N-ary Huffman Encoding using High-Degree Trees - A Performance Comparison

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ABSTRACT

In this paper an *n*-ary Huffman Encoding and Decoding application is implemented, using different degrees of tree structures. The goal is to compare the performance of the algorithm in terms of compression ratio, decompression speed and weighted path length when using higher degree trees, compared to the 2-ary Huffman Code. The Huffman tree degrees compared are 2-ary, 3-ary, 4-ary, 5-ary, 6-ary, 7-ary, 8-ary and 16-mal. The impact that branch prediction has on the performance of the *n*-ary Huffman Decoding is also presented.

General Terms

Information theory, Compression, Decompression, Data Communications

Keywords

Huffman Encoding, Decoding Speed, Compression Ratio, Octernary Huffman, Hexadecimal Huffman, Branch Prediction Rate

1. INTRODUCTION

In 1952, David A. Huffman proposed a new lossless compression technique [1] that allowed for "the construction of minimum-redundancy codes". In this paper he described an algorithm that encoded symbols from a message to be transmitted in such a way that symbols with higher frequency of appearance were encoded by shorter length sequences than symbols with lower frequency of appearance. The proposed algorithm created uniquely decodable variable length prefix-free codes.

Huffman Encoding utilizes the tree data structure to extract the code sequences for each distinct symbol that appears in the source message. In his paper, Huffman utilizes the 2-ary tree to showcase his proposed technique and offers a generalization of the process for higher degree trees (n > 2).

Extensive research has also been conducted on improving upon Huffman's Algorithm and implementing it in combination with other techniques to achieve higher compression ratios and encoding/decoding speeds, as well as extending its application to non-text data.

P. Suri and M. Goel [2], [3] described the implementation of the Huffman coding technique using ternary trees instead of binary, as well as a variation of it [4], the FGK Adaptive Huffman Algorithm.

R. Hashemian [5] proposes a method for clustering Huffman trees in order to achieve high efficiency in memory space as well as high-speed access to symbols.

Habib and Rahman [6] implemented the Huffman algorithm using the quaternary Huffman tree and compared the

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decompression rate between binary and quaternary trees; in their paper they concluded that quaternary trees offer much higher decoding speed compared to binary trees while resulting in negligibly smaller compression ratio.

In 2018, Habib et al. [7] published a paper on tribit (octernary tree) and quadbit (hexadecimal tree) Huffman Encoding and compared it with previously proposed dualbit (quaternary tree) and the Zopfli algorithm [8]. They did not coincide with the experimentation results presented in this paper. More on this topic will be discussed in chapter 4.

In this paper, an encoder/decoder is created, that utilizes the *n*-ary Huffman Encoding Algorithm for $n = \{2, 3, 4, 5, 6, 7, 8, 16\}$, in order to compare the performance for the various values of *n*. The application has been developed using the C programming language, focusing on the encoding and decoding of text data.

A term used in the following is the branch misprediction rate. In computer architecture, a branch predictor is a digital circuit that attempts to guess whether a branch-type instruction will be taken or not. This is implemented in pipelined processor architectures in order to allow speculative execution of parts of code that normally the processor would need to wait before initiating execution. In the case that the branch predictor does not guess correctly that a branch will be taken or not taken, we call this a branch prediction miss. Branch prediction misses cause considerable slowdown to the execution of routines. The ratio of branch prediction misses to the total branch predictions is called the Branch Misprediction Rate (BMR).

The effect of branch prediction on binary decision trees and specifically in Huffman trees has been noted in recent literature. However, sparse research has been conducted regarding the effects of branch prediction in decoding symbols that were encoded using the Huffman algorithm or variations of it.

In 2007, Baer presented [9] a theoretical model on the effects of static branch prediction in optimal alphabetic binary trees on microprocessor architectures such as the ARM CPU; in this paper an example of ONE-SHIFT Huffman Encoding [10] is used to showcase the proposed model.

In 2014, C. Jeong et al. [11] presented a method for improved branch prediction in Huffman Decoding for streams of data via elimination of indirect branching instructions.

Another important term is Weighted Path Length (WPL). WPL is a characteristic of *n*-ary tree structures. Given an *n*-ary tree *T*, with K leaf nodes and each leaf node an assigned weight value w_i and depth d_i the weighted path length is defined as the following:

WPL(T) =
$$\sum_{i=1}^{K} w_i * d_i$$
 (1)

where:

- T is the input tree
- w_i is the weight of a leaf node i
- d_i is the depth of a leaf node i
- K is the total number of leaf nodes

In the case of *n*-ary Huffman trees, the weighted value w_i associated with each leaf node is the number of occurrences of a symbol associated with the corresponding leaf node in a text. This value basically expresses the number of transitions from one node to another (including both external and internal nodes in the tree) that will occur when the decoding process takes place.

The key values, measured and compared in this paper, between the different degrees of Huffman trees are: the compression ratio, the decompression speed, the weighted path length of the tree structure and the branch misprediction rate of the application. Comparison of the encoding speed is not the main focus of this paper, however encoding speed measurements will also be presented.

2. THE n-ARY HUFFMAN ENCODING AND DECODING ALGORITHMS

In the following are presented the encoding and decoding algorithms for an arbitrary n; the algorithms do not depend on n.

2.1 *n*-ary Huffman Encoding algorithm

- First, a list of the distinct symbols and the frequency of appearance of each symbol in a message, say M, is created. Each symbol corresponds to a leaf node of a tree T, which grows up as described in the following.
- If the degree of the Huffman Encoder is greater than 2 (n > 2), a number (N_{pl}) of placeholder 0-probability leaf nodes must be used, in order for *T* to become a complete tree. This number is given by:

$$N_{pl} = (n-2) - ((S_{dst} + (n-3)) \mod (n-1))$$
(2)

where:

 N_{pl} is the number of placeholder nodes

 S_{dst} is the number of distinct symbols in M

n is the degree of the Huffman tree

The total number of leaf nodes of T is given by the sum $N_{pl}+S_{dsl}$.

- Using the *n* nodes with the lowest frequency of appearance, a new parent node is created, whose children are the *n* nodes mentioned. This new parent node will be assigned a frequency equal to the sum of frequencies of its children and, according to it, takes its place in the list of symbols; so, newly created internal nodes are taken into consideration when searching for the lowest frequency nodes. Nodes are continuously joined into a single node until all leaf nodes have been linked to a parent node. The final created internal node is the root of the Huffman tree *T*.
- As mentioned above, each internal node has a total number of *n* children nodes. For each internal node, the value i-1 in binary form is assigned to each edge leading to the i-th child node. So, for a 5-ary tree structure, all internal nodes will have the value '000' assigned to the edge leading to their left-most child node, the value '001' to the second child node from left to right and so on. It's obvious that, as many binary digits are used, as required to represent the full range of values, in order to ensure that the symbols remain uniquely decodable. The number b of digits needed is given by:

$$\mathbf{b} = \operatorname{ceil}(\log_2 n) \tag{3}$$

where:

b is the number of bits needed n is the degree of the Huffman tree utilized

For example, for n = 3, b = ceil(1.584) = 2.

Starting from the root node of T and following the path to each leaf node, the Huffman code is created for each symbol and consequently the Huffman table, containing every symbol and the corresponding code to it. Huffman codes leading to placeholder nodes are ignored. This way, the most probable symbols in M will be coded using fewer bits.

• Each symbol in *M* is replaced with its Huffman code sequence. Every 8 bits of gathered data are converted to a byte which is placed in a buffer. Special provision must be taken for the last bits gathered, since they will probably not complete a byte. In such a case, extra bits are added as padding, in order to complete a byte and the result is placed in the buffer. Obviously, it is very important for the decoder to know how many extra bits were added as padding, in order to omit them during the decoding process of the final encoded symbol.

The contents of the buffer, which, due to the coding used, consist of a compressed form of the original data, is the main information to be sent to the decoder. However, redundant information is needed to be transmitted. The structure of the final encoded message E, which includes the "useful" data and the redundant information, mentioned above, constitute a file whose structure is presented in Figure 1.

1 byte	1 byte	4 bytes	4 bytes	1 byte	Variable	Variable
Tree Degree	Extra Bits	Initial File Size	Compressed File Size	Number of Table Entries	Encoded Symbols	Huffman Table

Fig. 1: The structure of the transmitted file

The fields of this file are:

- i. The degree *n* of *T*.
- ii. The number of extra bits we added at the end of the encoding.
- iii. The number of bytes of the original file.
- iv. The number of bytes of the compressed file.
- v. The number of Huffman table entries.
- vi. The encoded symbols themselves.
- vii. The Huffman table.

2.2. *n*-ary Huffman Decoding algorithm

- Construct the *n*-ary Huffman tree from the Huffman table retrieved from the encoded message *E*. *n* is retrieved too.
- Starting from the root node of the constructed Huffman tree, read every b bits of the encoded message and navigate to the appropriate child node denoted from the value read. The number of b bits need to read, depends on the degree of Huffman tree used when the message was encoded and is given from equation 3. This is done for all encoded symbols, except the last one.
- When the last encoded symbol is reached, any bits, that may have been included at the end of the encoding process, are removed. Once again, this number is known, since it is included in the message during encoding.

2.3. Example of 16-mal Huffman Encoding-Decoding

2-ary Huffman Encoding methodology is well established. We showcase an example of *n*-ary Huffman Encoding and Decoding using tree degree n = 16. Suppose we want to encode the message "Mississippi River".

2.3.1. 16-mal Huffman Encoding Example

• List all symbols appearing in the message together with their corresponding frequency, as in Table 1.

Table 1. Symbol dist	ribution table
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Symbol	Absolute Frequency
i	5
S	4
р	2
R	1
М	1
r	1
e	1
v	1
SPACE	1
Total	17

- Starting tree degree is n > 2, therefore we use equation 2 to calculate the number N_{pl} of the placeholder nodes needed. $N_{pl} = (16 - 2) - ((9 + (16 - 3)) \mod (16 - 1)) = 7$
- So, 7 placeholder nodes are needed.
- Take the 16 nodes with the lowest frequency of appearance and link them into 1 new node, as in Figure 2. (PH nodes are the placeholders). The new node's frequency value is the sum of the frequencies of its children nodes. In this example, we do this only once (only 16 leaf nodes).

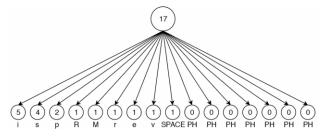


Fig. 2. Fully linked 16-mal Huffman tree

• Assign values to the edges of every parent node of the tree (see Figure 3). Starting from the leftmost edge assign values 0-15 (in binary form) using for each the appropriate amount of digits. Using equation 3, this number b is:

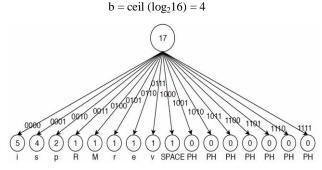


Fig. 3. 16-mal Huffman tree with binary values assigned to edges

• Following the path from the root node to all leaf nodes (ignoring the placeholder nodes), the following Huffman table

(Table 2) is obtained. This table is included in the final compressed and transmitted file.

Table 2. 16-mal	Huffman	table fo	or the	message	"Mississippi
		River '	,	-	

Symbol	Huffman Code
i	0000
S	0001
р	0010
R	0011
М	0100
r	0101
e	0110
V	0111
SPACE	1000

• Next, each symbol has to be replaced with its equivalent Huffman Code, converting every 8 bits to a byte value. Especially for the last symbol, there are 4 bits less than the necessary, in order to create the final 8-bit value, so the sequence is completed with four 0's, as seen in Figure 4. The number of extra added bits is included in the final transmitted file.

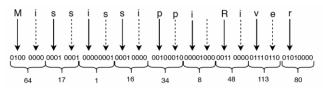


Fig. 4. Encoded message with zero-fill bits added

Final message size is 9 bytes. The comparison of it to the initial message, whose size is 17 bytes, gives a compression ratio of 17/9 = 1.88. Of course, since no predefined dictionaries are used, the Huffman table we be included in the final message, as well as other information about the encoding, such as the tree degree used and the extra bits added at the last encoded symbol. Practically, this means that for small text messages, Huffman Encoding without dictionaries, actually yields larger files than the original.

2.3.2. 16-mal Huffman Decoding Example

- Read the Huffman table from the compressed file and construct the 16-mal tree, using it as reference. The constructed tree is not necessarily a full Huffman tree, since it won't include the placeholder nodes, but this does not influence the decompression process.
- Starting from the root node, read from the encoded message b bits at a time, navigating to the node's children from left to right. When reaching a leaf node, move back to the root node and continue reading data. This is done for all the encoded symbols, except the last one.
- Read the number of extra bits, added during encoding of the message. In this case, 4 bits were added; therefore omit the 4 last bits of the last encoded symbol (see Figure 5).

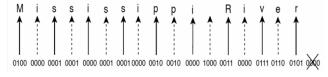


Fig. 5. Decoded Message. Last 4 bits are omitted

3. SYSTEM IMPLEMENTATION AND EVALUATION

3.1. Development Tools

The development platform was a Linux Ubuntu OS ver. 16.04.2 LTS computer, running on an Intel i3-3110M Quad-Core Processor clocked @ 2.4GHz. We developed the application to run in single-threaded mode. In order to measure the encoding and decoding speed the clock_gettime function was used, with the MONOTONIC_CLOCK argument, which allows for measurements with nanosecond precision. Moreover, to achieve increased accuracy, the measurements were repeated 100 times when decoding, and the average key values were extracted (runtime, throughput, misprediction rate).

In the following, measurements are presented, using the encoding/decoding application for a variety of degrees of Huffman trees, namely 2-ary, 3-ary, 4-ary, 5-ary, 6-ary, 7-ary, 8-ary and 16-mal. Specifically, the compression speed, decompression speed, weighted path length, branch misprediction rate and compression ratio for each of the above mentioned Huffman tree degrees are presented. The dataset used, consists of a single text-only file containing randomly generated words of the English language.

During decompression, a fluctuation in performance across the different Huffman tree degrees utilized was noticed. We suspected branch misprediction to be the cause of this fluctuation, thus the perf Linux command utility was used to measure the branch misprediction rate that occurred during the execution of decompressing application for the data set text files and for all tree degrees implemented. There was also implemented a Huffman decoder version of all tree degrees with minimal branch instructions, in order to eliminate branch misprediction as a performance variable.

In all cases, the -O3 optimization gcc argument was used when compiling the application. All variations were developed to run in single-threaded mode.

3.2. Dataset Acquisition

An online random word generator [12] was used to generate the text, consisting of 1388 Kbytes. Fig. 6 shows the letter frequency distribution of the sample text file used, together with that of the letters in the English language. As expected of random word generation, the frequencies of the letters in the randomly generated text closely follow these of the English language, reflecting the real-world data distribution. The reference frequency values were obtained from [13].

In the following, various quantities are presented, according to tree degree. These quantities are compression time and compression throughput, decompression time, decompression throughput and misprediction rate, as well as compression ratio, i.e. the ratio of the original file's size to the compressed file's size.

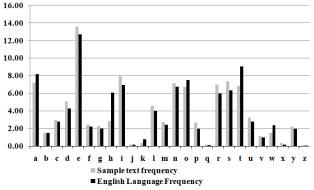


Fig. 6. Sample text data and English language characters (Percentage frequency distribution)

3.3. System Benchmark Results

3.3.1. Compression time and throughput

Compression time according to the tree degree utilized for the randomly generated text is presented in Fig. 7. It is obvious that lower values are better.

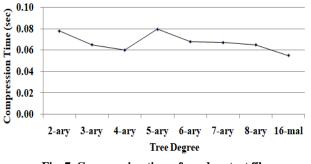


Fig. 7. Compression time of random text file

We can see that hexadecimal (16-mal) Huffman encoding offers the highest compression throughput, while quinary (5-ary) Huffman Encoding offers the lowest one. In Table 3, compression time appears together with compression throughput.

Table 3. Compression time and throughput of random text

1110.				
Tree Degree	Compression Time (sec)	Compression Throughput (Mbytes/sec)		
2-ary	0.0780	17.37		
3-ary	0.0651	20.82		
4-ary	0.0600	22.59		
5-ary	0.0797	17.00		
6-ary	0.0680	19.93		
7-ary	0.0671	20.20		
8-ary	0.0650	20.85		
16-mal	0.0550	24.64		

3.3.2. Decompression time, decompression throughput and misprediction rate.

Decompression time according to the tree degree for the randomly generated text is presented in Fig. 8 while in Fig. 9 misprediction rate according to the tree degree is presented for the same file. In both, lower values are better.

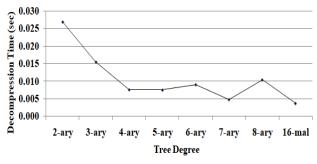


Fig. 8. Decompression time of random text file

In Table 4 and

Table 5, decompression time is presented, together with decompression throughput, misprediction rate and weighted path length. Throughput is calculated according to original file size. File I/O is not taken into account when measuring execution time

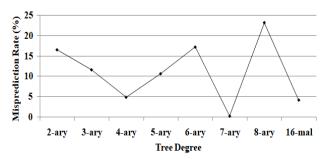


Fig. 9. Misprediction (%) rate for random text file

It is evident that a correlation exists between decompression times and misprediction rates for each Huffman tree structure, that is, low misprediction rate corresponds to short decompression time. Fastest decompression is achieved by 16mal tree with 7-ary being a close second, while 2-ary offers the slowest decompression speed.

Table 4. Decompression time and decompressionthroughput of random text file.

Tree Degree	Decompression Time (sec)	Decompression Throughput (Mbytes/sec)
2-ary	0.0270	50.22
3-ary	0.0156	87.09
4-ary	0.0077	176.65
5-ary	0.0076	178.45
6-ary	0.0091	148.90
7-ary	0.0049	278.10
8-ary	0.0105	129.33
16-mal	0.0038	359.16

Table 5. Misprediction rate and weighted path length for random text file

Tree Degree	Misprediction Rate (%)	Weighted Path Length
2-ary	16.48	6271254
3-ary	11.61	4037994
4-ary	4.92	3201120
5-ary	10.65	2783880
6-ary	17.15	2533878
7-ary	0.32	2369376
8-ary	23.09	2251044

16-mal	4.23	1668618
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3.3.3. Compression Ratio.

Compression ratio is the ratio of the original file's size to the compressed file's size. For example, if a file's size is 100 bytes and compressing reduces it to 40 bytes, the compression ratio is 2.50. Compression ratio is presented in Figure 10 (higher values are better) and Table 6, in correspondence to tree degree. Binary Huffman Encoding offers the best compression ratio, while the worst one is achieved by the 5-ary Huffman Encoding.

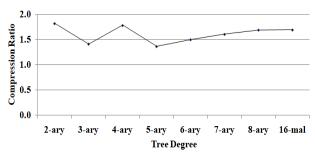


Fig. 10. Compression ratio for different tree degrees

Table 6. Compression Ratio of English language text file (1388 Kbytes) for each Huffman tree degree

Tree Degree	Compression Ratio	Compressed Size (Kbytes)
2-ary	1.824	761
3-ary	1.416	980
4-ary	1.786	777
5-ary	1.369	1014
6-ary	1.504	923
7-ary	1.610	862
8-ary	1.695	819
16-mal	1.697	818

3.3.4. Decompression measurements with minimal branch instructions.

In this chapter, in Table 7 and Figure 11, the performance of the modified decoding is application is presented, where the majority of branching instructions is removed, in order to deduce whether the branch prediction circuits of the Intel i3 processor impact the decoding performance of the application.

 Table 7. Decompression time, throughput and weighted path

 length of English language text, minimal branching

instructions.					
Tree Degree	Decompression Time (sec)	Decompression Throughput (Mbytes/sec)	Weighted Path Length		
2-ary	0.02817	48.12	6271254		
3-ary	0.01867	72.62	4037994		
4-ary	0.01474	91.94	3201120		
5-ary	0.01284	105.54	2783880		
6-ary	0.01166	116.24	2533878		
7-ary	0.01091	124.23	2369376		
8-ary	0.01037	130.69	2251044		
16-mal	0.00781	173.66	1668618		

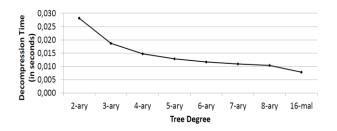


Fig. 11. Decompression time of English language text, minimal branching instructions

4. RESULTS AND DISCUSSION

The findings of [7] showed that utilizing 4-ary, 8-ary and 16mal trees in Huffman Encoding yielded 60% time enhancement compared to the Zopfli Algorithm when applied to the Enwik Corpus [14] and 90% speed improvement when applied to the Canterbury Corpus [15]. This increase was shown to be more or less the same for all 3 Huffman tree degrees.

Experimental data from our research however yielded results that do not coincide with these findings. For example, as the utilized Huffman tree degree was increased, the performance of the decoder changed. In most cases, performance increased but in cases where branch misprediction rate was high it actually decreased. A more consistent, albeit slower, implementation showcases the steady increase of the decoding algorithm's performance as tree degree increases and Weighted Path Length decreases.

As seen from Fig. 10 and Table 6, the 2-ary Huffman Encoding always achieves the best compression ratio when compared to Huffman tree structures with degree greater than 2 (n > 2). This difference tends to be negligible when comparing Huffman tree degrees that are powers of 2, such as 4-ary, 8-ary and 16-ary.

From Fig. 7 and Table 3it can be deduced that encoding data using 3-ary, 4-ary, 8-ary and 16-mal tree structures offer the best overall encoding throughput with mild difference in performance, compared to Huffman tree degrees 2, 5, 6 and 7.

On the other hand, Table 4 shows that *n*-ary Huffman decompression performance is inconsistent. Although WPL and compression ratio dictate the total size of the encoded message and thus the time it takes to parse it when decoding it, our results come to the conclusion that WPL is not the only factor in determining decompression speed. In cases where a higher degree Huffman decompression yields much higher misprediction rate, the benefits of lower WPL will be negated by the high misprediction rate.

For example, in the case of decompressing the language text file using 8-ary Huffman tree, there is a 115% decrease in performance, compared to 7-ary when there is a 5% increase in throughput, since weighted path length of the 8-ary Huffman tree is 5% smaller.

Estimating that branch prediction, coming from specific "if" statements in the application, may influence the performance, an implementation of the Huffman Decoding application was created, that replaces "if" statements and measured the performance.

These results can be seen in Fig. 11 and Table 7. In this case decompression throughput closely follows the values of the Huffman tree's weighted path length. For example, in Table 7 the decompression runtime of the English text file using 4-ary tree is 48% shorter and its weighted path length is 49% smaller than that of the 2-ary Huffman tree. Additionally, the graph in

Fig. 11 shows a more consistent increase in performance as the degree of the Huffman tree used was increased.

5. CONCLUSIONS

The results of this research show that the use of higher degree trees in Huffman Encoding/Decoding can be beneficial in encoding real-world text data, especially in the case of tree degrees that are powers of 2, where both high compression ratios and decompression throughputs are achieved, compared to tree degrees that are not powers of 2.

Higher degree trees would probably not offer much benefit for encoding single-byte text data, such as ASCII. Although decompression throughput may continue to show performance gains, compression ratio is almost certain to begin showing less and less gains due to the simple fact that for n > 16, we require a minimum of 5 bits per encoded symbol to represent.

This is not true however for UTF-8 encoded text data. Since UTF-8 is a variable-width encoding scheme, certain text data may contain symbols that require more than 1 bytes of data to represent. As such, encoding certain text data containing UTF-8 symbols with Huffman trees of degrees greater than 16, may continue to enable ever-increasing decompression throughput while keeping a high compression ratio.

However, even when plain ASCII text data is encoded, there is the option of encoding multiple symbols in a row as a single Huffman code. For example, instead of taking the frequency of single letters in a text before the Huffman tree is created, we could take the frequency of pairs of consecutive letters (digraphs) and encode them in a single Huffman code (e.g. encode the pair of letters 'et' or 'th' etc).

Further research on the effects of high-degree Huffman trees on encoding of non-textual data such as RGB images may result with better results (at least in decompression throughput) compared to utilizing 2-ary Huffman Encoding.

It is also shown that for specific files encoded, the Huffman tree that comes from that file, creates a decoding pattern that is favored by branch prediction circuitry in the CPU executing the decoding, thus resulting in a drastic increase in decoding performance.

Of course, it comes as no surprise that branch misprediction negatively impacts performance. Rather it is noteworthy that the rate of branch misprediction shows seemingly random fluctuations across Huffman tree degrees used. Future research in what leads to high misprediction rate as well as ways to consistently generate Huffman trees that yield low branch misprediction rate can lead to better future implementations of higher degree Huffman Encoding solutions.

6. REFERENCES

- Huffman, D. 1952. A Method for the Construction of Minimum-Redundancy Codes. In Proceedings of the I.R.E. 40 (9), 1098–1101.
- [2] Suri, P. R., and Goel, M. 2008. Ternary Tree & a Coding Technique. International Journal of Computer Science and Network Security 8 (10), 102–107.
- [3] Suri, P. R., and Goel, M., Ternary Tree and Memory -Efficient Huffman Decoding Algorithm. 2011. International Journal of Computer Science Issues 8 (1), 483–489.
- [4] Suri, P. R., and Goel, M. 2009. Ternary Tree & FGK Huffman Coding Technique. International Journal of Computer Science and Network Security 9 (1), 316–324.

- [5] Hashemian, R. 1995. Memory Efficient and High-Speed Search Huffman Coding. IEEE Trans. on Communications 43(10), 2576–2581.
- [6] Habib, A., and Rahman, M. S. 2017. Balancing Decoding Speed and Memory Usage for Huffman Codes Using Quaternary Tree. Applied Informatics 4 (1), 1–15.
- [7] Habib, A., Islam, M. J., and Rahman, M. S. 2018. Huffman Based Code Generation Algorithms: Data Compression Perspectives. Journal of Computer Science 14 (12), 1599–1610.
- [8] Alakuijala, J., and Vandevenne, L. 2013. Data Compression Using Zopfli. http://fh7922mg.bget.ru/ articles/compression/data-compression-using-zopfli.html.
- [9] Baer, M. B. 2007. On Conditional Branches in Optimal Decision Trees. In Proc. IEEE International Symposium on Information Theory, 436–440.
- [10] Moffat, A., and Turpin, A. 1997. On the Implementation

of Minimum Redundancy Prefix Codes. IEEE Trans. on Communications 45 (10), 1200–1207.

- [11] Jeong, C., Hang, S., and Burgstaller, B. 2014. Improved Branch Prediction for Just-in-Time Decompression of Canonical Huffman Bytecode Streams. Frontier and Innovation in Future Computing and Communications. Springer, 719–729.
- [12] Bibakis, V. Random Word Text Generator. http://randomtextgenerator.com/.
- [13] Rodriguez-Clark, D. Frequency Analysis: Breaking the Code. http://crypto.interactive-maths.com/frequency-analysis- breaking-the-code.html.
- [14] Mahoney, M. 2006. About the Test Data. https://cs.fit. edu/~mmahoney/compression/textdata.html.
- [15] Arnold, R., and Bell, T, 1997. A Corpus for the Evaluation of Lossless Compression Algorithms. In Proc. IEEE Data Compression Conference, 201-210.