lambda\psi_1{'}) + \lambda^2 u_0{'}(\psi_0 + \lambda\psi_1)\} \quad (53)$$

The pressure drop $\widehat{P}$ indicates the difference between the pressure at any point y in the flow field and that at the flat wall, with x- fixed and is given by

$$\tilde{P} = \hat{P}(x, y) - \hat{P}(x, 1) = Re\left[\epsilon i \frac{e^{i\lambda x}}{\lambda}\{Z(y) - Z(1)\}\right] \quad (54)$$

The constants are obtained but not given here due to brevity.

4.1.Discussions

In this problem, the effects of visco-elastic parameter on the free convective flow confined between a long vertical wavy wall and a moving flat wall is brought out. The visco-elastic effect is exhibited through the non-Newtonian parameter K_1 (K_1=0, .2, .4). The value 0 of K_1 corresponds to Newtonian fluid. In numerical calculations, the real part is implied throughout. In all the figures, the flow field is illustrated for the fixed values of the flow parameters M=.5, λ=.01, $= \frac{\pi}{4}$, ε=.01, m=5, P_r=5,G_r=10, U=1.

Figures 1 and 2 illustrate the perturbed part of the velocity profile u_1 along the channel for N=1 and α=-2 and α=2 respectively and figures 3 and 4 illustrate the perturbed part of the velocity profile u_1 along the channel for N=3 and α=-2 and α=2 respectively. It is noticed from the figures that the velocity profile u_1 has a diminishing trend near the wavy wall y=0 and an increasing trend near the moving flat wall y=1 in all the cases. It is also observed that the variation of velocity profile u_1 is less for heat source parameter (α=2) than that for heat sink parameter (α=-2).The figures also explain that the velocity profile u_1 diminishes with growth of the radiation parameter N.

Figures 5 and 6 illustrate the perturbed part of the velocity profile v_1 across the channel for N=1 and α=-2 and α=2 respectively and figures 7 and 8 illustrate the perturbed part of the velocity profile v_1 across the channel for N=3 and α=-2 and α=2 respectively. Figures 5 to 7 explain the parabolic nature of the velocity profile v_1 across the channel in all the figures, an enhancing trend is of the velocity profile v_1 with the growth of the visco-elastic parameter K_1 in comparison with that in Newtonian fluid flow. It is also observed that magnitude of the velocity profile v_1 is less for heat source (α=2) than that for heat sink (α=-2).The figures also explain that the velocity profile v_1 decreases with increase of the radiation parameter N.

Figures 9 and 11 illustrate the nature of skin-friction coefficient $\sigma_w$on the wavy wall y=0 for N=1 and N=3 respectively. In both the cases the skin-friction coefficient has an accelerating trend with the growth of the visco-elastic parameter K_1 in comparison with that in Newtonian fluid flow. It is also observed that the skin-friction coefficient decreases with the increase of the heat source parameter α.

Figures 10 and 12 illustrate the nature of skin-friction coefficient σ_1 on the moving flat wall y=1 for N=1 and N=3 respectively. In both the cases the skin-friction coefficient has a decelerating trend with the growth of the visco-elastic parameter K_1 in comparison with that in Newtonian fluid flow. It is also observed that the skin-friction coefficient increases with the increase of the heat source parameter α.

Figures 9, 11 and 10, 12 reveal that the skin-friction coefficient diminishes with the increase of the radiation parameter N.

Figures 13 and 14 illustrate the pressure drop profile across the channel for N=1 and α=-2 and α=2 respectively and figures 15 and 16 illustrate the pressure drop profile across the channel for N=3 and α=-2 and α=2 respectively. In all the figures it is observed that the pressure drop profile decreases with the increase of the visco-elastic parameter K_1 in comparison with that in Newtonian fluid flow. The variation in the pressure drop is more prominent for the value 3 of the radiation parameter N in comparison with that for the value 1 of the radiation parameter N.

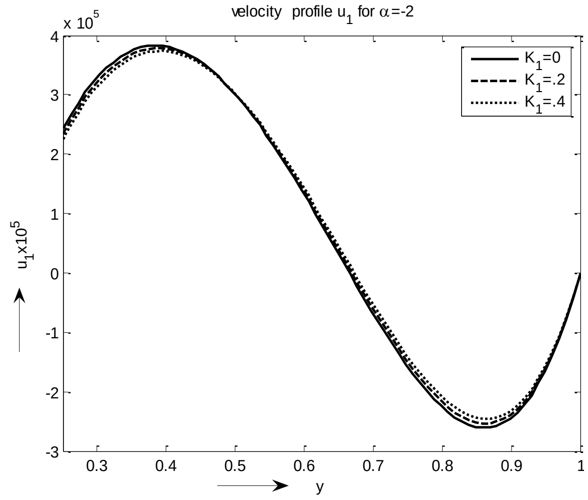

Fig-1: α=-2; N=1.

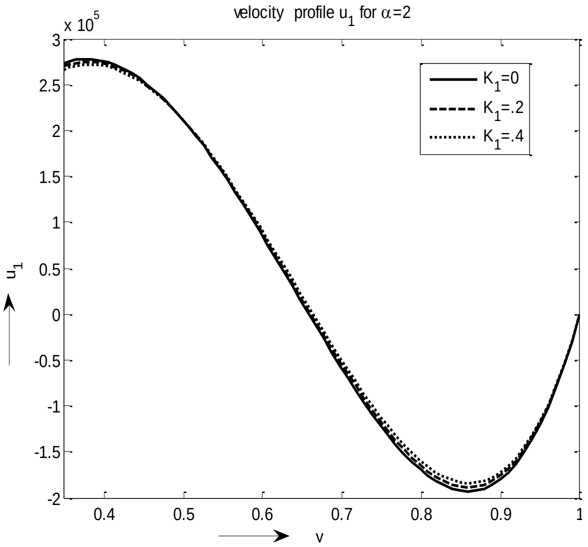

Fig-2: α=2; N=1.

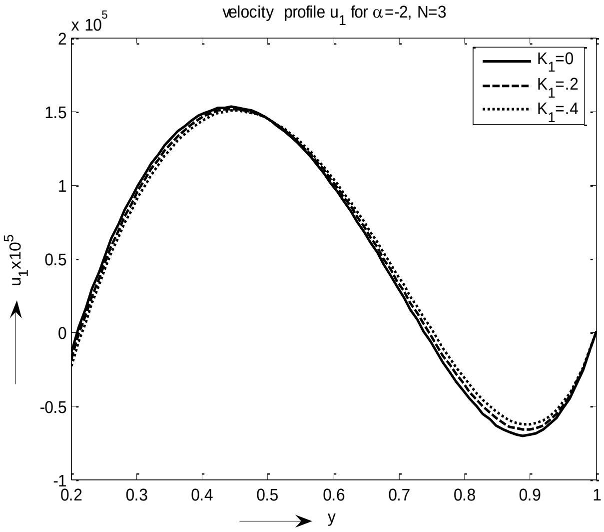

Fig -3: α=-2; N=3.

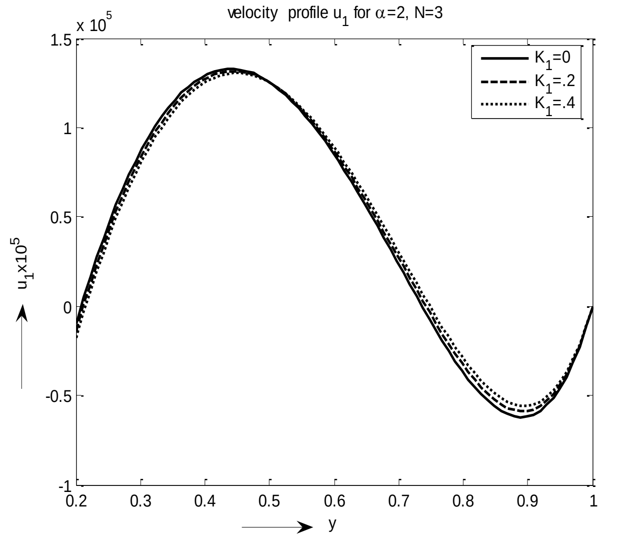

Fig -4: α=2; N=3.

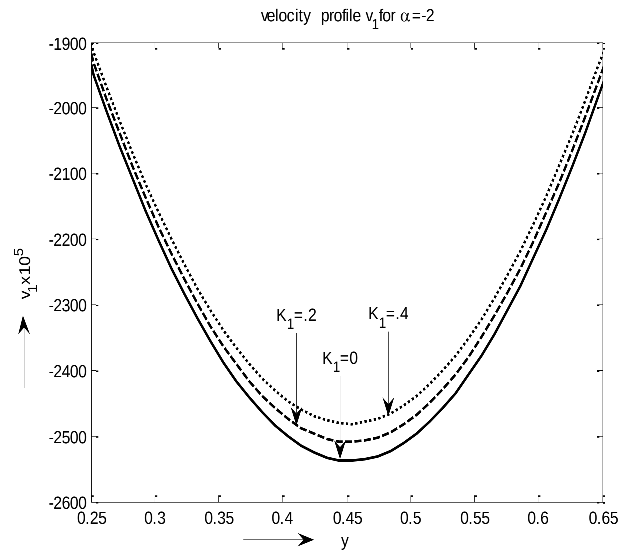

Fig-5: α=-2; N=1.

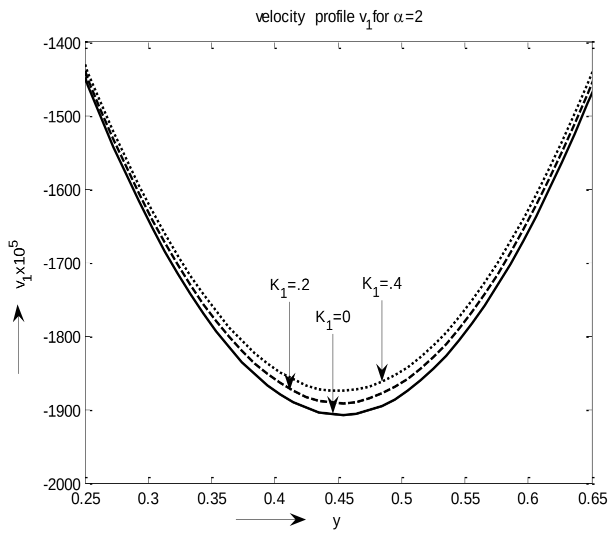

Fig-6: α=2; N=1.

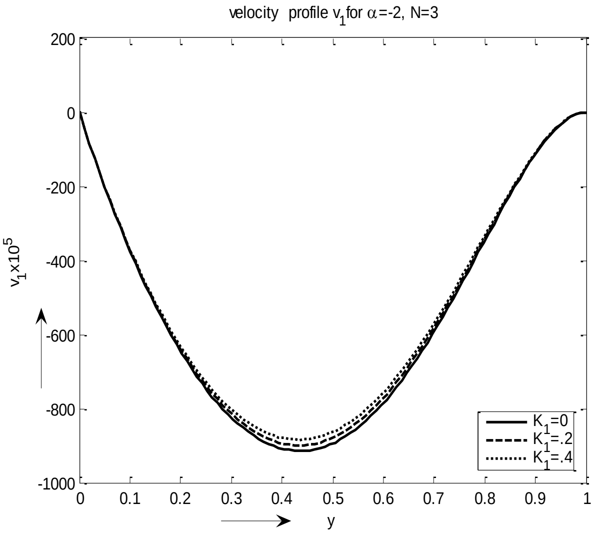

Fig-7: α=-2; N=3.

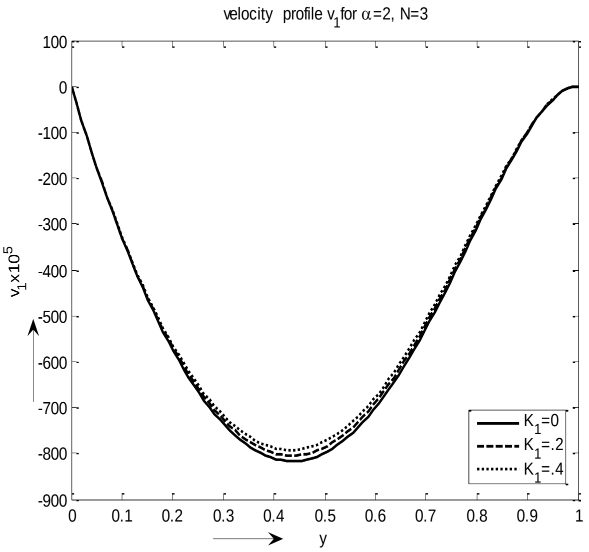

Fig-8: α=2; N=3.

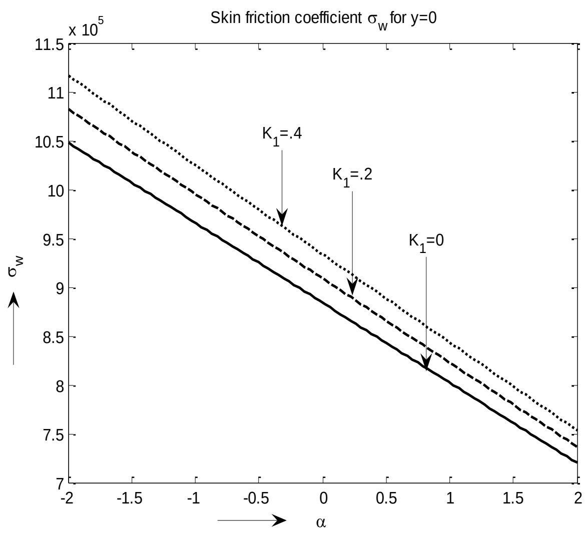

Fig-9: y=0; N=1.

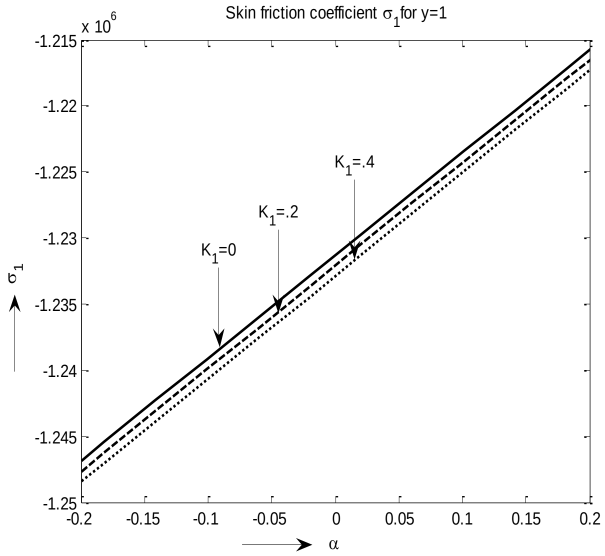

Fig-10: y=1; N=1.

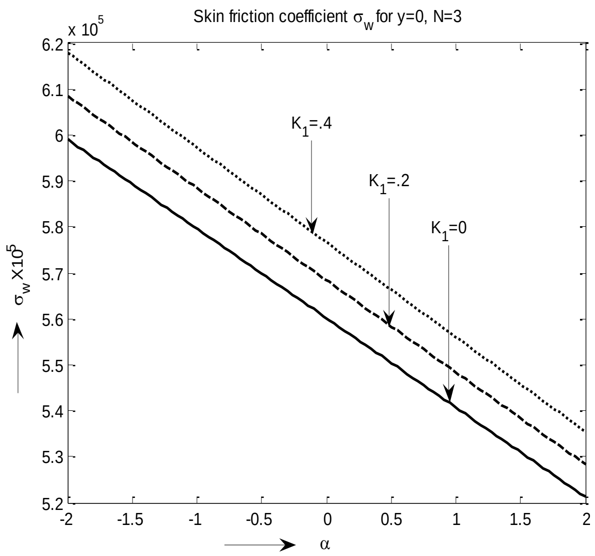

Fig-11: y=0; N=3.

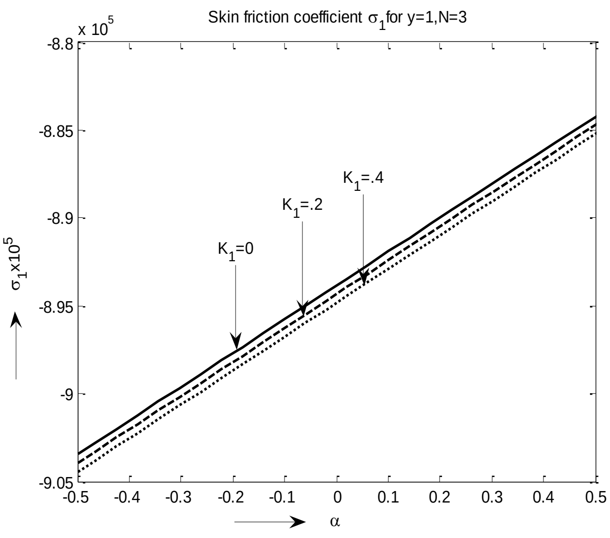

Fig-12: y=1; N=3.

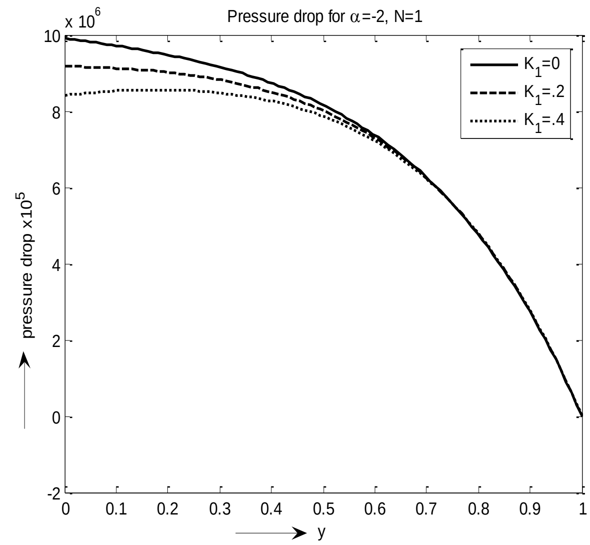

Fig-13: α=-2; N=1.

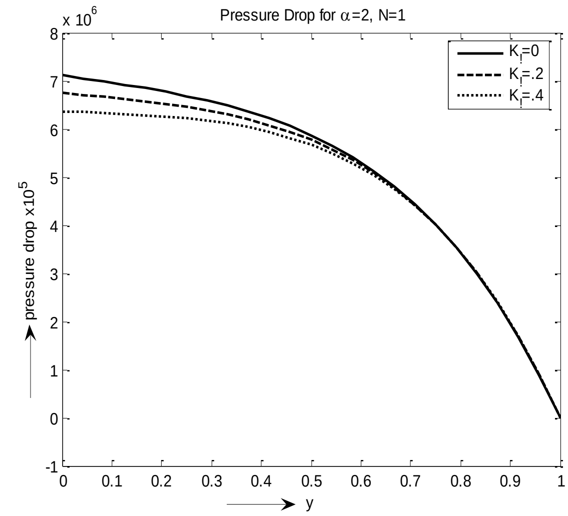

Fig-14: α=2; N=1.

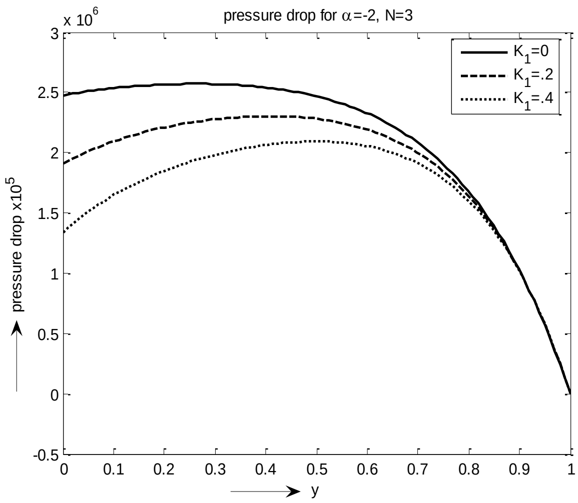

Fig-15: α=-2; N=3.

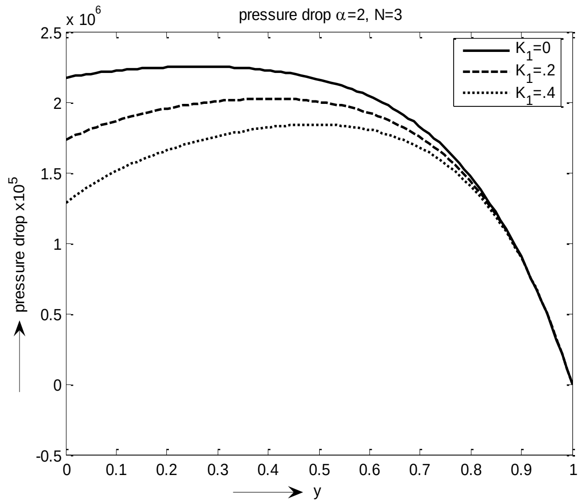

Fig-16: α=2; N=3.

5. CONCLUSION

An analysis of the effect of visco-elasticity on free convective flow confined between long vertical wavy wall and a flat wall has been presented for different values of visco-elastic parameter K_1 in combination of other flow parameters.

From this study, the following conclusions have been drawn:

- The effect of visco-elasticity on the velocity field is considerable.
- A mixed type of effect of visco-elasticity is observed in the velocity field along the channel.
- The velocity field across the channel enhances with the growth of the visco-elasticity.
- The skin friction coefficient against the heat source/sink parameter increases with the rising of the visco-elastic parameter at the wavy wall in comparison to that in Newtonian case.
- The skin friction coefficient against the heat source/sink parameter declines with the increase of the visco-elastic parameter at the moving flat wall in comparison to that in Newtonian case.
- A diminishing trend in pressure drop is observed with the growth of visco-elastic parameter in comparison with Newtonian fluid flow.

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