Engineering Education: Computer-Aided Engineering with MATLAB; Discrete Wavelet Transform as a Case Study

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ABSTRACT

Engineering education is to move engineering students along the progressive path from being novices toward becoming experts in design, problem-solving and application of knowledge. Engineering problem may require more computations than is possible by hand. Computer-aided engineering is the process of solving engineering problems with the aid of computer software. Engineering Lecturers need to help engineering students to develop expertise in Computer-aided engineering with examples. The development of Discrete Wavelet Transformation is used as a case study. A wavelet is a small wave whose energy is concentrated in time. Wavelets applications include signal processing, noise filtering, image processing, and document analysis. Among wavelet families, Haar wavelet is selected. Scientific representations of the problem and a logical plan of attacking the problem are presented. Necessary equations are derived. A modular approach to programming is demonstrated. A complex problem is broken down into simple tasks and steps which are coded into simple short MATLAB programs. A program calls another program to execute some specific tasks. Programs are checked against possible errors using a situation where the answers are known. Discrete wavelet transformation and inverse discrete wavelet transformation for 1D, 2D, and 3D discrete-time signals have been implemented. 2D gray level images and 3D color images are also considered. The use of similar examples is recommended for Engineering Lecturers.

General Terms

Digital Signal Processing, Digital Image Processing, Algorithm Development.

Keywords

Engineering Education, Problem-solving, Computer-aided engineering, MATLAB, Discrete Wavelet transformation.

1. INTRODUCTION

The objective of engineering education is to move engineering students along the progressive path from being novices toward becoming experts in design, problem-solving and application of knowledge [1]. The Engineering Lecturers have the duty to lead the engineering students from what they know to what they do not know. Studies have revealed that students have misunderstandings with regards to a range of concepts [2], [3], [4]. Improving the understanding and technical know-how of students are tasks that should be accomplished by Lecturers.

Problem-solving is required whenever there is a goal to reach and attainment of that goal is not possible either by direct action or by retrieving a sequence of previously learned steps from memory [4], [5]. During problem-solving, the path to the intended goal is uncertain.

Some of the problems to be solved may be well-defined or ill-defined [7]. Most problems in class are well defined; initial conditions, goal, means for generating and evaluating the solution, and the constraints on the solution are specified. For the ill-defined problems, students need to define some of the problems’ components on their own [7]. Regular practice is important [8]. Practice makes perfect.

Formulation of scientific representations of the problem to be solved in the form of pictures, diagrams, graphs, maps, models, and simulations facilitates problem-solving. A problem representation is the model of the problem constructed by the solver to summarize his/her understanding of the problem. This model may include the elements in the problem, the inter-relationship of the elements, the goals and the types of operations that can be performed on the elements, and any constraints on the solution process.

Yildirim, Shuman and Besterfield-Sacre in [9] recommend that the expert should guide the novice toward success. The expert needs to point out the strengths and weaknesses of the novice’s product or performance. Teamwork among students to solve open-ended, real-world problems can also improve students’ problem-solving skills.

Solution to an engineering problem may require more computations than is possible by hand. A digital computer is, therefore, an essential tool in solving engineering problems [10], [11], [12]. Engineering students need to be familiar with Computer-aided engineering which is the process of solving engineering problems with the aid of computer software.

MATLAB computer programming system is a problem-solving tool for scientists and engineers [13], [14]. It has a teach-yourself feature. MATLAB is based on the mathematical concept of a matrix. Every variable X is a matrix of order m-by-n; m rows and n columns. A matrix is a rectangular array. A vector is a list or a matrix with 1 column or with 1 row. A constant k is a scalar and it’s a 1-by-1 matrix. Regarding all variables as matrices is an official MATLAB mindset [13].

Students need to develop a logical plan of attack to solve a problem and learn exact rules for writing MATLAB statements. MATLAB can handle basic arithmetic operations such as addition, subtraction, division, multiplication, exponentiation, and all usual mathematical functions like Sin, Cos, Log, exp, square and square root just to mention a few. MATLAB can also handle logical operations. Values can be assigned to variables and arithmetic operations can be done on variables. It has numerous functions and commands. MATLAB can plot graphs. MATLAB can solve simultaneous equations. It has a very useful 'help' system.

MATLAB has program repetition feature which enables iteration. A set of instructions is executed a predefined number of times using the 'for' or the 'while' operator. The 'break' operator can terminate the repetition when a condition is met. The 'continue' operator is also used in the repetition feature to achieve some tasks.

Decision making is another important feature of MATLAB which uses logical 'if else statement'. A set S1 of instructions is executed if a logical statement is true otherwise another set S2 of instructions is executed. More than two sets of instructions can be accommodated by including one or more 'elseif' operators.

Solving a problem requires developing a logical plan of attack to solve the problem. The Engineer needs to understand the problem, develop scientific representations of the problem, and develop correct systematic steps for solving the problem. A clear understanding of the arithmetic and logical operations involved in the solution is essential. The correctness of these steps needs to be critically and carefully studied. These steps need to be further broken down into sub-steps that MATLAB can handle. Wrong steps will give the wrong solution. Repetitive or iterative nature of some of the steps should be carefully handled. If garbage goes in, garbage would come out. There will always be an answer; the answer may be correct or wrong. It's even worse if the Engineer does not know the answer is wrong or why the answer is wrong [13].

Checking algorithm against possible errors is essential. One error detecting method is running the program for special situations where the Engineer knows the answers [13]. For example, if the problem is the multiplication of two matrices, manually multiply a 3-by-3 matrix by a 3-by-2 matrix and compare the results with the output of the program. Manual calculation of a small scale version of the problem is a way of validating the output of a program. If the output of the program does not tally with the known answer, such program requires checking against possible arithmetic, logical and language errors.

An algorithm is a systematic logical procedure for solving a problem. The algorithm is represented by a structured plan. The algorithm is then coded into a MATLAB program often referred to as M-file; the sub-steps are faithfully translated into MATLAB language. The translation must be correct and faithful otherwise the correctness of the answer is compromised.

A modular approach to programming is recommended. Troubleshooting, debugging and testing the program becomes easier as modules can be debugged or tested module by module. A complex problem can be solved with several subprograms. A MATLAB program (main program) can call and execute other programs (subprograms) within it. A subprogram is a complete program on its own and can be executed if called within another program.

In the beginning, an Engineer may find it difficult. Over time, an Engineer builds experience in algorithm development and such development becomes easy or automatic. In this work, the development of Discrete Wavelet Transformation is taken as a case study.

A wavelet is defined as a small wave whose energy is concentrated in time [15], [16], [17], [18], [19]. Wavelets applications include signal processing, noise filtering, image processing, image compression, image recognition, document analysis, and feature extraction [18], [20], [21], [22]. Haar wavelet is selected. There are other wavelet families such as Daubechies (Db), Sinc, Gaussian and Mexican Hat as illustrated in Fig. 1 [15].

2. DISCRETE WAVELET TRANSFORM

2.1 Review of One-Dimensional DWT

Given a discrete-time signal X which is an N-by-1 matrix. It is required to determine Y, the one-dimensional discrete wavelet transform (1D DWT) of X at L_m^{th} decomposition level. For example, suppose X is given as in Eqn. (1).

$$X = \begin{bmatrix} 2 \\ 3 \\ 1 \\ 0 \\ 4 \\ 7 \\ 6 \\ 8 \end{bmatrix} \tag{1}$$

N is 8 in this example. Y is first set equal to X, $Y = X$. Y is broken into y_1 and y_2 as shown in Fig. 2. y_1 is set equal to Y as y_2 is empty. y_1 and y_2 are used to multiply the wavelet transformation matrix W and an Identity Matrix I respectively for the first decomposition level as illustrated in Fig. 2 and as described by Eqn. (2). The wavelet transformation matrix W is given by Eqn. (3). W_1 and W_2 are the approximate and detail wavelet transformation matrices respectively; they are regarded as low pass (lp) filter and high pass (hp) filter respectively. Y is also decomposed into two parts as described by Eqns. (4), (5), (6); the approximate part Ya which contains the low-frequency information of the signal and the details part Yd which contains the high-frequency information of the signal.

The output Y at the end of the first decomposition level is feedback and packaged as y_1 and y_2 which are used to multiply the wavelet transformation matrix W and an Identity Matrix I respectively for the second decomposition level as shown in Fig. 3. The output Y at the end of the second decomposition level is feedback and packaged as y_1 and y_2 which are used to multiply the wavelet transformation matrix W and an Identity Matrix I respectively for the third decomposition level as shown in Fig. 4. At the end of every decomposition level, Y is decomposed into the approximate part (Ya) and the details part (Yd) as described by Eqns. (4), (5), (6).

$$Y = \begin{bmatrix} Wy_1 \\ Iy_2 \end{bmatrix} = \begin{bmatrix} Wy_1 \\ y_2 \end{bmatrix} \tag{2}$$

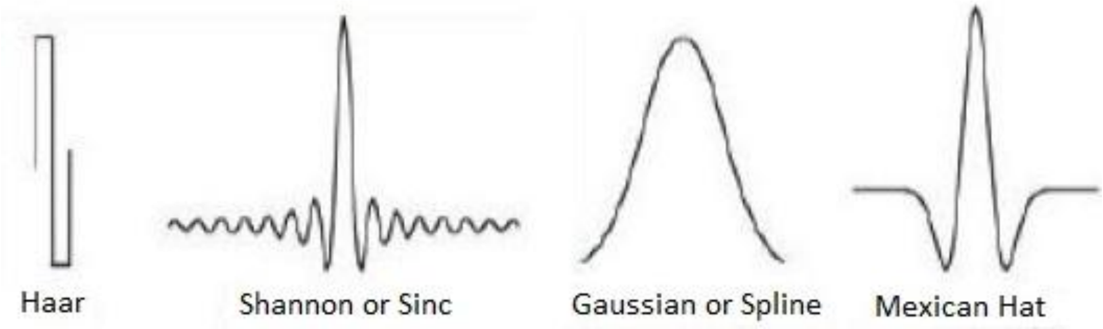
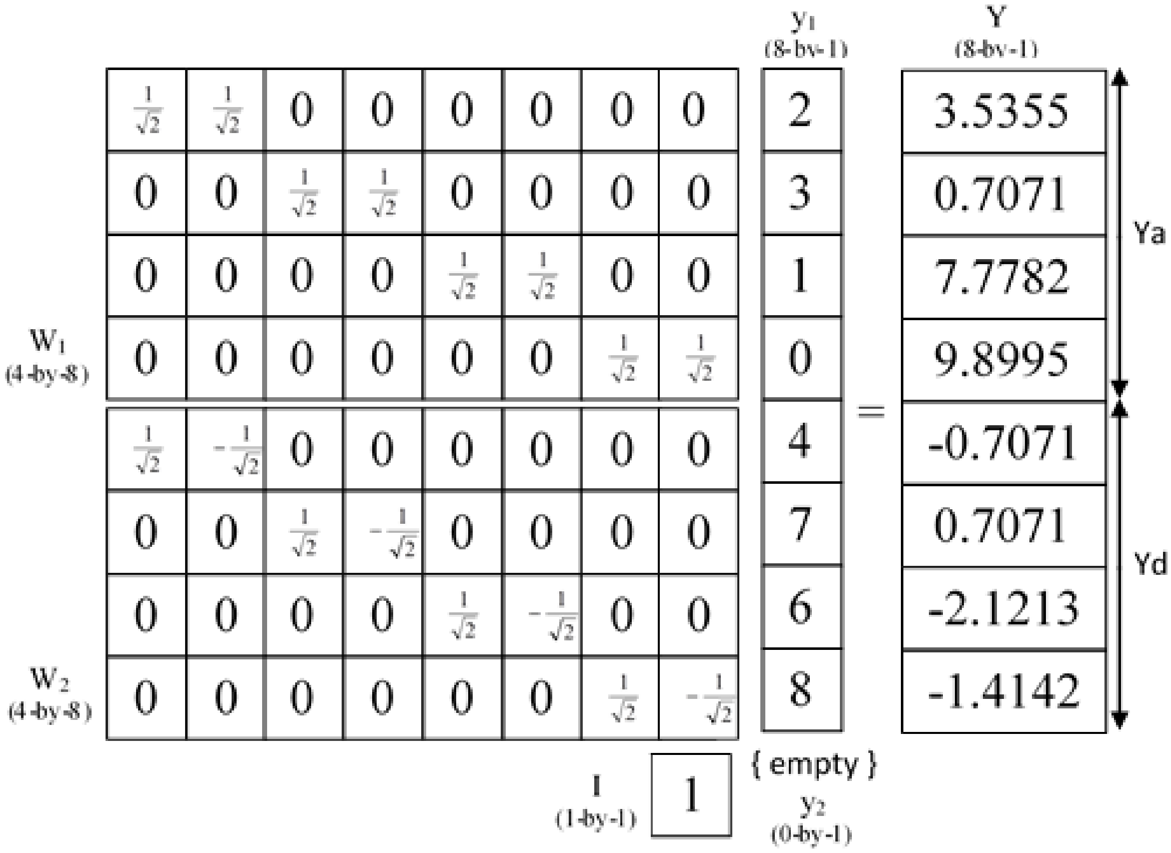


Fig 1: Some wavelet families



$$\begin{array}{c} \\ \\ \\ W_1 \\ (4\text{-by-}8) \\ \\ \\ \\ W_2 \\ (4\text{-by-}8) \end{array}\left[\begin{array}{cccccccc} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \hline \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{array}\right] \overset{\substack{y_1 \\ (8\text{-by-}1)}}{\begin{bmatrix} 2 \\ 3 \\ 1 \\ 0 \\ 4 \\ 7 \\ 6 \\ 8 \end{bmatrix}} = \overset{\substack{Y \\ (8\text{-by-}1)}}{\begin{bmatrix} 3.5355 \\ 0.7071 \\ 7.7782 \\ 9.8995 \\ -0.7071 \\ 0.7071 \\ -2.1213 \\ -1.4142 \end{bmatrix}} \begin{array}{l} \left.\begin{array}{c} \\ \\ \\ \\ \end{array}\right\} Ya \\ \left.\begin{array}{c} \\ \\ \\ \\ \end{array}\right\} Yd \end{array}$$

$$\underset{(1\text{-by-}1)}{I}\ \boxed{1}\ \ \underset{\substack{y_2 \\ (0\text{-by-}1)}}{\{\text{empty}\}}$$

Fig 2: First decomposition level ($L = 1$)

$$W = [W_1 ; W_2] = \begin{bmatrix} W_1 \\ W_2 \end{bmatrix} \qquad (3)$$

$$Ya = W_1 y_1 \qquad (4)$$

$$Yd = \begin{bmatrix} W_2 y_1 \\ y_2 \end{bmatrix} \qquad (5)$$

$$Y = [Ya ; \mathrm{Yd}] = \begin{bmatrix} Ya \\ Yd \end{bmatrix} \qquad (6)$$

Observations on the relationship between the dimensions of X, Y, W_1, W_2, y_1, y_2, Ya and Yd on one hand and the decomposition level (L) and the number of elements (N) in X on the other hand are summarized in Table 1. Both X and Y are N-by-1. Both W_1 and W_2 are r-by-c. r is usually half of c. y_1 is c-by-1, y_2 is (N-c)-by-1, Ya is r-by-1, and Yd is (N-r)-by-1. r and c are related to N and L as indicated in Table 1 and Eqns. (7) and (8). These equations are deduced from the trends in Table 1. W_1 and W_2 are concatenated to give W as in Eqn. (3) which is a MATLAB statement. Similarly, Wy_1 and Iy_2 are concatenated to give Y as in Eqn. (2).

$$r = \frac{N}{2^L} \qquad (7)$$

$$c = \frac{N}{2^{L-1}} \qquad (8)$$

$$\begin{array}{c|cccc|}
 & & & & \\
\begin{array}{c}W_1\\(2\text{-by-}4)\end{array} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\
 & 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\
\begin{array}{c}W_2\\(2\text{-by-}4)\end{array} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \\
 & 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}
\end{array}
\begin{array}{|c|}
\begin{array}{c}y_1\\(4\text{-by-}1)\end{array}\\
3.5355\\ 0.7071\\ 7.7782\\ 9.8995
\end{array}
=
\begin{array}{|c|c}
\begin{array}{c}Y\\(8\text{-by-}1)\end{array} & \\
3.0000 & Ya\\ 12.5000 & \\ 2.0000 & Yd\\ -1.5000 &
\end{array}$$

$$\begin{array}{c|cccc|}
 & 1 & 0 & 0 & 0 \\
 & 0 & 1 & 0 & 0 \\
 & 0 & 0 & 1 & 0 \\
\begin{array}{c}I\\(4\text{-by-}4)\end{array} & 0 & 0 & 0 & 1
\end{array}
\begin{array}{|c|}
-0.7071\\ 0.7071\\ -2.1213\\ -1.4142\\
\begin{array}{c}y_2\\(4\text{-by-}1)\end{array}
\end{array}
=
\begin{array}{|c|}
-0.7071\\ 0.7071\\ -2.1213\\ -1.4142\\ \\
\end{array}$$

Fig 3: Second decomposition level ($L = 2$ **)**

$$\begin{array}{c|cc|}
\begin{array}{c}W_1\\(1\text{-by-}2)\end{array} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\
\begin{array}{c}W_2\\(1\text{-by-}2)\end{array} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}
\end{array}
\begin{array}{|c|}
\begin{array}{c}y_1\\(2\text{-by-}1)\end{array}\\
3.0000\\ 12.5000
\end{array}
=
\begin{array}{|c|c}
\begin{array}{c}Y\\(8\text{-by-}1)\end{array} & \\
10.9602 & Ya\\ -6.7175 & Yd
\end{array}$$

$$\begin{array}{c|cccccc|}
 & 1 & 0 & 0 & 0 & 0 & 0 \\
 & 0 & 1 & 0 & 0 & 0 & 0 \\
 & 0 & 0 & 1 & 0 & 0 & 0 \\
 & 0 & 0 & 0 & 1 & 0 & 0 \\
 & 0 & 0 & 0 & 0 & 1 & 0 \\
\begin{array}{c}I\\(4\text{-by-}4)\end{array} & 0 & 0 & 0 & 0 & 0 & 1
\end{array}
\begin{array}{|c|}
2.0000\\ -1.5000\\ -0.7071\\ 0.7071\\ -2.1213\\ -1.4142\\
\begin{array}{c}y_2\\(6\text{-by-}1)\end{array}
\end{array}
=
\begin{array}{|c|}
2.0000\\ -1.5000\\ -0.7071\\ 0.7071\\ -2.1213\\ -1.4142\\ \\
\end{array}$$

Fig 4: Third decomposition level ($L = 3$ **)**

Table 1. Observations on dimensions of variables

Decomposition Level	W₁	W₂	W	I	y₁	y₂	Ya	Yd
L=1	4-by-8	4-by-8	8-by-8	-	8-by-1	0-by-1	4-by-1	4-by-1
L=2	2-by-4	2-by-4	4-by-4	4-by-4	4-by-1	4-by-1	2-by-1	6-by-1
L=3	1-by-2	1-by-2	2-by-2	6-by-6	2-by-1	6-by-1	1-by-1	7-by-1
Generally for L and N	r-by-c $r=\frac{N}{2^L}$ $c=\frac{N}{2^{L-1}}$	r-by-c $r=\frac{N}{2^L}$ $c=\frac{N}{2^{L-1}}$	c-by-c $c=\frac{N}{2^{L-1}}$	(N-c)-by-(N-c)	c-by-1 $c=\frac{N}{2^{L-1}}$	(N-c)-by-1 $c=\frac{N}{2^{L-1}}$	r-by-1 $r=\frac{N}{2^L}$	(N-r)-by-1 $r=\frac{N}{2^L}$
X is N-by-1; ($N=8$ in this example.)			$W=\begin{bmatrix}W_1\\W_2\end{bmatrix}$; $Y=\begin{bmatrix}Wy_1\\Iy_2\end{bmatrix}$; $Y=\begin{bmatrix}Ya\\Yd\end{bmatrix}$; Y is also N-by-1					

W_1 and W_2 are the approximate and detail wavelet transformation matrices respectively. Most of the elements of W_1 and W_2 are zeroes. Few of the elements of W_1 and W_2 are $\frac{1}{\sqrt{2}}$ or $-\frac{1}{\sqrt{2}}$. Eqns. (9) and (10) set all elements of W_1 and W_2 to zero. A careful review of W_1 in Figs. 4, 5 and 6 shows that on the kth row, the (2k)th column and the (2k-1)th column are both $\frac{1}{\sqrt{2}}$. A careful review of W_2 in Figs. 4, 5 and 6 shows that on the kth row, the (2k)th column and the (2k-1)th column are $-\frac{1}{\sqrt{2}}$ and $\frac{1}{\sqrt{2}}$ respectively. Therefore, Eqns. (11), (12), (13) and (14) set specific elements of W_1 and W_2 to $-\frac{1}{\sqrt{2}}$ or $\frac{1}{\sqrt{2}}$ as appropriate. The solution is repetitive for each decomposition level L=1, 2, 3, up to the specified level $L=L_m$.

$$W_1 = zeros(r,c) \tag{9}$$

$$W_2 = zeros(r,c) \tag{10}$$

$$W_1(k,(2k-1)) = \frac{1}{\sqrt{2}} \tag{11}$$

$$W_1(k,2k) = \frac{1}{\sqrt{2}} \tag{12}$$

$$W_2(k,(2k-1)) = \frac{1}{\sqrt{2}} \tag{13}$$

$$W_2(k,2k) = \frac{1}{\sqrt{2}} \tag{14}$$

2.2 Basic Haar Wavelet Program

Given the Y at the end of last decomposition level, it is required to determine r, c, y_1, y_2, W_1, and W_2 using Eqns. (7) to (14) and update Y using Eqns. (2) and (3) for the next decomposition level (L). A program is developed to carry out this task. This program is named *haarbasic*. Its flow chart and MATLAB codes are presented in Figs. 5 and 6 respectively.

The inverse transform program *INVhaarbasic* is presented in Fig. 7. *INVhaarbasic* is similar to *haarbasic* except for the fact that Eqn. (2) is replaced by Eqn. (15).

$$Y = \begin{bmatrix}W^{-1}y_1\\Iy_2\end{bmatrix} = \begin{bmatrix}W^{-1}y_1\\y_2\end{bmatrix} \tag{15}$$

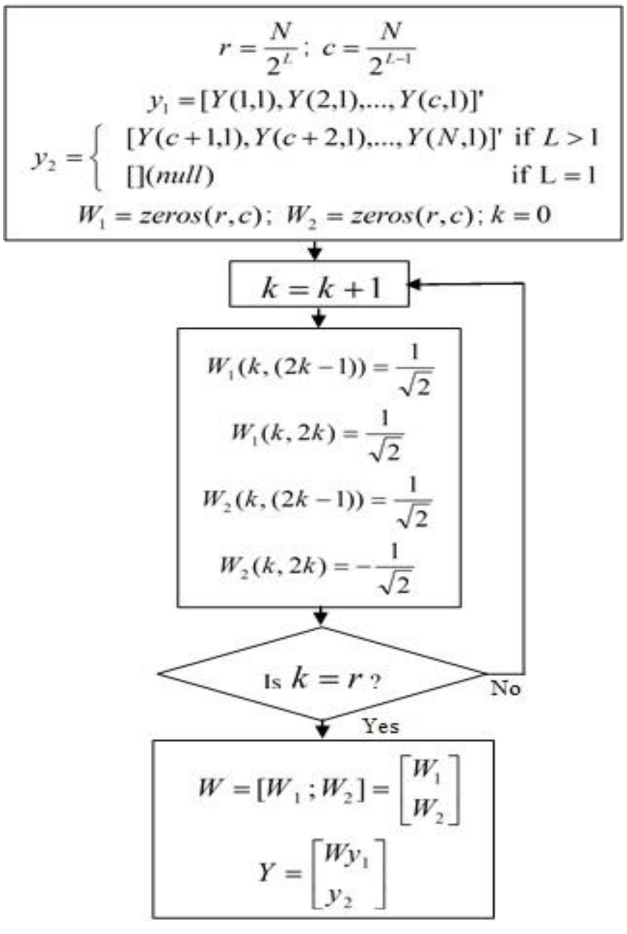
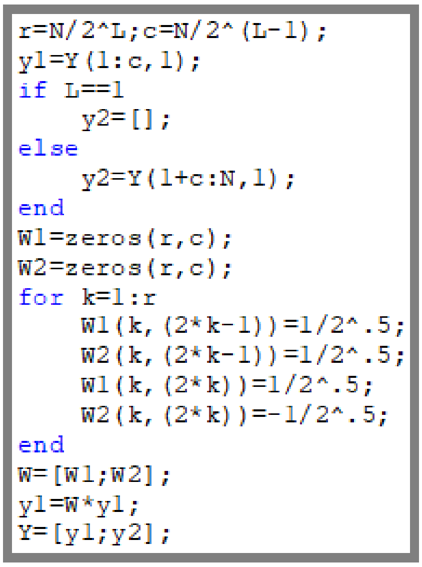
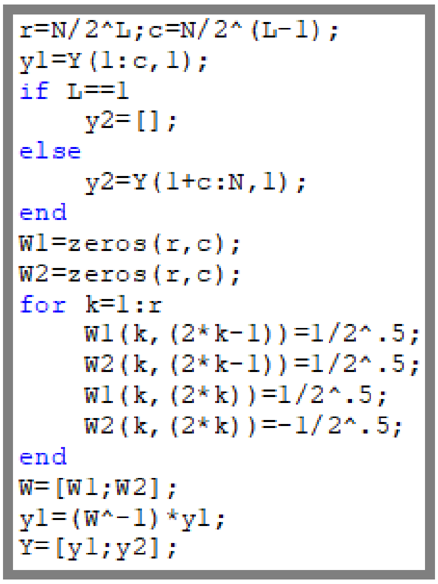

$r=\frac{N}{2^L}$; $c=\frac{N}{2^{L-1}}$
$y_1=[Y(1,1),Y(2,1),....,Y(c,1)]'$
$y_2=\begin{cases}[Y(c+1,1),Y(c+2,1),....,Y(N,1)]' & \text{if } L>1\\ [](null) & \text{if L = 1}\end{cases}$
$W_1=zeros(r,c)$; $W_2=zeros(r,c)$; $k=0$
$k=k+1$
$W_1(k,(2k-1))=\frac{1}{\sqrt{2}}$
$W_1(k,2k)=\frac{1}{\sqrt{2}}$
$W_2(k,(2k-1))=\frac{1}{\sqrt{2}}$
$W_2(k,2k)=-\frac{1}{\sqrt{2}}$
Is $k=r$?
No
Yes
$W=[W_1;W_2]=\begin{bmatrix}W_1\\W_2\end{bmatrix}$
$Y=\begin{bmatrix}Wy_1\\y_2\end{bmatrix}$

Fig 5: Flow chart of the *haarbasic* program for decomposition level L



```
r=N/2^L;c=N/2^(L-1);
y1=Y(1:c,1);
if L==1
    y2=[];
else
    y2=Y(1+c:N,1);
end
W1=zeros(r,c);
W2=zeros(r,c);
for k=1:r
    W1(k,(2*k-1))=1/2^.5;
    W2(k,(2*k-1))=1/2^.5;
    W1(k,(2*k))=1/2^.5;
    W2(k,(2*k))=-1/2^.5;
end
W=[W1;W2];
y1=W*y1;
Y=[y1;y2];
```

Fig 6: *haarbasic* program for decomposition level L



```
r=N/2^L;c=N/2^(L-1);
y1=Y(1:c,1);
if L==1
    y2=[];
else
    y2=Y(1+c:N,1);
end
W1=zeros(r,c);
W2=zeros(r,c);
for k=1:r
    W1(k,(2*k-1))=1/2^.5;
    W2(k,(2*k-1))=1/2^.5;
    W1(k,(2*k))=1/2^.5;
    W2(k,(2*k))=-1/2^.5;
end
W=[W1;W2];
y1=(W^-1)*y1;
Y=[y1;y2];
```

Fig 7: *INVhaarbasic* program for decomposition level L

2.3 One-Dimensional DWT Program

2.3.1 Column Signal

Given an input column signal X (N-by-M) where $M = 1$, it is required to determine Y, the one-dimensional discrete wavelet transform (1D DWT) of X at L_m^{th} decomposition level. Initialize Y as $Y = X$. Starting with $L = 1$, call the *haarbasic* program, increase L and repeat the call of the *haarbasic* program up to L=L_m as illustrated in the flowchart of Fig. 8. Y obtained at the end of $L = L_m$ is the required 1D DWT of X.

The flow chart of Fig. 8 is coded into a MATLAB program named *dwt1Dc* and presented in Fig. 9. The inverse transform program *INVdwt1Dc* presented in Fig. 10 is similar to *dwt1Dc* except that L decreases from L_m to 1 in *INVdwt1Dc* whereas L increases from 1 to L_m in *dwt1Dc*.

2.3.2 Row Signal

Given an input row signal X (M-by-N) where $M = 1$, It is required to determine Y, the one-dimensional discrete wavelet transform (1D DWT) of X at L_m^{th} decomposition level. Change X from being a row signal into a column signal. Call *dwt1Dc* program. Change the final output Y from being a column signal to a row signal. The new program is named *dwt1Dr* and is presented in Fig 11. The inverse transform program *INVdwt1Dr* presented in Fig. 12 is similar to *dwt1Dr*. Change Y from being a row signal into a column signal. Call *INVdwt1Dc* program. Change the final output X from being a column signal to a row signal.

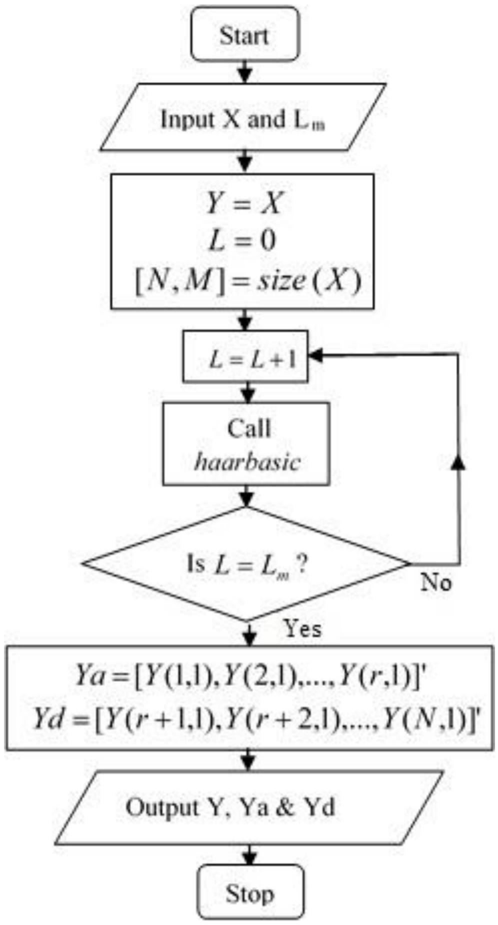
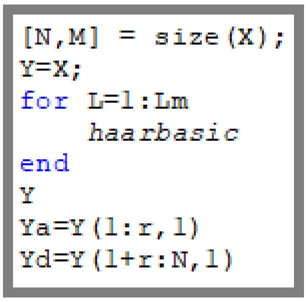
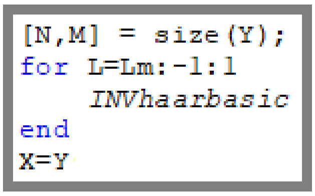


Fig 8: Flow chart of the *dwt1Dc* program for a column 1D signal



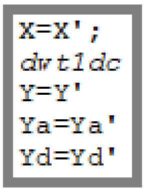
```
[N,M] = size(X);
Y=X;
for L=1:Lm
    haarbasic
end
Y
Ya=Y(1:r,1)
Yd=Y(1+r:N,1)
```

Fig 9: *dwt1Dc* program for a column 1D signal



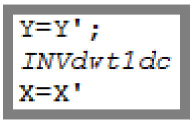
```
[N,M] = size(Y);
for L=Lm:-1:1
    INVhaarbasic
end
X=Y
```

Fig 10: *INVdwt1Dc* program for a column 1D signal



```
X=X';
dwt1dc
Y=Y'
Ya=Ya'
Yd=Yd'
```

Fig 11: *dwt1Dr* program for a row 1D signal



```
Y=Y';
INVdwt1dc
X=X'
```

Fig 12: *INVdwt1Dr* program for a row 1D signal

2.4 Two-Dimensional DWT Program

Given a two-dimensional signal or gray level image XX (NN-by-MM), It is required to determine YY, the two-dimensional discrete wavelet transform (2D DWT) of XX at LL$_m^{th}$ decomposition level. Initialize YY as $YY = XX$. If XX is a gray level image, XX should be converted from uint8 format to double format before applying it as input. The output YY should be converted from double format to uint8 format before displaying it as an image.

The 2D DWT of a signal or image can be partitioned into four regions or components namely: Approximation (A), Horizontal details (HD), Vertical details (VD), and Diagonal details (DD) as illustrated in Fig. 13. A, HD, VD, and DD are the Low/Low (LL), Low/High (LH), High/Low (HL) and the High/High (HH) frequency components of the signal respectively. At the next decomposition level, the Approximation part is further divided into another set of A, HD, VD, and DD as shown in Fig. 13.

1D DWT and inverse 1D DWT programs are required here as subprograms. The outputs of these subprograms constitute intermediate results in 2D DWT program and such outputs may be disabled from being published with ';' .

Starting with the decomposition level $LL = 1$, work on the entire YY (NN-by-MM). Take the first row in YY as X and call *dwt1Dr* with $L_m = 1$. Update YY by replacing the first row of YY with the Y obtained from *dwt1Dr*. Repeat for all the rows in YY. This is illustrated in Fig. 14 (a) tagged as the first step. Take the first column in the updated YY as X and call *dwt1Dc* with $L_m = 1$. Update YY by replacing the first column of YY with the Y obtained from *dwt1Dc*. Repeat for all the columns in YY as illustrated in Fig. 14 (b) tagged as the second step. The YY at the end of the second step is the two-dimensional discrete wavelet transform (2D DWT) for $LL = 1$.

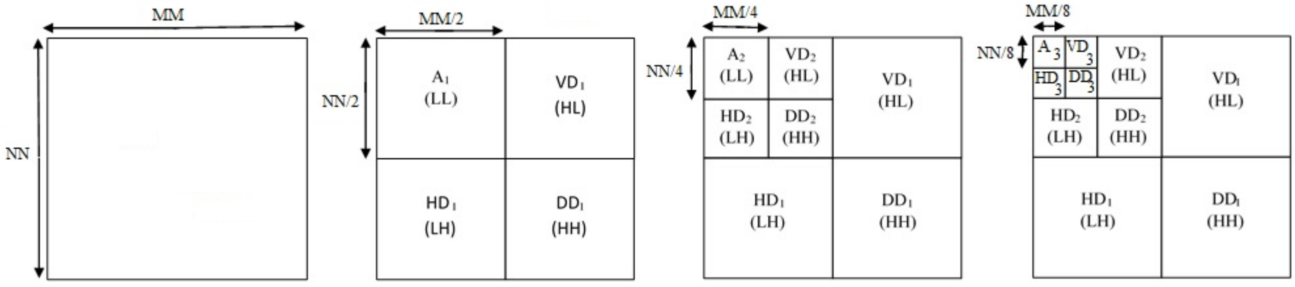
For $LL = LL$, select the Approximation component which is the NN/LL-by-MM/LL top-left-portion of the updated YY and repeat the procedure above. The remaining portions of YY remain unchanged. For $LL = 2$, 2-by-2 top-left-portion of YY is taken and the procedure is repeated as illustrated in Fig. 14 (c) and (d) tagged third and fourth step respectively. Increase LL from 1 up to LL$_m$. In Fig. 14, the operations stopped at $LL = 2$ because LL$_m$ is given as 2. A program named as *dwt2D* is developed and presented in Fig. 15. MATLAB code 'clear' is introduced in this program during test running when an error was observed. This code deletes a variable in the workspace or memory. For example, when LL changed from 1 to 2 in Fig. 14, the dimension of both X and Y changed from 1-by-4 to 1-by-2. If X and Y are not cleared before the change in LL, X and Y will remain 1-by-4 after the change in LL instead of 1-by-2 which actually led to an error during test running.

The steps for the inverse two-dimensional discrete wavelet transform program *INVdwt2D* of Fig. 16 are similar. In *dwt2d*, LL increases from 1 up to LL$_m$ and rows are operated before columns. In *INVdwt2D*, LL decreases from LL$_m$ down to 1 and columns are operated before rows.

2.5 Three-Dimensional DWT Program

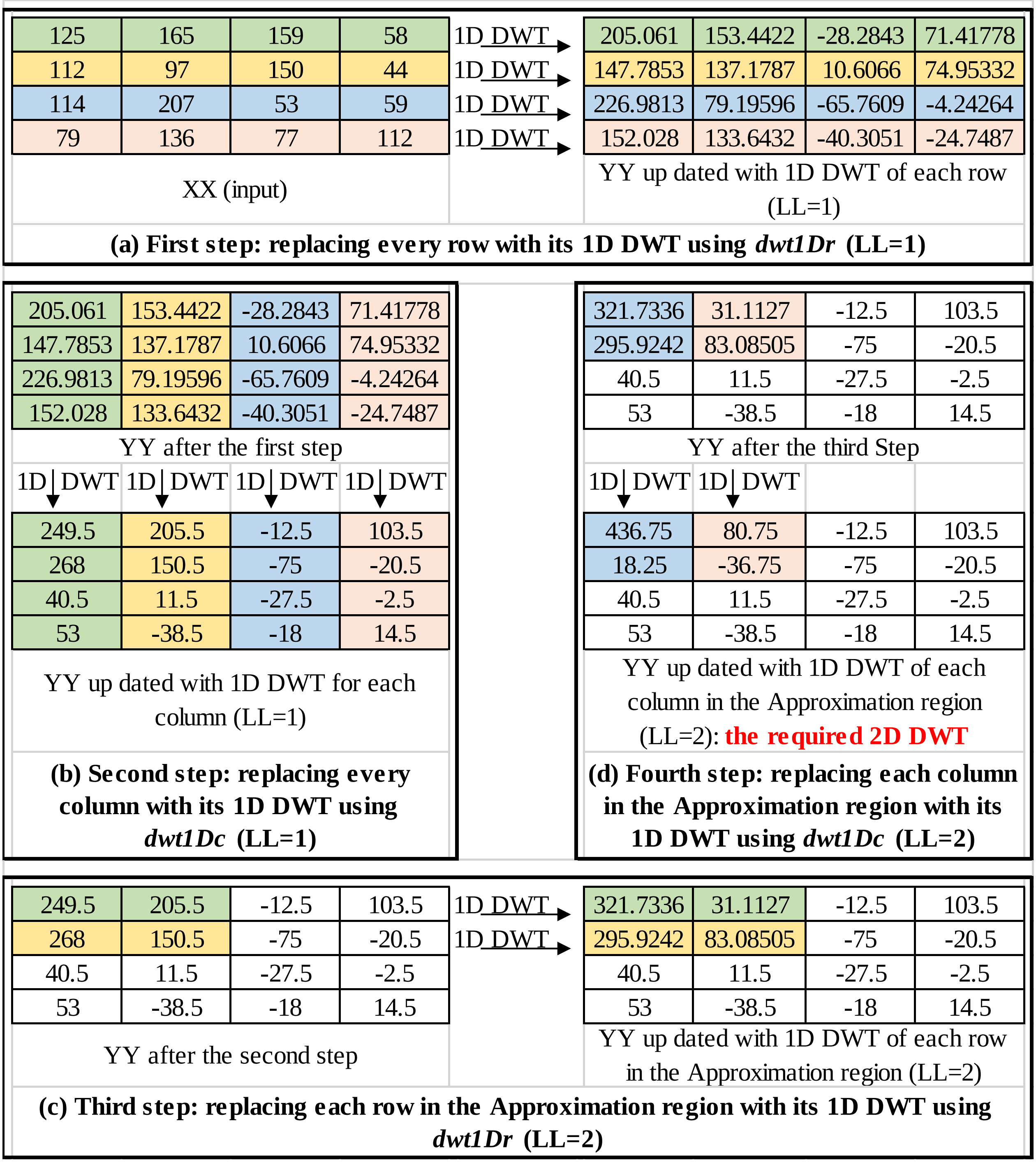
Given a three-dimensional signal or color image x (NN-by-MM-by-3), It is required to determine y, the three-dimensional discrete wavelet transform (3D DWT) of y at LL$_m^{th}$ decomposition level. This is accomplished by simply finding the 2D DWT of each of the red, green and blue components of x. *dwt2D* is executed three times as indicated in the *dwt3D* program of Fig. 17.

The first line in Fig. 17 converts the color image from the uint8 format to the double format. The output 3D DWT (y) should not be converted from the double format to the uint8 format otherwise original x will not be recoverable from y via the *INVdwt3D* program of Fig. 18 which is the inverse 3D DWT program. The last line in Fig. 17 shows the image of uint8 version of y on a fresh figure. The second to the last line in Fig. 18 converts the inverse 3D DWT from the double format back to the uint8 format.



XX YY after $LL = 1$ YY after $LL = 2$ YY after $LL = 3$

Fig 13: Regions or components of 2D DWT of a 2D signal XX



125	165	159	58
112	97	150	44
114	207	53	59
79	136	77	112

XX (input)

1D DWT →
1D DWT →
1D DWT →
1D DWT →

205.061	153.4422	-28.2843	71.41778
147.7853	137.1787	10.6066	74.95332
226.9813	79.19596	-65.7609	-4.24264
152.028	133.6432	-40.3051	-24.7487

YY up dated with 1D DWT of each row (LL=1)

(a) First step: replacing every row with its 1D DWT using *dwt1Dr* (LL=1)

205.061	153.4422	-28.2843	71.41778
147.7853	137.1787	10.6066	74.95332
226.9813	79.19596	-65.7609	-4.24264
152.028	133.6432	-40.3051	-24.7487

YY after the first step

1D↓DWT	1D↓DWT	1D↓DWT	1D↓DWT
249.5	205.5	-12.5	103.5
268	150.5	-75	-20.5
40.5	11.5	-27.5	-2.5
53	-38.5	-18	14.5

YY up dated with 1D DWT for each column (LL=1)

(b) Second step: replacing every column with its 1D DWT using *dwt1Dc* (LL=1)

321.7336	31.1127	-12.5	103.5
295.9242	83.08505	-75	-20.5
40.5	11.5	-27.5	-2.5
53	-38.5	-18	14.5

YY after the third Step

1D↓DWT	1D↓DWT		
436.75	80.75	-12.5	103.5
18.25	-36.75	-75	-20.5
40.5	11.5	-27.5	-2.5
53	-38.5	-18	14.5

YY up dated with 1D DWT of each column in the Approximation region (LL=2): **the required 2D DWT**

(d) Fourth step: replacing each column in the Approximation region with its 1D DWT using *dwt1Dc* (LL=2)

249.5	205.5	-12.5	103.5
268	150.5	-75	-20.5
40.5	11.5	-27.5	-2.5
53	-38.5	-18	14.5

YY after the second step

1D DWT →
1D DWT →

321.7336	31.1127	-12.5	103.5
295.9242	83.08505	-75	-20.5
40.5	11.5	-27.5	-2.5
53	-38.5	-18	14.5

YY up dated with 1D DWT of each row in the Approximation region (LL=2)

(c) Third step: replacing each row in the Approximation region with its 1D DWT using *dwt1Dr* (LL=2)

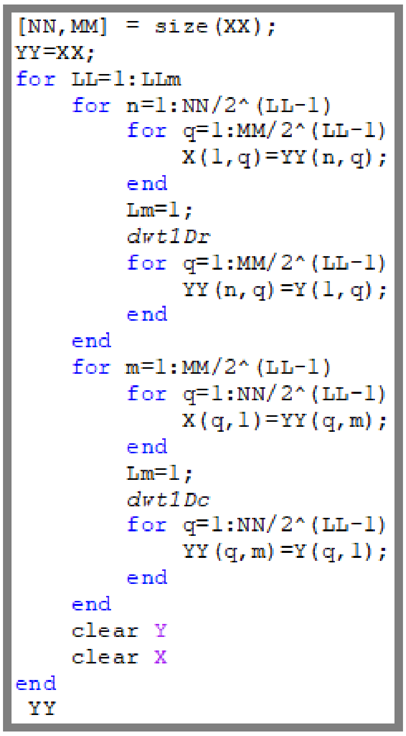
Fig 14: Illustration of 2D DWT for a 4-by-4 input at decomposition level $LL_m = 2$

3. RESULTS AND DISCUSSIONS

Validation tests for the 1D and 2D DWT and inverse DWT programs were carried out for situations when the results are known. As shown in Figs. 19, 20, 21 and 22, the outputs of the 1D and 2D DWT and inverse DWT programs are found to be valid. When the output of the DWT program is supplied as input to the corresponding inverse DWT program, the output of the inverse DWT program is found in all cases to be equal to the initial input to the DWT program as illustrated in Figs. 19, 20, 21 and 22.

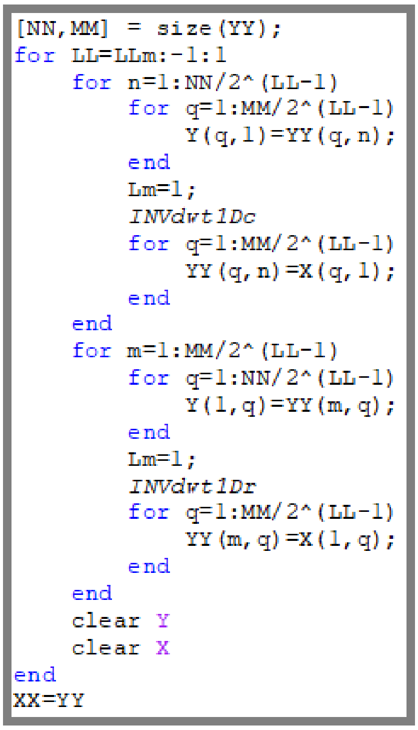
2D DWT and inverse 2D DWT programs are also tested with a gray level image. The results are presented in Figs. 23 (a) and (b). YY in Fig. 23(b) is in double format which gives Fig. 23(c) when converted to uint8 format. Application of image enhancement on the Approximation component gives Fig. 23(d). The image enhancement is accomplished by replacing each element e of the approximation component with e^p where $p = 0.7$. The approximation component is thus a version of the original input image but with reduced size (dimension). The VD, HD, and DD give the vertical, horizontal and diagonal edges (details) of the input image.

3D DWT and inverse 3D DWT programs are tested with a color image. The results are presented in Figs. 24 (a) and (b). y in Fig. 24(b) is in the double format which gives Fig. 24(c) when converted to the uint8 format. Application of image enhancement on the Approximation component gives Fig. 24(d). The image enhancement is accomplished by replacing each element e of the approximation component with e^p where $p = 0.8$.



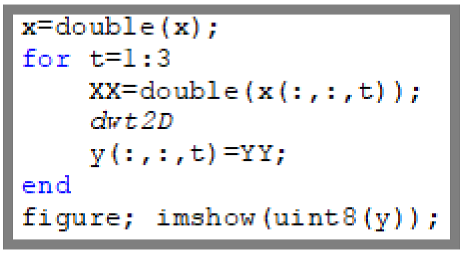
```
[NN,MM] = size(XX);
YY=XX;
for LL=1:LLm
    for n=1:NN/2^(LL-1)
        for q=1:MM/2^(LL-1)
            X(1,q)=YY(n,q);
        end
        Lm=1;
        dwt1Dr
        for q=1:MM/2^(LL-1)
            YY(n,q)=Y(1,q);
        end
    end
    for m=1:MM/2^(LL-1)
        for q=1:NN/2^(LL-1)
            X(q,1)=YY(q,m);
        end
        Lm=1;
        dwt1Dc
        for q=1:NN/2^(LL-1)
            YY(q,m)=Y(q,1);
        end
    end
    clear Y
    clear X
end
 YY
```

Fig 15: *dwt2D* program for 2D signal



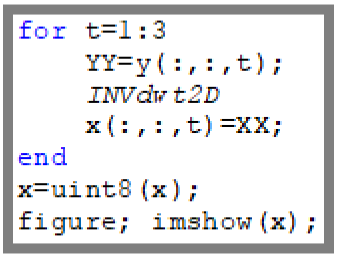
```
[NN,MM] = size(YY);
for LL=LLm:-1:1
    for n=1:NN/2^(LL-1)
        for q=1:MM/2^(LL-1)
            Y(q,1)=YY(q,n);
        end
        Lm=1;
        INVdwt1Dc
        for q=1:MM/2^(LL-1)
            YY(q,n)=X(q,1);
        end
    end
    for m=1:MM/2^(LL-1)
        for q=1:NN/2^(LL-1)
            Y(1,q)=YY(m,q);
        end
        Lm=1;
        INVdwt1Dr
        for q=1:MM/2^(LL-1)
            YY(m,q)=X(1,q);
        end
    end
    clear Y
    clear X
end
XX=YY
```

Fig 16: *INVdwt2D* program for 2D signal



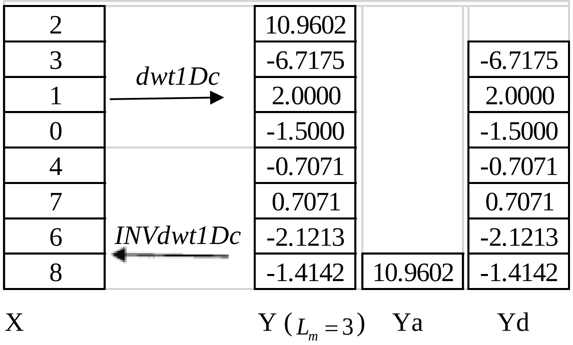
```
x=double(x);
for t=1:3
    XX=double(x(:,:,t));
    dwt2D
    y(:,:,t)=YY;
end
figure; imshow(uint8(y));
```

Fig 17: *dwt3D* program for 3D signal



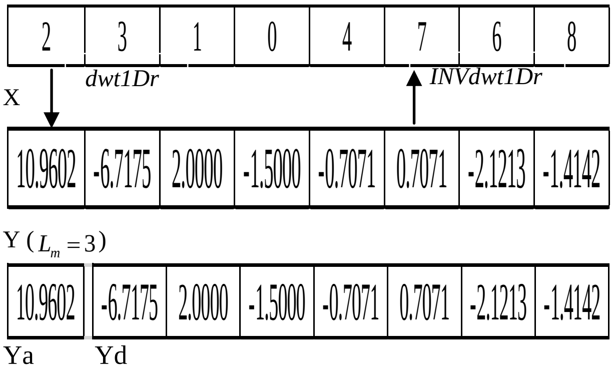
```
for t=1:3
    YY=y(:,:,t);
    INVdwt2D
    x(:,:,t)=XX;
end
x=uint8(x);
figure; imshow(x);
```

Fig 18: *INVdwt3D* program for 3D signal



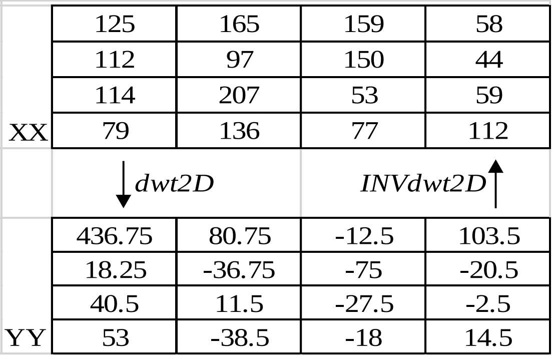
X	Y ($L_m = 3$)	Ya	Yd
2	10.9602		
3	-6.7175		-6.7175
1	2.0000		2.0000
0	-1.5000		-1.5000
4	-0.7071		-0.7071
7	0.7071		0.7071
6	-2.1213		-2.1213
8	-1.4142	10.9602	-1.4142

Fig 19: *dwt1Dc* and *INVdwt1Dc* results



X	2	3	1	0	4	7	6	8
Y ($L_m = 3$)	10.9602	-6.7175	2.0000	-1.5000	-0.7071	0.7071	-2.1213	-1.4142
Ya / Yd	10.9602	-6.7175	2.0000	-1.5000	-0.7071	0.7071	-2.1213	-1.4142

Fig 20: *dwt1Dr* and *INVdwt1Dr* results



	125	165	159	58
	112	97	150	44
	114	207	53	59
XX	79	136	77	112
	436.75	80.75	-12.5	103.5
	18.25	-36.75	-75	-20.5
	40.5	11.5	-27.5	-2.5
YY	53	-38.5	-18	14.5

Fig 21: *dwt2D* and *INVdwt2D* results for $LL_m = 2$

208	245	108	174	71	112	181	245
231	247	234	194	12	98	193	87
33	41	203	190	25	196	71	150
233	248	245	101	210	203	174	58
162	245	168	168	178	48	168	192
25	124	10	44	81	125	42	66
72	205	217	181	243	114	31	130
140	37	239	9	9	165	128	179

XX

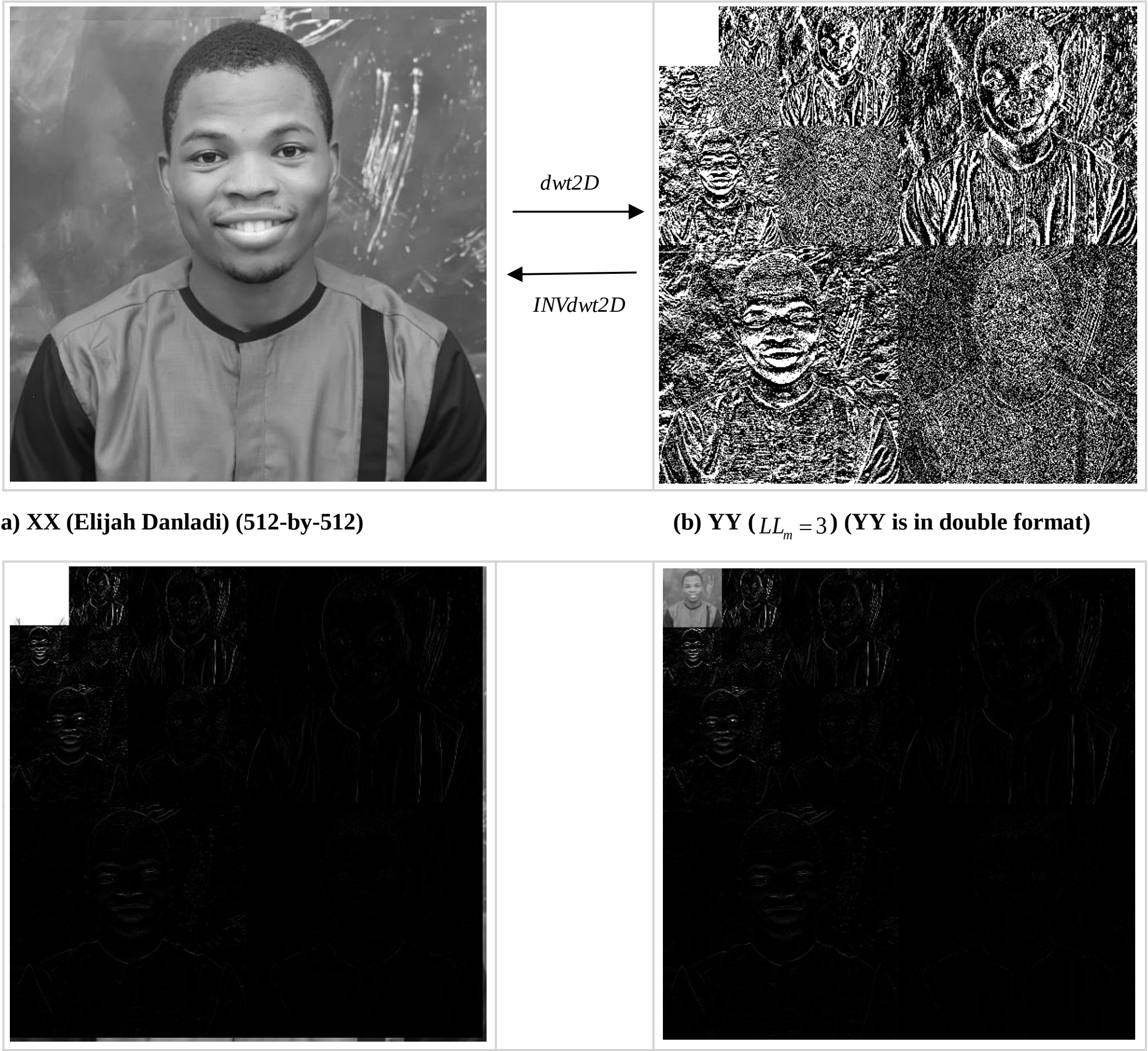
$\xrightarrow{dwt2D}$

$\xleftarrow{INVdwt2D}$

1120.75	124.5	9.25	-58	-26.5	-13	-63.5	21
134.5	87.75	-6.5	6.75	-11.5	78.5	-82	18.5
86.75	-22	101.25	-148.5	-91	-17	43	-24
-38.5	-24.75	89.5	-24.75	-15	133	-13.5	-75
-12.5	-73	36.5	73	-10.5	-53	22.5	-85
-203.5	23.5	-96	-5.5	3.5	-65.5	-89	-97.5
129	141	10	126	8	17	87	0
50	75	91.5	-73	-118	-97	142.5	-24

YY ($LL_m = 3$**)**

Fig 22: *dwt2D* and *INVdwt2D* results for $LL_m = 3$



(a) XX (Elijah Danladi) (512-by-512)

(b) YY ($LL_m = 3$**) (YY is in double format)**

(c) YY converted from double to uint8 format

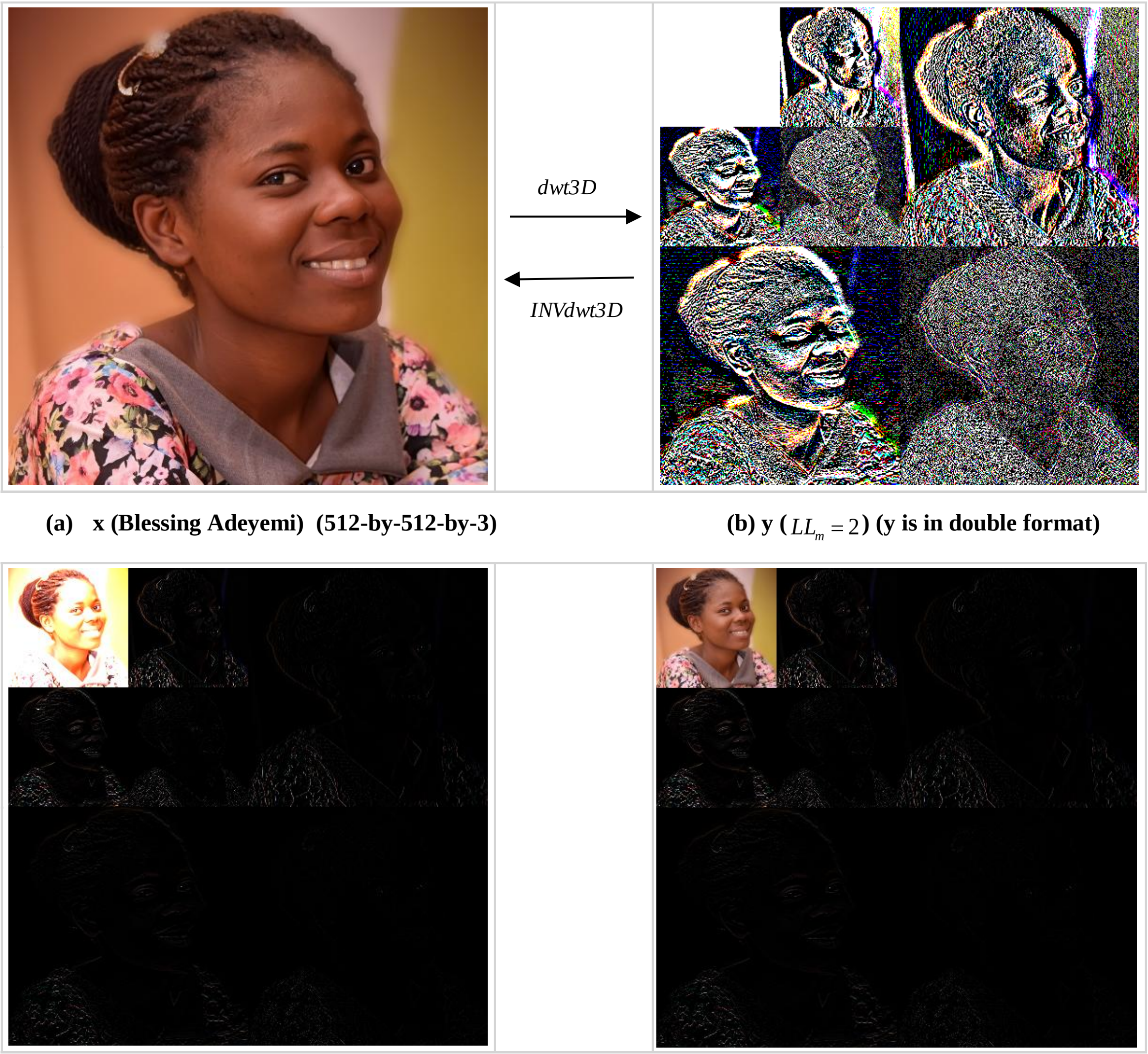
(d) YY converted from double to uint8 format and the approximation component is enhanced

Fig 23: *dwt2D* and *INVdwt2D results* for $LL_m = 3$

The approximation component is thus a version of the original input image but with reduced size. The VD, HD, and DD give the vertical, horizontal and diagonal edges (details) of the input image.

4. CONCLUSIONS

Development of a logical plan of attack in solving a problem has been demonstrated. Scientific representations of the problem in the form of diagrams were presented. Arithmetic and logical relationships between the input, output and intermediate variables in the form of mathematical equations were derived. Simple relevant tools of MATLAB were used in developing the programs for solving the problem. Programs were developed in modules. A program may call another program for some specific tasks. A complex problem is solved by simple programs instead of one long complex program. Simple short programs are easier to develop and debug.



(a) x (Blessing Adeyemi) (512-by-512-by-3)

(b) y ($LL_m = 2$) (y is in double format)

(c) y converted from double to uint8 format

(d) y converted from double to uint8 format and the approximation component enhanced

Fig 24: *dwt3D* and *INVdwt3D* results for $LL_m = 2$

Programs are validated with situations where the answers are known. Discrete wavelet transformation and inverse discrete wavelet transformation have been demystified. Discrete wavelet transformation and inverse discrete wavelet transformation for 1D, 2D, and 3D discrete-time signals in general and gray level images and color images in particular have been implemented and can be applied in Discrete Signal Processing and Digital Image Processing. The use of similar examples is recommended for Engineering Lecturers for transforming Engineering students from novices to experts vis-a-vis computer-aided engineering and problem-solving.

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