Regular Generalized $\mathcal{G}^{\omega}-$Closed Sets in Grill Topological Spaces

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ABSTRACT

In this paper we study the generalization and regularity properties of $\mathcal{G}^{\omega}-$closed sets in grill topological spaces by developing the concept of $\mathcal{G}^{\omega}-$closed set and giving the notion of regular generalized $\mathcal{G}^{\omega}-$closed set.

General Terms

AMS classification, Primary 54A05, 54A10

Keywords

Open set, Generalized closed set, Grill topological spaces

1. INTRODUCTION

For closed sets in topological spaces, Levine [6], studied the generalization property by giving the notion of generalized closed sets. Next Palaniappan and Rao [7] studied the regularity property by giving the notion of regular generalized closed sets. For $\omega-$closed sets in topological spaces which introduced by Hdeib [5]. Al-Zoubi [2] introduced the generalization property by giving the notion of generalized $\omega-$closed sets. Next Al-Omari and Noorani [1] studied the regularity property by giving the notion of regular generalized $\omega-$closed sets.

In grill topological spaces, under the notion of $\omega-$closed sets, [9] Saif, et al., introduced the notion $\mathcal{G}^{\omega}-$closed sets and studied the generalization property of $\mathcal{G}^{\omega}-$closed set by giving the notion of generalized $\mathcal{G}^{\omega}-$closed sets.

In this paper, Section 3 develops the notion of generalized $\mathcal{G}^{\omega}-$closed sets. In Section 4, we introduce regularity property of $\mathcal{G}^{\omega}-$closed sets in grill topological spaces by giving the notion of regular generalized $\mathcal{G}^{\omega}-$closed set.

2. PRELIMINARIES

For any topological space (X, τ) and $A \subseteq X$, we mear by $Cl(A)$ and $Int(A)$ the closure operator A and the interior operator of A, respectively.

LEMMA 1. *[4] For every open U in a topological space X and every $A \subseteq X$,*

$$Cl(U \cap A) = Cl(U \cap Cl(A)).$$

THEOREM 2. *[4] For a topological space (X, τ),*

(1) $Cl(X - A) = X - Int(A)$ *for all* $A \subseteq X$.
(2) $Int(X - A) = X - Cl(A)$ *for all* $A \subseteq X$.

For a topological space (X, τ) and $E \subseteq X$, the relativization topology of τ to E is denoted by $\tau|_E$ and defined by:

$$\tau|_E = \{G \cap E : G \text{ is an open set in} X\}.$$

We say the pair $(E, \tau|_E)$ is a subspace of (X, τ). Let $(E, \tau|_E)$ be a subspace of a topological space (X, τ). For a subset A of E, the $\tau|_E$-closure operator of A is a set defined as the intersection of all closed subsets of E containing A and denoted by $Cl|_E(A)$. The $\tau|_E$-interior operator of A is a set defined as the union of all open subsets of E contained in A and denoted by $Int|_E(A)$.

THEOREM 3. *[4] Let $(E, \tau|_E)$ be a subspace of a topological space (X, τ). For a subset A of E:*

(1) *A is a closed in E if and only if $A = F \cap E$ for some closed set F in X.*
(2) $Cl|_E(A) = Cl(A) \cap E$.
(3) $Int(A) \subseteq Int|_E(A)$.

A subset A of a topological space (X, τ) is called regular open set [4] if $A = Int(Cl(A))$. The compliment of regular open set is called regular closed set.

THEOREM 4. *[10] Let Y be an open subset of a topological space (X, τ). If A is regular open set in $(Y, \tau|_Y)$ then $A = G \cap Y$ for some a regular open set G in X.*

DEFINITION 5. *[5] A subset A of a space X is called ω-open set if for each $x \in A$, there is an open set U_x containing x such that $U_x - A$ is a countable set. The complement of ω-open set is called ω-closed set.*

By $Cl_{\omega}(A)$ and $Int_{\omega}(A)$ we mean the $\omega-$closure set and the $\omega-$interior set of A in topological space (X, τ), respectively.

DEFINITION 6. *A subset A of a topological space (X, τ) is called:*

(1) *generalized closed set (briefly, $g-$closed) [6] if $Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an open in X.*
(2) *generalized $\omega-$closed set (briefly, $g\omega-$closed) [2] if $Cl_\omega(A) \subseteq U$ whenever $A \subseteq U$ and $U \in \tau$.*
(3) *regular generalized closed set (briefly $rg-$closed) [7] if $Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in (X, τ).*
(4) *regular generalized $\omega-$closed set (briefly $rg\omega-$closed) [1] if $Cl_\omega(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in (X, τ).*

A collection $\mathcal{G}$ of subsets of a topological spaces (X, τ) is said to be a grill [3] on X if $\mathcal{G}$ satisfies the following conditions:

(1) $\emptyset \notin \mathcal{G}$;
(2) $A \in \mathcal{G}$ and $A \subseteq B$ implies that $B \in \mathcal{G}$;
(3) $A, B \subseteq X$ and $A \cup B \in \mathcal{G}$ implies that $A \in \mathcal{G}$ or $B \in \mathcal{G}$.

For a grill $\mathcal{G}$ on a topological space X, an operator from the power set $P(X)$ of X to $P(X)$ was defined in [8] the following manner : For any $A \in P(X)$,

$\Phi(A) = \{x \in X : U \cap A \in \mathcal{G}, \text{for each open neighborhood} U \text{of} x\}$.

Then the operator $\Psi : P(X) \to P(X)$, given by: $\Psi(A) = A \cup \Phi(A)$, for $A \in P(X)$, was also shown in [8] to be a Kuratowski closure operator, defining a unique topology $\tau_{\mathcal{G}}$ on X such that $\tau \subseteq \tau_{\mathcal{G}}$. This topology defined by

$$\tau_{\mathcal{G}} = \{U \subseteq X : (X - U) = X - U\},$$

where $\tau \subseteq \tau_{\mathcal{G}}$ and for any $A \subseteq X$, $\Psi(A) = {}_{\mathcal{G}}Cl(A)$ such that ${}_{\mathcal{G}}Cl(A)$ denotes the set of all closure points of A in topological space $(X, \tau_{\mathcal{G}})$. The set of all interior points of A in topological space $(X, \tau_{\mathcal{G}})$ denoted by ${}_{\mathcal{G}}Int(A)$.
If (X, τ) is a topological space and $\mathcal{G}$ is a grill on X then the triple $(X, \tau, \mathcal{G})$ will be called a grill topological space.

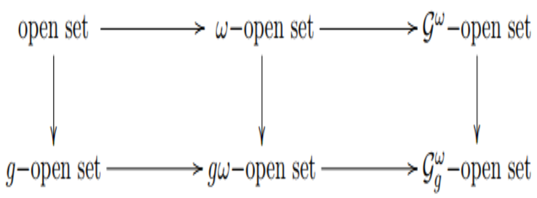
DEFINITION 7. *[9] A subset A of a grill topological space $(X, \tau, \mathcal{G})$ is called $\mathcal{G}^\omega-$open set if $A \subseteq Cl(Int_\omega(\Psi(A)))$ in grill topological space $(X, \tau, \mathcal{G})$. The compliment of $\mathcal{G}^\omega-$open set is called $\mathcal{G}^\omega-$closed set.*

By ${}_{\mathcal{G}^\omega}Cl(A)$ and ${}_{\mathcal{G}^\omega}Int(A)$ we mean the $\mathcal{G}^\omega-$closure set and the $\mathcal{G}^\omega-$interior set of A in topological space (X, τ), respectively.

DEFINITION 8. *[9] A subset A of a grill topological space $(X, \tau, \mathcal{G})$ is called generalized $\mathcal{G}^\omega-$closed set (simply $\mathcal{G}^\omega_g-$closed) if ${}_{\mathcal{G}^\omega}Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an open subset of $(X, \tau, \mathcal{G})$. The complement of $\mathcal{G}^\omega_g-$closed set is called generalized $\mathcal{G}^\omega-$open set (simply $\mathcal{G}^\omega_g-$open set).*

THEOREM 9. *[9] A subset A of a grill topological space $(X, \tau, \mathcal{G})$ is $\mathcal{G}^\omega_g-$open set if and only if $F \subseteq {}_{\mathcal{G}^\omega}Int(A)$ whenever $F \subseteq A$ and F is closed subset of $(X, \tau, \mathcal{G})$.*

Recall [9] that we have the following graph:



3. ON $\mathcal{G}^\omega_G-$CLOSED SETS

THEOREM 10. *If A is $\mathcal{G}^\omega_g-$closed set in a grill topological space $(X, \tau, \mathcal{G})$ and B is a closed set in X then $A \cap B$ is $\mathcal{G}^\omega_g-$closed set.*

PROOF. Let $A \in \mathcal{G}^\omega_g C(X, \tau, \mathcal{G})$ and B be closed set in $(X, \tau, \mathcal{G})$. Let U be an open set in (X, τ) such that $A \cap B \subseteq U$. Since B is closed set. Then $A \subseteq U \cup (X - B) \in \tau$. Since $A \in \mathcal{G}^\omega_g C(X, \tau, \mathcal{G})$ and $A \subseteq U \cup (X - B) \in \tau$ then ${}_{\mathcal{G}^\omega}Cl(A) \subseteq U \cup (X - B)$. Since ${}_{\mathcal{G}^\omega}Cl(A \cap B) \subseteq {}_{\mathcal{G}^\omega}Cl(A) \cap {}_{\mathcal{G}^\omega}Cl(B)$ and ${}_{\mathcal{G}^\omega}Cl(A) \cap {}_{\mathcal{G}^\omega}Cl(B) \subseteq {}_{\mathcal{G}^\omega}Cl(A) \cap Cl(B)$. Then ${}_{\mathcal{G}^\omega}Cl(A \cap B) \subseteq {}_{\mathcal{G}^\omega}Cl(A) \cap Cl(B)$. Therefor

$$\begin{aligned} {}_{\mathcal{G}^\omega}Cl(A \cap B) \subseteq {}_{\mathcal{G}^\omega}Cl(A) \cap Cl(B) &= {}_{\mathcal{G}^\omega}Cl(A) \cap B \subseteq [U \cup (X - B)] \cap \\ &\subseteq U \cap B \subseteq U. \end{aligned}$$

Hence $A \cap B \in \mathcal{G}^\omega_g C(X, \tau, \mathcal{G})$. □

THEOREM 11. *Let Y be an open subspace of a grill topological space $(X, \tau, \mathcal{G})$ and $A \subseteq Y$. If A is $\mathcal{G}^\omega_g-$closed subset in X then A is $\mathcal{G}^\omega_g-$closed set in Y.*

PROOF. Let O be an open subset in Y such that $A \subseteq O$. Then $O = U \cap Y$ for some open set U in X and so $A \subseteq U$. Since A is $\mathcal{G}^\omega_g-$closed subset of X, then ${}_{\mathcal{G}^\omega}Cl(A) \subseteq U$. By Theorem (3),

$${}_{\mathcal{G}^\omega}Cl|_Y(A) = {}_{\mathcal{G}^\omega}Cl(A) \cap Y \subseteq U \cap Y = O.$$

Hence A is $\mathcal{G}^\omega_g-$closed set in Y. □

THEOREM 12. *Let Y be an open subspace of a grill topological space $(X, \tau, \mathcal{G})$ and $A \subseteq Y$. If A is $\mathcal{G}^\omega_g-$closed subset in Y and Y is $\mathcal{G}^\omega_g-$closed in X then A is $\mathcal{G}^\omega_g-$closed set in X.*

PROOF. Let U be an open subset in X such that $A \subseteq U$. Then $A \subseteq U \cap Y$ and $U \cap Y$ an is open set in Y. Since A is $\mathcal{G}^\omega_g-$closed subset in Y, then ${}_{\mathcal{G}^\omega}Cl|_Y(A) \subseteq U \cap Y$. Since Y is an open set in X and it is $\mathcal{G}^\omega_g-$closed in X then By Theorem (3),

$$\begin{aligned} {}_{\mathcal{G}^\omega}Cl(A) &= {}_{\mathcal{G}^\omega}Cl(A \cap Y) \subseteq {}_{\mathcal{G}^\omega}Cl(A) \cap {}_{\mathcal{G}^\omega}Cl(Y) = {}_{\mathcal{G}^\omega}Cl(A) \cap Y \\ &= {}_{\mathcal{G}^\omega}Cl|_Y(A) \subseteq U \cap Y \subseteq U. \end{aligned}$$

Hence A is $\mathcal{G}^\omega_g-$closed set in X. □

THEOREM 13. *Let A be $\mathcal{G}^\omega-$generalized closed in a grill topological space $(X, \tau, \mathcal{G})$. If $B \subseteq X$ such that $A \subseteq B \subseteq {}_{\mathcal{G}^\omega}Cl(A)$, then $B \in \mathcal{G}^\omega_g C(X, \tau)$.*

PROOF. Let $U \in \tau$ such that $B \subseteq U$. Then $A \subseteq B \subseteq U$. Since $A \in \mathcal{G}^\omega_g C(X, \tau)$, ${}_{\mathcal{G}^\omega}Cl(A) \subseteq {}_{\mathcal{G}^\omega}Cl(B) \subseteq {}_{\mathcal{G}^\omega}Cl({}_{\mathcal{G}^\omega}Cl(A)) = {}_{\mathcal{G}^\omega}Cl(A) \subseteq U$. Hence $B \in \mathcal{G}^\omega_g C(X, \tau)$. □

To show that a intersection of two $\mathcal{G}^\omega_g-$closed sets in grill topological space $(X, \tau, \mathcal{G})$ need not be $\mathcal{G}^\omega_g-$closed set we consider the following example.

EXAMPLE 1. *Let $(\Re, \tau, \mathcal{G})$ be a grill topological space on the set of real numbers $\Re$ with*

$$\tau = \{\emptyset, \Re, \Re - (0, 1)\}$$

and $\mathcal{G} = P(\Re) - \{\emptyset\}$. The sets $A_0 = \Re - (0, \frac{1}{2}]$ and $A_1 = \Re - [\frac{1}{2}, 1)$ are $\mathcal{G}^\omega_g-$closed sets but $A_0 \cap A_1 = \Re - (0, 1)$ is not $\mathcal{G}^\omega_g-$closed set. Since ${}_{\mathcal{G}^\omega}Cl(A_0 \cap A_1) = {}_{\mathcal{G}^\omega}Cl(\Re - (0, 1)) = \Re$ is not subset or equal of $\Re - (0, 1)$.

To show that a union of two $\mathcal{G}^\omega_g-$closed sets in grill topological space $(X, \tau, \mathcal{G})$ need not be $\mathcal{G}^\omega_g-$closed set we consider the following example.

EXAMPLE 2. *In Example(1). The sets* $A_0 = \Re - [0,1)$ *and* $A_1 = \Re - (0,1]$ *are* $\mathcal{G}^{\omega}_{g}-$*closed sets but* $A_0 \cup A_1 = \Re - (0,1)$ *is not* $\mathcal{G}^{\omega}_{g}-$*closed set. because* $\Re - (0,1)$ *is an open set and* $\Re - (0,1) \subseteq \Re - (0,1)$ *but* ${}_{\mathcal{G}^{\omega}}Cl(A_0 \cup A_1) = {}_{\mathcal{G}^{\omega}}Cl(\Re - (0,1)) = \Re$ *is not subset or equal of* $\Re - (0,1)$.

4. $\mathcal{G}^{\omega}_{RG}-$CLOSED SETS

DEFINITION 14. *A subset* A *of a grill topological space* $(X, \tau, \mathcal{G})$ *is called regular generalized* $\mathcal{G}^{\omega}-$*closed set (simply* $\mathcal{G}^{\omega}_{rg}-$*closed) if* ${}_{\mathcal{G}^{\omega}}Cl(A) \subseteq U$ *whenever* $A \subseteq U$ *and* U *is regular−open subset of* $(X, \tau, \mathcal{G})$*. The compliment of* $\mathcal{G}^{\omega}_{rg}-$*closed set is called regular generalized* $\mathcal{G}^{\omega}-$*open set (simply* $\mathcal{G}^{\omega}_{rg}-$*open).*

In any grill topological space $(X, \tau, \mathcal{G})$ we denote of all regular generalized $\mathcal{G}^{\omega}-$closed subset by $\mathcal{G}^{\omega}_{rg}C(X, \tau, \mathcal{G})$ and denote of all regular generalized $\mathcal{G}^{\omega}-$open subset by $\mathcal{G}^{\omega}_{rg}O(X, \tau, \mathcal{G})$. Observe in any grill topological space $(X, \tau, \mathcal{G})$ with a countable set X that $\mathcal{G}^{\omega}_{rg}C(X, \tau, \mathcal{G}) = \mathcal{G}^{\omega}_{rg}O(X, \tau, \mathcal{G}) = P(X)$.

EXAMPLE 3. *Let* $(\Re, \tau, \mathcal{G})$ *be a grill topological space on the set of real numbers* $\Re$ *with* $\tau = \{\emptyset, \Re, \{1,2\}\}$ *and* $\mathcal{G} = P(\Re) - \{\emptyset\}$*. Any subset of* $\Re$ *is a both* $\mathcal{G}^{\omega}_{rg}-$*closed set and* $\mathcal{G}^{\omega}_{rg}-$*open set.*

EXAMPLE 4. *Any subset of a grill topological space* $(X, \tau, \mathcal{G})$ *with a countable set* X *is a both* $\mathcal{G}^{\omega}_{rg}-$*closed set and* $\mathcal{G}^{\omega}_{rg}-$*open set.*

THEOREM 15. *Every* $\mathcal{G}^{\omega}_{g}-$*closed set is* $\mathcal{G}^{\omega}_{rg}-$*closed set.*

PROOF. Let A be a $\mathcal{G}^{\omega}_{g}-$closed subset of a grill topological space $(X, \tau, \mathcal{G})$ and $A \subseteq U$, where U is any regular open set in X. Since A is $\mathcal{G}^{\omega}_{g}-$closed set and U is an open set then

$$A = {}_{\mathcal{G}^{\omega}}Cl(A) \subseteq U.$$

That is, A is $\mathcal{G}^{\omega}_{rg}-$closed set. □

The converse of above theorem no need to be true.

EXAMPLE 5. *Let* $(\Re, \tau, \mathcal{G})$ *be a grill topological space on the set of real numbers* $\Re$ *with* $\tau = \{\emptyset, \Re, (1,2)\}$ *and* $\mathcal{G} = P(\Re) - \{\emptyset\}$*. The set* $(1,2)$ *is* $\mathcal{G}^{\omega}_{rg}-$*closed set and not* $\mathcal{G}^{\omega}_{g}-$*closed.*

THEOREM 16. *Every* $rg\omega-$*closed set is* $\mathcal{G}^{\omega}_{rg}-$*closed set.*

PROOF. Let A be a $rg\omega-$closed subset of grill topological space $(X, \tau, \mathcal{G})$ and $A \subseteq U$, where U is any regular open set in X. Since A is a $rg\omega-$closed set, then

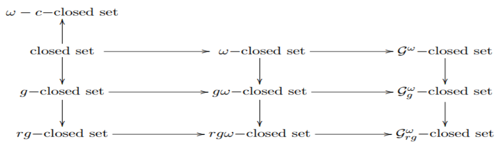
$$A = Cl_{\omega}(A) \subseteq U.$$

Since ${}_{\mathcal{G}^{\omega}}Cl(A) \subseteq Cl_{\omega}(A)$, then A is $\mathcal{G}^{\omega}_{rg}-$closed set. □

The converse of above theorem no need to be true.

EXAMPLE 6. *Let* $(\Re, \tau, \mathcal{G})$ *be a grill topological space on the set of real numbers* $\Re$ *with* $\tau = \{\emptyset, \Re, (0,1), [2,3], (0,1) \cup [2,3]\}$ *and* $\mathcal{G} = P(\Re) - \{\emptyset\}$*. Take* $A = (\frac{1}{3}, \frac{1}{2})$*. Since* $\Re - (\frac{1}{3}, \frac{1}{2})$ *is* $\mathcal{G}^{\omega}-$*open set then* A *is* $\mathcal{G}^{\omega}_{rg}-$*closed set but it is not* $rg\omega-$*closed set because* $(0,1)$ *is regular−open set and* $(\frac{1}{3}, \frac{1}{2}) \subseteq (0,1)$ *but* $Cl_{\omega}(A) = \Re - [2,3]$ *is not subset or equal of* $(0,1)$.

The following figure is an enlargement of figure from [1], figure(2) and theorems above we have the following figure:



THEOREM 17. *A subset* A *of grill topological space* $(X, \tau, \mathcal{G})$ *is* $\mathcal{G}^{\omega}_{rg}-$*open set if and only if* $F \subseteq {}_{\mathcal{G}^{\omega}}Int(A)$ *whenever* F *is regular closed set in* X *and* $F \subseteq A$.

PROOF. Let A be a $\mathcal{G}^{\omega}_{rg}-$open subset of X and let F be regular closed subset of X such that $F \subseteq A$. Then $X - A$ is $\mathcal{G}^{\omega}_{rg}-$closed set and $X - A \subseteq X - F$. Since $X - A$ is $\mathcal{G}^{\omega}_{rg}-$closed set, then

$$X - {}_{\mathcal{G}^{\omega}}Int(A) = {}_{\mathcal{G}^{\omega}}Cl(X - A) \subseteq X - F.$$

That is, $F \subseteq {}_{\mathcal{G}^{\omega}}Int(A)$.

Conversely, let U be any regular open subset such that $X - A \subseteq U$. Then we have $X - U \subseteq A$. That is,

$$X - {}_{\mathcal{G}^{\omega}}Int(A) = {}_{\mathcal{G}^{\omega}}Cl(X - A) \subseteq U.$$

Therefore $X - A$ is $\mathcal{G}^{\omega}_{rg}-$closed set. □

THEOREM 18. *Let* A *be a* $\mathcal{G}^{\omega}_{rg}-$*closed subset of a grill topological space* $(X, \tau, \mathcal{G})$*. Then* ${}_{\mathcal{G}^{\omega}}Cl(A) - A$ *does not contain any nonempty regular closed set.*

PROOF. Let F be a regular closed subset of grill topological space$(X, \tau, \mathcal{G})$ such that $F \subseteq {}_{\mathcal{G}^{\omega}}Cl(A) - A$. Then $F \subseteq X - A$ and hence $A \subseteq X - F$. Since A is $\mathcal{G}^{\omega}_{rg}-$closed set and $X - F$ is regular open subset of $(X, \tau, \mathcal{G})$, ${}_{\mathcal{G}^{\omega}}Cl(A) \subseteq X - F$ and so $F \subseteq X - {}_{\mathcal{G}^{\omega}}Cl(A)$. Therefore $F \subseteq {}_{\mathcal{G}^{\omega}}Cl(A) \cap (X - {}_{\mathcal{G}^{\omega}}Cl(A)) = \emptyset$. □

THEOREM 19. *Let* A *be a* $\mathcal{G}^{\omega}_{rg}-$*closed subset of grill topological space* $(X, \tau, \mathcal{G})$*. If* $B \subseteq X$ *such that* $A \subseteq B \subseteq {}_{\mathcal{G}^{\omega}}Cl(A)$*, then* B *is* $\mathcal{G}^{\omega}_{rg}-$*closed set.*

PROOF. Let B be a subset of grill topological space $(X, \tau, \mathcal{G})$ and let A be a $\mathcal{G}^{\omega}_{rg}-$open subset such that ${}_{\mathcal{G}^{\omega}}Int(A) \subseteq B \subseteq A$. Then B is $\mathcal{G}^{\omega}_{rg}-$open set. □

THEOREM 20. *Let* A *be a* $\mathcal{G}^{\omega}_{rg}-$*closed set in grill topological space* $(X, \tau, \mathcal{G})$*. Then* $A = {}_{\mathcal{G}^{\omega}}Cl({}_{\mathcal{G}^{\omega}}Int(A))$ *if and only if* ${}_{\mathcal{G}^{\omega}}Cl({}_{\mathcal{G}^{\omega}}Int(A)) - A$ *is regular closed set.*

PROOF. If $A = {}_{\mathcal{G}^{\omega}}Cl({}_{\mathcal{G}^{\omega}}Int(A))$, then ${}_{\mathcal{G}^{\omega}}Cl({}_{\mathcal{G}^{\omega}}Int(A)) - A = \emptyset$ and hence ${}_{\mathcal{G}^{\omega}}Cl({}_{\mathcal{G}^{\omega}}Int(A)) - A$ is regular closed.

Conversely, let ${}_{\mathcal{G}^{\omega}}Cl({}_{\mathcal{G}^{\omega}}Int(A)) - A$ be regular closed, since ${}_{\mathcal{G}^{\omega}}Cl(A) - A$ contains the regular closed set ${}_{\mathcal{G}_{\omega}}Cl({}_{\mathcal{G}^{\omega}}IntA)) - A$. By Theorem(18), ${}_{\mathcal{G}^{\omega}}Cl({}_{\mathcal{G}^{\omega}}Int(A)) - A = \emptyset$ and hence $A = {}_{\mathcal{G}^{\omega}}Cl({}_{\mathcal{G}^{\omega}}Int(A))$.

□

THEOREM 21. *If* A *be a* $\mathcal{G}^{\omega}_{rg}-$*closed subset of* $(X, \tau, \mathcal{G})$*, then* ${}_{\mathcal{G}^{\omega}}Cl(A) - A$ *is* $\mathcal{G}^{\omega}_{rg}-$*open set.*

PROOF. Let A be a $\mathcal{G}^{\omega}_{rg}-$closed subset of (X, τ) and let F be a regular closed subset such that $F \subseteq {}_{\mathcal{G}^{\omega}}Cl(A) - A$. By Theorem(18), $F = \emptyset$ and thus $F \subseteq {}_{\mathcal{G}^{\omega}}Int({}_{\mathcal{G}^{\omega}}Cl(A))$. By Theorem 4.8, ${}_{\mathcal{G}^{\omega}}Cl(A) - A$ is $\mathcal{G}^{\omega}_{rg}-$open set. □

To show that a intersection of two $\mathcal{G}^{\omega}_{rg}-$closed sets in grill topological space $(X, \tau, \mathcal{G})$ need not be $\mathcal{G}^{\omega}_{rg}-$closed set we consider the following example.

EXAMPLE 7. *Let* $(\Re, \tau, \mathcal{G})$ *be a grill topological space on the set of real numbers* $\Re$ *with*

$$\tau = \{\emptyset, \Re, \Re-(0,1], \{\frac{1}{2}\}, (Re-(0,1])\cup\{\frac{1}{2}\}, \Re-(0,1), (\Re-(0,1))\cup\{\frac{1}{2}\}\}$$

and $\mathcal{G} = P(\Re) - \{\emptyset\}$. *The sets* $A = \Re - (0, \frac{1}{2}]$ *and* $B = \Re - [\frac{1}{2}, 1]$ *are* $\mathcal{G}^{\omega}_{rg}-$*closed sets but* $A \cap B = \Re - (0,1]$ *is not* $\mathcal{G}^{\omega}_{rg}-$*closed set. Since* $\Re - (0,1)$ *is regular−open set,* $\Re - (0,1] \subseteq \Re - (0,1)$ *and* ${}_{\mathcal{G}^\omega}Cl(A \cap B) = {}_{\mathcal{G}^\omega}Cl(\Re - (0,1]) = \Re - \{\frac{1}{2}\}$ *is not subset or equal of* $\Re - (0,1)$.

To show that a union of two $\mathcal{G}^{\omega}_{rg}-$closed sets in grill topological space $(X, \tau, \mathcal{G})$ need not be $\mathcal{G}^{\omega}_{rg}-$closed set we consider the following example.

EXAMPLE 8. *In Example(7). The sets* $A = \Re - [0,1]$ *and* $B = \Re - (0,1] \cup \{2\}$ *are* $\mathcal{G}^{\omega}_{rg}-$*closed sets but* $A \cup B = \Re - (0,1]$ *is not* $\mathcal{G}^{\omega}_{rg}-$*closed set. Since* $\Re - (0,1)$ *is regular−open set,* $\Re - (0,1] \subseteq \Re - (0,1)$ *and* ${}_{\mathcal{G}^\omega}Cl(A \cup B) = {}_{\mathcal{G}^\omega}Cl(\Re - (0,1]) = \Re - \{\frac{1}{2}\}$ *is not subset or equal of* $\Re - (0,1)$.

5. CONCLUSION

The notion of $\mathcal{G}^{\omega}_{rg}-$closed sets is developing to the notion of $\mathcal{G}^{\omega}_{g}-$closed sets in grill topological spaces and this notion of $\mathcal{G}^{\omega}_{rg}-$closed sets is developing to the notion of $rg\omega-$closed sets in topological spaces. The fundamental notions in topological spaces such as the continuity, connectedness, compactness and separation axioms also may be introduced and investigated by using the class $\mathcal{G}^{\omega}_{rg}-$closed set and also by using the class $\mathcal{G}^{\omega}_{g}-$closed set in grill topological spaces.

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