Stochastic Analysis of Markov Chain Model with different Types of Repair Facility

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ABSTRACT
The paper deals with cost-benefit and reliability analysis of two non-identical units considering all random variables are statistically independent, by using regenerative point technique. Initially, one unit is operative and other unit is warm standby, a single repair facility is always available with the system to perform all repair activities. The failure time distribution of both units taken as an exponential distribution with different parameters whereas all repair time distributions are taken as general. Various important characteristics of reliability like steady-state availability, mean sojourn time, mean time to system failure (MTSF), busy period and cost-benefit of the system have been studied.

Keywords
Two non-identical units, Transition probability, Mean sojourn time, MTSF, Availability, Busy period and Cost-benefit analysis

1. INTRODUCTION
The configuration of the stochastic model is concerned with the development and application of the technique for increasing the system efficiency by reducing the frequency of failures and minimizing the high maintenance cost. The configuration and design of industrial systems such as communication system, satellite system, power plant system mechanical engineering, aeronautical engineering software engineering and gaming systems are more complex to design in the current scenario. Two-unit redundant system model has been analyzed widely in the literature of reliability by several authors, such as El-Said [1], Haggag [2], Mahmoud and Moshref [3] and Kumar *et al.* [4] due to their vital existence in modern business and industries. The reliability of the system may further be enhanced by introducing the concept of repair and preventive maintenance. Malik [5] in the field of stochastic theory analyzed redundant system models under different sets of assumptions such as fails unit need minor or major repair facility, availability of repairman for repair facility, priority and non-priority unit and imperfect switching device etc. by using regenerative point technique. Kumar and Kadyan [6] studied a system of non-identical units with degradation and replacement initially original unit is operative and other is kept as spare in cold standby. The failure time of the units are exponentially distributed whereas the distributions of inspection time, replacement time of the duplicate unit and repair time of the original/duplicate/degraded unit are taken as arbitrary with different probability density functions. Also, Bhardwaj *et al.* [7] discussed a two non-identical system with the concept; the standby system goes under inspection to check the feasibility for its maintenance or replacement after completion of pre-specified time. Mahmoud and Mashrefa [8] deal with the study of the stochastic analysis of a two unit cold standby system considering hardware failure, human error failure and preventive maintenance. In the field of reliability theory, many researchers including Borzadaran and Asadi [9], Sultan and Moshref [10] and Pundir *et al.* [11] have considered two or more units repairable reliability models by using continuous distribution for different types of repair mode.

The present paper deals with two non-identical units have been designed with the concept of single repairman play triple role fault detection, minor repair or major repair of the failed unit. Initially, one unit is operative and other unit is warm standby. If operative unit failed, standby unit comes in operative mode by using switching device and a repairman attend the failed unit immediately to detect whether the failed unit need minor or major repair whose probabilities are fixed as ‘p’ and ‘q’ (p+q=1) after diction of failure type the same repairman start the repair of the failed unit. A repaired unit always works as well as new. The system is considered to be in failure state if both units are in failure mode. The mathematical expression of reliability measures such as steady-state availability, reliability, availability, mean time to system failure (MTSF), busy period in different repair facility and cost benefit function have been derived. The graphical representation shows the behaviors of MTSF, availability and cost benefit at the different sets of values.

2. SYSTEM DESCRIPTION AND ASSUMPTIONS
i) The system consists of two non-identical units. Initially one is operative and other is warm standby.
ii) Each unit has two possible modes: normal (N) and total failure (F).
iii) The failure time distributions are taken exponential distribution with different parameters. Whereas the fault detection, minor repair and major repair time distribution are taken as general.
iv) There is a single repair facility with the system which plays triple role of fault detection, minor repair and major repair**.**
v) As soon as a unit fails, it is attended by the repairman immediately. The repairman detects whether the failed unit need minor repair or major repair whose probabilities are fixed as ‘p’ and ‘q’ (p+q=1).
vi) The switching device used to put the standby unit into operation is always perfect and instantaneous.
vii) The system failure occurs when both the units are in F-mode**.**
viii) A repaired unit always works as good as new.

3. NOTATION AND SYMBOLS

α_1 / α_2	Constant failure rate of operating unit-1/unit-2
$F_1(\bullet)/F_2(\bullet)$	cdf of time to fault detection of a failed unit-1/unit-2
$G_1(\bullet)/G_2(\bullet)$	cdf of time to minor repair of a failed unit-1/unit-2
$H_1(\bullet)/H_2(\bullet)$	cdf of time to major repair of a failed unit-1/unit-2

p	Probability that a failed unit requires minor repair
q	Probability that a failed unit requires major repair
N_o^1 / N_o^2	Unit-1/unit-2 is in Normal (N) mode
N_s^2	Unit-2 is in standby (S) mode
F_d^1 / F_d^2	A unit-1/unit-2 is in total failure mode (F) and is under fault detection
F_r^1/F_r^2	A unit-1/unit-2 is in total failure mode (F) and is under minor repair
F_R^1/F_R^2	A unit-1/unit-2 is in total failure mode (F) and is under major repair
F_w^1 / F_w^2	A unit-1/unit-2 is in total failure mode (F) and is waiting for fault detection
©	Symbol for ordinary convolution.
*, ~	Symbols for Laplace and Laplace-Stieltjes transforms.

Considering the above symbols, we have the following states of the system:

Up States

$$S_0 = \left(N_o^1, N_s^2\right),\qquad S_1 = \left(F_d^1, N_o^2\right),\qquad S_2 = \left(F_r^1, N_o^2\right),$$

$$S_3 = \left(F_R^1, N_o^2\right),\qquad S_7 = \left(N_o^1, F_d^2\right),\qquad S_8 = \left(N_o^1, F_r^2\right),$$

$$S_9 = \left(N_o^1, F_R^2\right)$$

Failed States

$$S_4 = \left(F_d^1, F_w^2\right),\qquad S_5 = \left(F_r^1, F_w^2\right),\qquad S_6 = \left(F_R^1, F_w^2\right),$$

$$S_{10} = \left(F_w^1, F_d^2\right),\qquad S_{11} = \left(F_w^1, F_r^2\right),\qquad S_{12} = \left(F_w^1, F_R^2\right)$$

The transition diagram of the system model is shown in **Figure. 1**

4. TRANSITION PROBABILITIES

Simple probabilistic reasoning, the non-zero elements of $Q_{ij}(t)$ may be obtained in the following manner:

$$Q_{01}(t) = \int_0^t \alpha_1 e^{-\alpha_1 u} du = \left(1 - e^{-\alpha_1 t}\right)$$

Similarly,

$$Q_{12}(t) = \int_0^t p e^{-\alpha_2 u} dF_1(u) \qquad Q_{13}(t) = \int_0^t q e^{-\alpha_2 u} dF_1(u)$$

$$Q_{15}^{(4)}(t) = p\int_0^t \left(1 - e^{-\alpha_2 v}\right) dF_1(v) \qquad Q_{16}^{(4)}(t) = q\int_0^t \left(1 - e^{-\alpha_2 v}\right) dF_1(v)$$

$$Q_{20}(t) = \int_0^t e^{-\alpha_2 u} dG_1(u) \qquad Q_{27}^{(5)}(t) = \int_0^t \left(1 - e^{-\alpha_2 v}\right) dG_1(v)$$

$$Q_{30}(t) = \int_0^t e^{-\alpha_2 u} dH_1(u) \qquad Q_{37}^{(6)}(t) = \int_0^t \left(1 - e^{-\alpha_2 v}\right) dH_1(v)$$

$$Q_{57}(t) = \int_0^t dG_1(u) \qquad Q_{67}(t) = \int_0^t dH_1(u)$$

$$Q_{78}(t) = p\int_0^t e^{-\alpha_1 u} dF_2(u) \qquad Q_{79}(t) = q\int_0^t e^{-\alpha_1 u} dF_2(u)$$

$$Q_{7,11}^{(10)}(t) = p\int_0^t \left(1 - e^{-\alpha_1 v}\right) dF_2(v) \qquad Q_{7,12}^{(10)}(t) = q\int_0^t \left(1 - e^{-\alpha_1 v}\right) dF_2(v)$$

$$Q_{80}(t) = \int_0^t e^{-\alpha_1 u} dG_2(u) \qquad Q_{81}^{(11)}(t) = \int_0^t \left(1 - e^{-\alpha_1 v}\right) dG_2(v)$$

$$Q_{90}(t) = \int_0^t e^{-\alpha_1 u} dH_2(u) \qquad Q_{91}^{(12)}(t) = \int_0^t \left(1 - e^{-\alpha_1 v}\right) dH_2(v)$$

$$Q_{11,1}(t) = \int_0^t dG_2(u) \qquad Q_{12,1}(t) = \int_0^t dH_2(u) \qquad (1\text{-}21)$$

The steady state transition probabilities can be obtained from (1-21) by using:

$$p_{ij} = \lim_{t\to\infty} Q_{ij}(t) \quad \text{and} \quad p_{ij}^{(k)} = \lim_{t\to\infty} Q_{ij}^{(k)}(t)$$

Thus,

$$p_{01} = \int_0^\infty \alpha_1 e^{-\alpha_1 t} dt = 1 \qquad p_{12} = p\tilde{F}_1(\alpha_2)$$

$$p_{13} = q\tilde{F}_1(\alpha_2) \qquad p_{15}^{(4)} = p\left[1 - \tilde{F}_1(\alpha_2)\right]$$

$$p_{16}^{(4)} = q\left[1 - \tilde{F}_1(\alpha_2)\right] \qquad p_{20} = \tilde{G}_1(\alpha_2)$$

$$p_{27}^{(5)} = 1 - \tilde{G}_1(\alpha_2) \qquad p_{30} = \tilde{H}_1(\alpha_2)$$

$$p_{37}^{(6)} = 1 - \tilde{H}_1(\alpha_2) \qquad p_{57} = 1$$

$$p_{67} = 1 \qquad p_{78} = p\tilde{F}_2(\alpha_1)$$

$$p_{79} = q\tilde{F}_2(\alpha_1) \qquad p_{7,11}^{(10)} = p\left[1 - \tilde{F}_2(\alpha_1)\right]$$

$$p_{7,12}^{(10)} = q\left[1 - \tilde{F}_2(\alpha_1)\right] \qquad p_{80} = \tilde{G}_2(\alpha_1)$$

$$p_{81}^{(11)} = 1 - \tilde{G}_2(\alpha_1) \qquad p_{90} = \tilde{H}_2(\alpha_1)$$

$$p_{91}^{(12)} = 1 - \tilde{H}_2(\alpha_1) \qquad p_{11,1} = 1$$

$$p_{12,1} = 1$$

It can be easily verified that

$$p_{01} = p_{57} = p_{67} = p_{11,1} = p_{12,1} = 1$$

$$p_{12} + p_{13} + p_{15}^{(4)} + p_{16}^{(4)} = 1 \qquad p_{20} + p_{27}^{(5)} = 1$$

$$p_{30} + p_{37}^{(6)} = 1 \qquad p_{78} + p_{79} + p_{7,11}^{(10)} + p_{7,12}^{(10)} = 1$$

$$p_{80} + p_{81}^{(11)} = 1 \qquad p_{90} + p_{91}^{(12)} = 1 \qquad (22\text{-}28)$$

4.1 Mean sojourn times

Mean sojourn time Ψ_i in state S_i is defined as the expected time taken by the system in state S_i before transition to any other state.

$$Q_{16}^{(4)}(t) = q\int_0^t \left(1 - e^{-\alpha_2 v}\right) dF_1(v)$$

$$\Psi_i = \int_0^\infty P\left[T_0 > T\right] dt$$

Thus,

$$\Psi_0 = \int_0^\infty e^{-\alpha_1 t} dt = \frac{1}{\alpha_1}$$

Similarly,

$$\Psi_1 = \int_0^\infty e^{-\alpha_2 t}\bar{F}_1(t) dt \qquad \Psi_2 = \int_0^\infty e^{-\alpha_2 t}\bar{G}_1(t) dt$$

$$\Psi_3 = \int_0^\infty e^{-\alpha_2 t}\bar{H}_1(t) dt \qquad \Psi_5 = \int_0^\infty \bar{G}_1(t) dt$$

$$\Psi_6 = \int_0^\infty \bar{H}_1(t) dt \qquad \Psi_7 = \int_0^\infty e^{-\alpha_1 t}\bar{F}_2(t) dt$$

$$\Psi_8 = \int_0^\infty e^{-\alpha_1 t}\bar{G}_2(t)dt \qquad \Psi_9 = \int_0^\infty e^{-\alpha_1 t}\bar{H}_2(t)dt$$

$$\Psi_{11} = \int_0^\infty \bar{G}_2(t)dt \qquad \Psi_{12} = \int_0^\infty \bar{H}_2(t)dt \qquad (29\text{-}39)$$

5. RELIABILITY OF THE SYSTEM AND MEAN TIME TO SYSTEM FAILURE (MTSF)

Let the random variable T_i denotes the time to system failure (TSF) when the system initially starts functioning from state $S_i \in E$. Then the system reliability is given by:

$$R_i(t) = P[T_i > t]$$

We have the following set of convolution equations:

$$R_0(t) = e^{-\alpha_1 t} + \int_0^t q_{01}(u)duR_1(t-u)$$
$$= Z_0(t) + q_{01}(t)©R_1(t)$$

Similarly,

$$R_1(t) = Z_1(t) + q_{12}(t)©R_2(t) + q_{13}(t)©R_3(t)$$
$$R_2(t) = Z_2(t) + q_{20}(t)©R_0(t)$$
$$R_3(t) = Z_3(t) + q_{30}(t)©R_0(t) \qquad (40\text{-}43)$$

Where,

$$Z_1(t) = e^{-\alpha_2 t}\bar{F}_1(t) \qquad Z_2(t) = e^{-\alpha_2 t}\bar{G}_1(t)$$
$$Z_3(t) = e^{-\alpha_2 t}\bar{H}_1(t)$$

Taking the Laplace Transform of the above set of equations (40-43) and the resulting set of algebraic equation for $R_0^*(s)$, we get

$$R_0^*(s) = \frac{Z_0^* + q_{01}^*\left(Z_1^* + q_{12}^*Z_2^* + q_{13}^*Z_3^*\right)}{1 - q_{01}^*q_{12}^*q_{20}^* - q_{01}^*q_{13}^*q_{30}^*} \qquad (44)$$

Taking the Inverse Laplace Transform of (44), we can get reliability of the system for known values of the parameters.

The mean time to system failure (MTSF) is given by:

$$E(T_0) = \lim_{s\to 0} R_0^*(s) = \frac{N_1(0)}{D_1(0)}$$

To determine $N_1(0)$ and $D_1(0)$, we use the results

$Z_i^*(0) = \Psi_i$ and $q_{ij}(0) = p_{ij}$

We get,

$$MTSF = \frac{\Psi_0 + \Psi_1 + p_{12}\Psi_2 + p_{13}\Psi_3}{1 - p_{12}p_{20} - p_{13}p_{30}}$$

6. AVALIBILITY ANALYSIS

Let $A_i(t)$ be the probability that the system is up at epoch t, when initially it starts operation from state $S_i \in E$. By using similar probabilistic arguments, as in case of reliability, we observe the following recurrence relations can be easily developed for $A_i(t)$:

$$A_0(t) = Z_0(t) + q_{01}(t)©A_1(t)$$
$$A_1(t) = Z_1(t) + q_{12}(t)©A_2(t) + q_{13}(t)©A_3(t) + q_{15}^{(4)}(t)©A_5(t) + q_{16}^{(4)}(t)©A_6(t)$$
$$A_2(t) = Z_2(t) + q_{20}(t)©A_0(t) + q_{27}^{(5)}(t)©A_7(t)$$
$$A_3(t) = Z_3(t) + q_{30}(t)©A_0(t) + q_{37}^{(6)}(t)©A_7(t)$$
$$A_5(t) = q_{57}(t)©A_7(t)$$
$$A_6(t) = q_{67}(t)©A_7(t)$$
$$A_7(t) = Z_7(t) + q_{78}(t)©A_8(t) + q_{79}(t)©A_9(t) + q_{7,11}^{(10)}(t)©A_{11}(t) + q_{7,12}^{(10)}(t)©A_{12}(t)$$
$$A_8(t) = Z_8(t) + q_{80}(t)©A_0(t) + q_{81}^{(11)}(t)©A_1(t)$$
$$A_9(t) = Z_9(t) + q_{90}(t)©A_0(t) + q_{91}^{(12)}(t)©A_1(t)$$
$$A_{11}(t) = q_{11,1}(t)©A_1(t)$$
$$A_{12}(t) = q_{12,1}(t)©A_1(t) \qquad (45\text{-}55)$$

Where,

$$Z_7(t) = e^{-\alpha_1 t}\bar{F}_2(t), \qquad Z_2(t) = e^{-\alpha_1 t}\bar{G}_2(t),$$
$$Z_3(t) = e^{-\alpha_1 t}\bar{H}_2(t)$$

Taking Laplace Transforms of the above set of equations (45-55) and solving the resulting set of algebraic equations for $A_0^*(s)$ by Cramers rule, we have

$$A_0^*(s) = \frac{N_2(s)}{D_2(s)}$$

Where,

$$N_2(s) = [1 - (q_{7,11}^{(10)*}q_{11,1}^* + q_{7,12}^{(10)*}q_{12,1}^*)(q_{12}^*q_{27}^{(5)*} + q_{13}^*q_{37}^{(6)*} + q_{15}^{(4)*}q_{57}^* + q_{16}^{(4)*}q_{67}^*) - q_{81}^{(11)*}q_{78}^*(q_{12}^*q_{27}^{(5)*} + q_{13}^*q_{37}^{(6)*} + q_{15}^{(4)*}q_{57}^* + q_{16}^{(4)*}q_{67}^*)$$
$$- q_{91}^{(12)*}q_{79}^*(q_{12}^*q_{27}^{(5)*} + q_{13}^*q_{37}^{(6)*} + q_{15}^{(4)*}q_{57}^* + q_{16}^{(4)*}q_{67}^*)]Z_0^* + q_{01}^*Z_1^*$$
$$+ q_{01}^*q_{12}^*Z_2^* + q_{01}^*q_{13}^*Z_3^* + q_{01}^*q_{12}^*q_{27}^{(5)*}Z_7^* + q_{01}^*q_{78}^*(q_{12}^*q_{27}^{(5)*} + q_{13}^*q_{37}^{(6)*}$$
$$+ q_{15}^{(4)*}q_{57}^* + q_{16}^{(4)*}q_{67}^*)Z_8^* + q_{01}^*q_{79}^*(q_{12}^*q_{27}^{(5)*} + q_{13}^*q_{37}^{(6)*} + q_{15}^{(4)*}q_{57}^*$$
$$+ q_{16}^{(4)*}q_{67}^*)Z_9^*$$

and

$$D_2(s) = [1 - (q_{7,11}^{(10)*}q_{11,1}^* + q_{7,12}^{(10)*}q_{12,1}^*)(q_{12}^*q_{27}^{(5)*} + q_{13}^*q_{37}^{(6)*}$$
$$+ q_{15}^{(4)*}q_{57}^* + q_{16}^{(4)*}q_{67}^*) - q_{81}^{(11)*}q_{78}^*(q_{12}^*q_{27}^{(5)*} + q_{13}^*q_{37}^{(6)*}$$
$$+ q_{15}^{(4)*}q_{57}^* + q_{16}^{(4)*}q_{67}^*) - q_{91}^{(12)*}q_{79}^*(q_{12}^*q_{27}^{(5)*} + q_{13}^*q_{37}^{(6)*}$$
$$+ q_{15}^{(4)*}q_{57}^* + q_{16}^{(4)*}q_{67}^*)] - q_{01}^*q_{12}^*q_{20}^* - q_{01}^*q_{13}^*q_{30}^* - q_{01}^*$$
$$q_{78}^*q_{80}^*(q_{12}^*q_{27}^{(5)*} + q_{13}^*q_{37}^{(6)*} + q_{15}^{(4)*}q_{57}^* + q_{16}^{(4)*}q_{67}^*)$$
$$- q_{01}^*q_{79}^*q_{90}^*(q_{12}^*q_{27}^{(5)*} + q_{13}^*q_{37}^{(6)*} + q_{15}^{(4)*}q_{57}^* + q_{16}^{(4)*}q_{67}^*)$$

Now, the steady state availability i.e. the probability that the system will be operative in long run is given by:

$$A_0 = \lim_{t\to\infty} A_0(t) = \lim_{s\to 0} sA_0^*(s) = \lim_{s\to 0} s\frac{N_2(s)}{D_2(s)}$$

Since, $D_2(0) = 0$, therefore by applying L-Hospital rule, the steady state availability is given by:

$$A_0 = \lim_{s\to 0}\frac{N_2(s)}{D_2'(s)} = \frac{N_2(0)}{D_2'(0)}$$

Where,

$$N_2(0)=\left[\left(p_{78}p_{80}+p_{79}p_{90}\right)\left(1-p_{12}p_{20}-p_{13}p_{30}\right)\right]\Psi_0$$
$$+\Psi_1+p_{12}\Psi_2+p_{13}\Psi_3+p_{12}p_{27}\Psi_7$$
$$+\left(p_{78}\Psi_8+p_{79}\Psi_9\right)\left(1-p_{12}p_{20}-p_{13}p_{30}\right)$$

and

$$D_2'(0)=[p_{12}p_{20}+p_{13}p_{30}+\left(1-p_{12}p_{20}-p_{13}p_{30}\right)$$
$$\left(p_{78}p_{80}+p_{79}p_{90}\right)]\Psi_0+n_1+p_{12}n_2+p_{13}n_3$$
$$+p_{15}^{(4)}n_2+p_{16}^{(4)}n_3+\left(1-p_{12}p_{20}-p_{13}p_{30}\right)n_7$$
$$+\left(p_{78}+p_{7,11}^{(10)}\right)\left(1-p_{12}p_{20}-p_{13}p_{30}\right)n_8$$
$$+\left(p_{79}+p_{7,12}^{(10)}\right)\left(1-p_{12}p_{20}-p_{13}p_{30}\right)n_9$$

Where (n_1, n_2, n_3, n_7 n_8 and n_9) is the mean repair time.

7. BUSY PERIOD ANALYSIS

7.1 Due to fault detection

Let $B_i^F(t)$ be the probability that the repairman is busy in fault detection of a failed unit at epoch t, when the system starts from state $S_i \in E$. Now for $B_0^F(t)$, we have the sum of the probabilities of the following contingencies:

$$B_0^F(t)=q_{01}(t)©B_1^F(t)$$
$$B_1^F(t)=Z_1^F(t)+q_{12}(t)©B_2^F(t)+q_{13}(t)©B_3^F(t)+q_{15}^{(4)}(t)©B_5^F(t)$$
$$+q_{16}^{(4)}(t)©B_6^F(t)$$
$$B_2^F(t)=q_{20}(t)©B_0^F(t)+q_{27}^{(5)}(t)©B_7^F(t)$$
$$B_3^F(t)=q_{30}(t)©B_0^F(t)+q_{37}^{(6)}(t)©B_7^F(t)$$
$$B_5^F(t)=q_{57}(t)©B_7^F(t)$$
$$B_6^F(t)=q_{67}(t)©B_7^F(t)$$
$$B_7^F(t)=Z_7^F(t)+q_{78}(t)©B_8^F(t)+q_{79}(t)©B_9^F(t)+q_{7,11}^{(10)}(t)©B_{11}^F(t)$$
$$+q_{7,12}^{(10)}(t)©B_{12}^F(t)$$
$$B_8^F(t)=q_{80}(t)©B_0^F(t)+q_{81}^{(11)}(t)©B_1^F(t)$$
$$B_9^F(t)=q_{90}(t)©B_0^F(t)+q_{91}^{(12)}(t)©B_1^F(t)$$
$$B_{11}^F(t)=q_{11,1}(t)©B_1^F(t)$$
$$B_{12}^F(t)=q_{12,1}(t)©B_1^F(t) \qquad (56\text{-}66)$$

Where,

$$Z_1^F(t)=e^{-\alpha_2 t}\bar{F}_1(t) \qquad Z_7^F(t)=e^{-\alpha_1 t}\bar{F}_2(t)$$

Taking Laplace transforms of the above equations (56-66) and solving them, we get the following result for $B_0^{F*}(s)$

$$B_0^{F*}(s)=\frac{N_3(s)}{D_2(s)}$$

Here, $D_2(0)=0$

Therefore, by L-hospital rule, the steady state busy period of the repairman due to fault detection is given by:

$$B_0^F=\lim_{s\to 0}\frac{N_3(s)}{D_2'(s)}=\frac{N_3(0)}{D_2'(0)}$$

Where,

$$N_3(0)=\psi_1+\left(p_{12}p_{27}^{(5)}+p_{13}p_{37}^{(6)}+p_{15}^{(4)}+p_{16}^{(4)}\right)\psi_7$$

and $D_2'(0)$ is same as given in availability analysis.

7.2 Due to miner repair

Let $B_i^r(t)$ be the probability that the repairman is busy in miner repair of a failed unit at epoch t, when the system starts from state $S_i \in E$. Now for $B_0^r(t)$, we have the sum of the probabilities of the following contingencies:

$$B_0^r(t)=q_{01}(t)©B_1^r(t)$$
$$B_1^r(t)=q_{12}(t)©B_2^r(t)+q_{13}(t)©B_3^r(t)+q_{15}^{(4)}(t)©B_5^r(t)$$
$$+q_{16}^{(4)}(t)©B_6^r(t)$$
$$B_2^r(t)=Z_2^r(t)+q_{20}(t)©B_0^r(t)+q_{27}^{(5)}(t)©B_7^r(t)$$
$$B_3^r(t)=q_{30}(t)©B_0^r(t)+q_{37}^{(6)}(t)©B_7^r(t)$$
$$B_5^r(t)=Z_5^r(t)+q_{57}(t)©B_7^r(t)$$
$$B_6^r(t)=q_{67}(t)©B_7^r(t)$$
$$B_7^r(t)=q_{78}(t)©B_8^r(t)+q_{79}(t)©B_9^r(t)+q_{7,11}^{(10)}(t)©B_{11}^r(t)$$
$$+q_{7,12}^{(9)}(t)©B_{12}^r(t)$$
$$B_8^r(t)=Z_8^r(t)+q_{80}(t)©B_0^r(t)+q_{81}^{(11)}(t)©B_1^r(t)$$
$$B_9^r(t)=q_{90}(t)©B_0^r(t)+q_{91}^{(12)}(t)©B_1^r(t)$$
$$B_{11}^r(t)=Z_{11}^r(t)+q_{11,1}(t)©B_1^r(t)$$
$$B_{12}^r(t)=q_{12,1}(t)©B_1^r(t) \qquad (67\text{-}77)$$

Where,

$$Z_2^r(t)=e^{-\alpha_2 t}\bar{G}_1(t) \qquad Z_5^r(t)=\bar{G}_1(t)$$
$$Z_8^r(t)=e^{-\alpha_1 t}\bar{G}_2(t) \qquad Z_{11}^r(t)=\bar{G}_2(t)$$

Taking Laplace transforms of the above equations (67-77) and solving them, we get the following result for $B_0^{r*}(s)$

$$B_0^{r*}(s)=\frac{N_4(s)}{D_2(s)}$$

Here, $D_2(0)=0$

Therefore, by L-hospital rule, the steady state busy period of the repairman due to minor repair is given by:

$$B_0^r=\lim_{s\to 0}\frac{N_4(s)}{D_2'(s)}=\frac{N_4(0)}{D_2'(0)}$$

Where,

$$N_4(0)=p_{15}^{(4)}n_1+p_{12}\psi_2+\left(p_{12}p_{27}^{(5)}+p_{13}p_{37}^{(6)}+p_{15}^{(4)}+p_{16}^{(4)}\right)$$
$$p_{7,11}^{(10)}n_2+\left(p_{12}p_{27}^{(5)}+p_{13}p_{37}^{(6)}+p_{15}^{(4)}+p_{16}^{(4)}\right)p_{78}\psi_8$$

and $D_2'(0)$ is same as given in availability analysis.

7.3 Due to major repair

Let $B_i^R(t)$ be the probability that the repairman is busy in major repair of a failed unit at epoch t, when the system starts from state $S_i \in E$. Now for $B_0^R(t)$, we have the sum of the probabilities of the following contingencies:

$$B_0^R(t)=q_{01}(t)©B_1^R(t)$$

$$B_1^R(t) = q_{12}(t) © B_2^R(t) + q_{13}(t) © B_3^R(t) + q_{15}^{(4)}(t) © B_5^R(t) + q_{16}^{(4)}(t) © B_6^R(t)$$

$$B_2^R(t) = q_{20}(t) © B_0^R(t) + q_{27}^{(5)}(t) © B_7^R(t)$$

$$B_3^R(t) = Z_3^R(t) + q_{30}(t) © B_0^R(t) + q_{37}^{(6)}(t) © B_7^R(t)$$

$$B_5^R(t) = q_{57}(t) © B_7^R(t)$$

$$B_6^R(t) = Z_6^R(t) + q_{67}(t) © B_7^R(t)$$

$$B_7^R(t) = q_{78}(t) © B_8^R(t) + q_{79}(t) © B_9^R(t) + q_{7,11}^{(10)}(t) © B_{11}^R(t) + q_{7,12}^{(10)}(t) © B_{12}^R(t)$$

$$B_8^R(t) = q_{80}(t) © B_0^R(t) + q_{81}^{(11)}(t) © B_1^R(t)$$

$$B_9^R(t) = Z_9^R(t) + q_{90}(t) © B_0^R(t) + q_{91}^{(12)}(t) © B_1^R(t)$$

$$B_{11}^R(t) = q_{11,1}(t) © B_1^R(t)$$

$$B_{12}^R(t) = Z_{12}^R(t) + q_{12,1}(t) © B_1^R(t) \qquad (78\text{-}88)$$

Where,

$$Z_3^R(t) = e^{-\alpha_2 t}\overline{H}_1(t) \qquad Z_6^R(t) = \overline{H}_1(t)$$

$$Z_9^R(t) = e^{-\alpha_1 t}\overline{H}_2(t) \qquad Z_{12}^R(t) = \overline{H}_2(t)$$

Taking Laplace transforms of the above equations (78-88) and solving them, we get the following result for $B_0^{R*}(s)$

$$B_0^{R*}(s) = \frac{N_5(s)}{D_2(s)}$$

Here, $D_2(0) = 0$

Therefore, by L-hospital rule, the steady state busy period of the repairman due to fault detection is given by:

$$B_0^R = \lim_{s \to 0} \frac{N_5(s)}{D_2'(s)} = \frac{N_5(0)}{D_2'(0)}$$

Where,

$$N_5(0) = p_{16}^{(4)} n_3 + p_{13}\psi_3 + \left(p_{12}p_{27}^{(5)} + p_{13}p_{37}^{(6)} + p_{15}^{(4)} + p_{16}^{(4)}\right) p_{7,12}^{(10)} n_4 + \left(p_{12}p_{27}^{(5)} + p_{13}p_{37}^{(6)} + p_{15}^{(4)} + p_{16}^{(4)}\right) p_{79}\psi_9$$

and $D_2'(0)$ is same as given in availability analysis.

8. COST BENEFIT ANALYSIS

In steady state, the net-expected profit earned to the system model during time interval (0,t) as given below:

$$P(t) = K_0\mu_{up}(t) - K_1\mu_b^F(t) - K_2\mu_b^r(t) - K_3\mu_b^R(t)$$

Where K_0 per-unit up time revenue by the system due to the operation of any unit and K_1 , K_2 and K_3 are repair cost per-unit of time when unit is under fault detection, minor repair and major repair respectively.

The expected total cost per-unit time in steady state is given by:

$$P = \lim_{t \to \infty} \frac{P(t)}{t}$$

$$= K_0A_0 - K_1B_0^F - K_2B_0^r - K_3B_0^R$$

Where A_0, $\mathbf{B_0^F}$, $\mathbf{B_0^r}$ and $\mathbf{B_0^R}$ have been already defined.

9. CONCLUSION

This paper concludes the stochastic modeling analysis of various reliability measures like MTSF, availability and busy period due to fault detection, miner repair and major repair, and cost benefit analysis by different levels of performance. Let us suppose that the different random variables follow an exponential distribution with different probability density function given as $f(t) = \beta e^{-\beta t}$, $g(t) = \theta e^{-\theta t}$ and $h(t) = \lambda e^{-\lambda t}$. The numerical analysis of MTSF and cost benefit analysis have been studied at different levels of failure rate (α_2) of unit-2 by fixing the values of certain parameters $K_0 = 10000$, $K_1 = 1000$, $K_2 = 800$ $K_3 = 600$ as shown in Table 1 and Table 2. The variation in MTSF with respect to the failure rate (α_2) of unit-2 for different values of p , q , β_1, θ_1 and λ_1 is shown in Figures 2. The variation in cost benefit analysis with respect to the failure rate (α_2) of unit-2 for different values p , q , β_1, β_2, θ_1, θ_2, λ_1 and λ_2 is shown in Figures 3.

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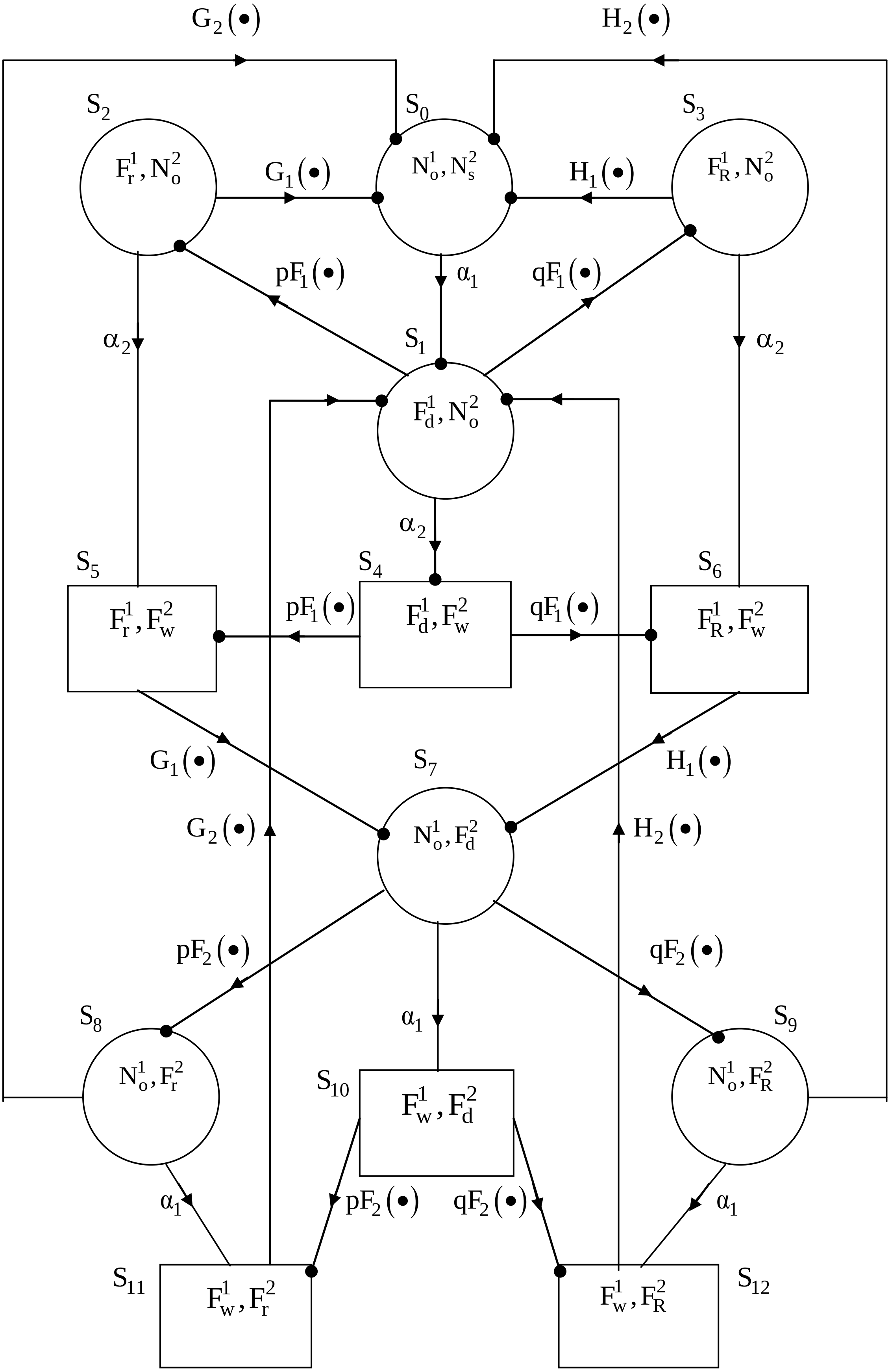
11. APPENDIX


$G_2(\bullet)$
$H_2(\bullet)$
S_2
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F_r^1, N_o^2
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$H_1(\bullet)$
F_R^1, N_o^2
$pF_1(\bullet)$
α_1
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α_2
S_1
α_2
F_d^1, N_o^2
α_2
S_5
S_4
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F_d^1, F_w^2
$qF_1(\bullet)$
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$G_1(\bullet)$
S_7
$H_1(\bullet)$
$G_2(\bullet)$
N_o^1, F_d^2
$H_2(\bullet)$
$pF_2(\bullet)$
$qF_2(\bullet)$
S_8
α_1
S_9
N_o^1, F_r^2
S_{10}
F_w^1, F_d^2
N_o^1, F_R^2
α_1
$pF_2(\bullet)$
$qF_2(\bullet)$
α_1
S_{11}
F_w^1, F_r^2
F_w^1, F_R^2
S_{12}

Figure 1

Table 1. Effect of p, q, β_1, θ_1 and λ_1 on system performance with respect to α_2

α_1	α_2	β_1=0.1,θ_1=0.3, λ_1=0.5,p=0.5, q=0.5	β_1=0.2,θ_1=0.4, λ_1=0.6,p=0.8, q=0.2	β_1=0.3,θ_1=0.5, λ_1=0.7,p=0.3, q=0.7	β_1=0.4, θ_1=0.6, λ_1=0.8,p=0.2, q=0.8	β_1=0.5,θ_1=0.7, λ_1=0.9,p=0.1, q=0.9
0.4	0.31	5.8003	5.9893	6.1582	6.3830	6.6253
0.41	0.32	5.6322	5.8074	5.9648	6.1753	6.4028
0.42	0.33	5.4737	5.6364	5.7834	5.9808	6.1947
0.43	0.34	5.3241	5.4755	5.6128	5.7983	5.9997
0.44	0.35	5.1826	5.3237	5.4523	5.6267	5.8166
0.45	0.36	5.0486	5.1802	5.3008	5.4651	5.6444
0.46	0.37	4.9215	5.0445	5.1577	5.3126	5.4821
0.47	0.38	4.8008	4.9159	5.0223	5.1685	5.3288
0.48	0.39	4.6859	4.7938	4.8939	5.0321	5.1840
0.49	0.4	4.5766	4.6777	4.7721	4.9028	5.0469

Table 2. Effect of p, q, β_1, β_2, θ_1, θ_2, λ_1 and λ_2 on system performance with respect to α_2

α_1	α_2	β_1=0.1,θ_1=0.3, λ_1=0.5β_2=0.1, θ_2=0.3,λ_2=0.5, p=0.5,q=0.5	β_1=0.2,θ_1=0.4, λ_1=0.6,β_2=0.2, θ_2=0.4,λ_2=0.6, p=0.8,q=0.2	β_1=0.3,θ_1=0.5, λ_1=0.7β_2=0.3, θ_2=0.5,λ_2=0.7, p=0.3,q=0.7	β_1=0.4,θ_1=0.6, λ_1=0.8,β_2=0.4, θ_2=0.6,λ_2=0.8, p=0.2, q=0.8	β_1=0.5,θ_1=0.7, λ_1=0.9,β_2=0.5, θ_2=0.7,λ_2=0.9, p=0.1,q=0.9
0.4	0.31	24997.41	17413.11	12824.40	10368.77	8702.73
0.41	0.32	24372.18	17132.34	12628.54	10235.48	8605.76
0.42	0.33	23767.31	16849.88	12430.59	10099.24	8505.79
0.43	0.34	23182.82	16566.97	12231.44	9960.70	8403.34
0.44	0.35	22618.49	16284.71	12031.85	9820.50	8298.91
0.45	0.36	22074.01	16004.01	11832.50	9679.17	8192.93
0.46	0.37	21548.94	15725.67	11633.97	9537.19	8085.79
0.47	0.38	21042.77	15450.34	11436.76	9394.99	7977.83
0.48	0.39	20554.93	15178.57	11241.31	9252.96	7869.39
0.49	0.4	20084.82	14910.80	11047.97	9111.42	7760.74

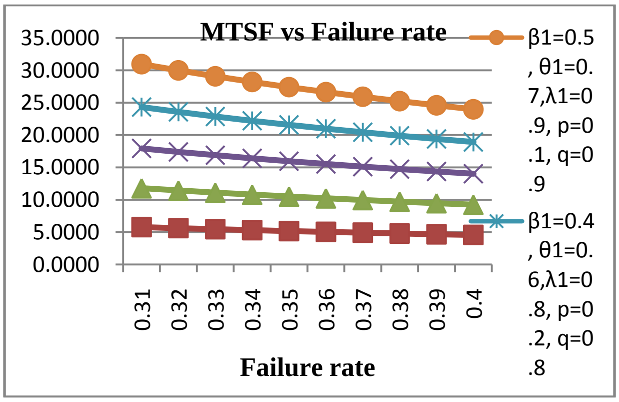

Figure 2

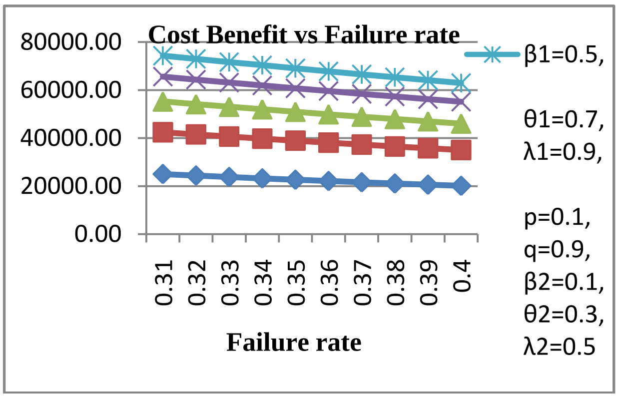

Figure 3