International Journal of Computer Applications (0975 - 8887)
Volume 184 - No.31, October 2022

A Morphological Pyramids Approach to Grayscale Image Enhancement

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ABSTRACT

Medical image processing algorithms significantly affect the precision of disease diagnostic process. This makes it crucial to improve the quality of a medical image with the goal to enhance perceivability of the points of interest in order to obtain accurate diagnosis of a patient. Despite the reliance of various medical diagnostics on utilize X-rays, they are usually plagued by dark and low contrast properties. Sought-after details in X-rays can only be accessed by means of digital image processing techniques, despite the fact that these techniques are far from being perfect. In this paper, we implement a wavelet decomposition and reconstruction technique to enhance radiograph properties, some of which include contrast and noise, by using a series of morphological erosion and dilation to improve the visual quality of the chest radiographs for the detection of cancer nodules.

General Terms

Medical image processing, Morphological Pyramid

Keywords

Chest radiograph, image enhancement, mathematical morphology, wavelet decomposition

1. INTRODUCTION

Signals and images are made up of features at different scales, as such, multiresolution methods are critical in most signal and image processing applications [16]. Moreover, computational algorithms based on multiresolution methods have several advantages over competitors for being robust. Multiresolution analysis represents a signal at multiple scales. Thus, it is better suited for extracting information represented by singularities and other irregular structures (possessed by images) through a time-frequency representation [10].

The discovery and application of wavelets has ushered into prominence multiresolution techniques in image processing in the form of pyramid schemes that are geared at avoiding redundancy. The pyramid scheme "consists of a (finite or infinite) number of levels such that the information content decreases towards higher levels and each step toward a higher level is implemented by an (information-reducing) analysis operator, whereas each step toward a lower level is implemented by an (information-preserving) synthesis operator" [**?**]. The pyramid scheme is based on the principle that, an image subjected to a low-pass filter does not contain details beyond the cut-off frequency of the low-pass filter any more. As such, the image can be subsampled or decimated without any loss of information. The pyramid decomposition scheme with j levels is thus effected by applying the following three steps iteratively:

(1) Apply a Low Pass filter to the image I_j to obtain $F(I_j)$
(2) Compute the difference $Dn = I_j - F(I_j)$ (This produces the detail at level j)
(3) Apply a High Pass filter to subsample $F(I_j)$ in order to obtain I_{j+1}.
The result is a series of decreasing resolution images I_j and a series of decreasing resolution details D_j.

Implementing morphological filters at each resolution of the pyramid scheme enables the characterization of image objects based on their geometric features [13, 34]. In addition, the properties of idempotence and differentiation of target backgrounds inherrent in morphological filters enables the efficiency of their application to medical images such as detecting cancer nodules in chest X-Rays.

Since mathematical morphology (MM) deals with the shapes of objects, it naturally makes it a more efficient and an effective technique for object recognition and feature extraction [18]. The morphological pyramid scheme exploits the properties of the nonlinear operators provided by MM [13, 3]. Most existing pyramid schemes are based on linear filters. The nonlinear morphological filters are faster and are object edge preserving. As has been illustrated above, the computation process is an iterative analysis involving smoothing by the morphological filter, then computing the details lost in the smoothing, down-sampling the current image, and finally computing the details lost in the down-sampling process. Being a time domain analysis, MM has the limitation that noise and disturbance characteristics are not easily captured compared to frequency domain analysis [19]. Combining MM with frequency domain analysis provided by the wavelet technique, significantly improves the results.

X-Rays are the most cost effective and affordable medical imaging technique available. Consequently, they are used routinely in diagnostic tests to reveal some unsuspected pathologic alterations such as pulmonary nodules [11, 2]. Most often, medical images have a low dynamic range and many of its features are difficult to see. The different intensity transformations that improve the appearance of an image do not merely serve as an aesthetic role but often, help to improve the performance of image segmentation algorithms and feature recognition. Unfortunately, X-ray images have a low contrast due to the subtle distinction of attenuation coefficients and scatter effect, which makes it difficult to distinguish signals from background. There are also subtle differences between the attenuation coefficients and scatter effect between images and the background. Lung cancer nodules can appear anywhere in the lung field and can also be hidden by ribs, the mediastinum and structures beneath the diaphragm resulting in a huge variation of contrast to the background as shown in Figure 1 of the Chest radiograph with certified Nodules. Mining of nodules help to detect them in chest radiographs and may serve as an early detection system to reveal signs of lung cancer in any X-Ray film [27, 17, 1, 35, **?**].

Fig. 1. X-ray Image with cancer nodules

In order to eliminate the problems listed above, a new technique using the pyramid scheme combined with MM is proposed to remove anatomical noise while preserving details. First, the wavelet transform that has the capability to locally decompose X-ray images via the pyramid scheme to remove the unwanted details is applied. Then, the image is reconstructed using the derived wavelet coefficients.This is followed by the application of morphological erosion and dilation for several iterations to enhance the cancer nodules and realize a better appearance, using a small and ellipsoidal structuring element. Combined with the wavelet pyramid scheme, the MM algorithm can process pixel-level gray-scale chest radiograph images to extract image features of interest such as cancer nodules [36]. The technique is demonstrated on a database of a set of 247 chest X-ray images from a standard Public Database of the Japanese Society of Radiological Technology.

2. MORPHOLOGICAL OPENINGS AND CLOSINGS

Morphological opening is defined as the dilation of the erosion of a set A by a structuring element B. It is denoted by $A \circ B$ and is given by

$$A \circ B = (A \ominus B) \oplus B$$

where $\ominus$ denotes erosion and $\oplus$ denotes dilation. An opening visually smoothens contours and eliminates small islands in the foreground of an image, thereby making them a part of the image background [26]. This operation can also be used to locate edges and corners or slopes that a particular structuring element can fit.

A closing on the other hand is the dual operator to the morphological opening and is denoted by $A \bullet B$, is a dilation of A by the structuring element B followed by an erosion of the result. This can be written as

$$A \bullet B = (A \oplus B) \ominus B$$

This smoothens contours, fills narrow gulfs and eliminates small holes. Morphological closing serves as a tool to preserve regions in the background of an image that fits the structuring element used. Morphological opening and closing inherit the effects of erosion operator which has the effect of eroding away the boundaries of regions of the foreground pixels and dilation with the effect of gradually enlarging the boundaries of the image foreground pixels respectively [4]. Morphological opening and closing as well as variations of these operations have been applied in various applications. These include algorithm for the detection of license plates [14]. Suero et al [30], have used a set of the morphological opening and closing operations to locate a pixel in the optic disc of retinal images. Their method proved to outstrip other methods in the literature. A multiscale approach can also be used where these operations are applied repeatedly either with the same or varying structuring elements [31]. They have also been applied to independent components of an image or to the whole image [21]. These operators can be further used for image segmentations by applying a series of the operators with increasing sizes of a specific structuring element or by using different structuring elements of the same family [22, 9].

The morphological opening and closing operations as operators satisfy the pyramid scheme. **Definition** *[15]: Let X and Y denote two partially ordered sets and let $\varepsilon : X \to Y, \delta : Y \to X$ denote two operators. The pair (ε, δ) is and adjunction between X and Y if*

$$\delta(y) \leq x \Leftrightarrow y \leq \varepsilon(x), \forall x \in X, y \in Y. \tag{1}$$

Both operators δ and ε are increasing. In this case, the operators ε and δ denote morphological erosion and dilation.

Definition: *Let ψ denote an operator from a partially ordered set X onto itself. Then (a) ψ is an idempotent if $\psi^2 = \psi$*
(b) If ψ is increasing and an idempotent then ψ is a filter
(c) A filter ψ which satisfies $\psi \leq id$ is called an opening
(d) A filter ψ which satisfies $\psi \geq id$ is called a closing.

PROPOSITION 1. *[15]: Let X and Y be two partially ordered sets and lets (ε, δ) be an adjunction between them $\varepsilon \circ \delta$ is a closing on Y and $\delta \circ \varepsilon$ is an opening on X. Denoting these respectively by $\psi_j^{\downarrow}$ and $\psi_j^{\uparrow}$, then these form the analysis and synthesis operators respectively sasisfying the condition $\psi_j^{\uparrow}\psi_j^{\downarrow} = id$ on V_{j+1}.*

Applying a morphological filter at each resolution in a multiresolution analysis enables the characterization of geometric features of pertinent image artifacts such as cancer nodules in X-Ray images [12]. The idempotence property of morphological filters facilitates the extraction of important image features at any given scale with only a single application. The wavelets technique is implemented to remove unwanted artifacts from an image while preserving important characteristics [4, 20].

A multiresolution analysis (MRA) or multiscale approximation is the design method which is useful for analysing signals according to scale. An MRA is a decomposition of the Hilbert space $H = L^2(\mathbf{R})$ into a chain of closed subspaces $(V_j)_{j\in\mathbf{Z}}$ which form a sequence of successive approximation subspaces of H such that the following hold:

(1) $V_j \subset V_{j+1}$ for all $j \in \mathbb{Z}$

(2) $\cup_{j=-\infty}^{\infty} V_j$ is dense in $L^2(\mathbb{R})$ and $\cap_{j=-\infty}^{\infty} V_j = \{0\}$

(3) $f(x) \in V_j \Leftrightarrow f(2x) \in V_{j+1}$ for all $j \in \mathbf{Z}$

(4) $f(x) \in V_j \Leftrightarrow f(x-k) \in V_j$ for all $j, k \in \mathbf{Z}$

(5) Each subspace V_j is spanned by integer translates of a single function $f(x)$. That is, for any $f \in L^2(\mathbb{R})$ and any $k \in \mathbb{Z}, f(x) \in V_0 \Leftrightarrow f(x-k) \in V_0$. All subspaces are therefore scaled versions of the central space V_0.

(6) There exists a function $\psi(x)$, belonging to V_0, such that the sequence $(\psi(x-k)), k \in \mathbf{Z}$ forms a Riesz basis or unconditional basis for V_0.

Most of the time, the filters relied upon in such applications are real halfband, so perfect reconstruction can be achieved. Sometimes the filters may not be ideal so it is difficult to achieve perfect reconstruction. One of the the most common ideal filters are the ones developed by Ingrid Daubechies, and are known as Daubechies' Wavelets. In Figure 1, a typical multiresolution algorithm is displayed.

MultiResolution2.PDF

Fig. 2. Multiresolution Algorithm

The pyramid scheme requires that each analysis operator acting on a signal s_j at level j results in a coarser signal at level $j+1$, with reduced information. The pyramid scheme produces the coarser signal together with the detail signal at each level [16]. A morphological skeleton representation in terms of the morphological operations of dilation, erosion, opening, and closing, constitute a special case of the pyramid scheme. In this case the signal/image spaces are complete lattices and the analysis and synthesis operations applied are adjunctions. The pyramid is made up of a number of levels, which for practical applications is considered to be finite, with the information content decreasing towards higher levels. A higher level is arrived at by a synthesis operator and each step towards a lower level of the pyramid is achieved by an analysis operator [**?**].

3. MORPHOLOGICAL PYRAMIDS AND THE WAVELET LIFTING SCHEME

Pyramids are one of the many morphological techniques that are relied upon for very effective image analysis. Combined with wavelets, they provide a very efficient approach for image coding. A simplified schematic representation of the application is illustrated in Figure 2.

3.1 Morphological Pyramids

The use of a morphological smoothing filter in the pyramidal analysis usually leads to a subsampled filtered image in which loss of information is risked. This problem can be solved by keeping the details possibly lost in the down-sampling operation. The Morphological Pyramid is a powerful approach to such a decomposition.

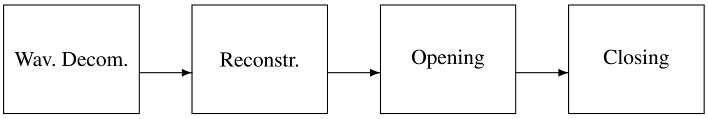

Fig. 3. Schematic representation of the algorithm

The computational process consists of an iterative analysis involving smoothing by the morphological filter, computing the details lost in the smoothing, down-sampling the current image, and computing the details lost in the down-sampling. Some basic concepts about morphological operators for grey-scale images after which we discuss multiresolution signal decomposition are now introduced.

3.1.1 Morphological Operators. Given a signal $f \in$ Fun($\mathbb{Z}^d$,$\mathbb{T}$) and a vector $k = (k_1, k_2, ..., k_d) \in \mathbb{Z}^d$, define the translation operator $T = T(k_1, k_2, ..., k_d)$ by $(Tf)(n) = (Tf)(n_1, n_2, ..., n_d) = f(n_1 - k_1, n_2 - k_2, ..., n_d - k_d) = f(n-k)$ where $n, k \in \mathbb{Z}^d$ [**?**]. The two basic morphological operators on Fun($\mathbb{Z}^d$,$\mathbb{T}$), the (flat) dilation and the (flat) erosion can be explained as follows:
Given an image function $f(x)$ and a structuring function $S(x)$, the gray-scale dilation and erosion of f by S is defined as

$$\delta_S f(x) = D(f,s) = (f \oplus S)(x) = supz \in S(f(x+z)) \quad (2)$$

$$\epsilon_S f(x) = E(f,s) = (f \ominus S)(x) = infz \in S(f(x+z)) \quad (3)$$

where $S \subseteq \mathbb{Z}^d$, is the support of the structuring function [33]. The pair (ϵ_S, δ_S) constitute an adjunction on Fun($\mathbb{Z}^d$,$\mathbb{T}$) and the composition $\alpha_S = \delta_S \epsilon_S$ is an opening and $\beta_S = \epsilon_S \delta_S$ is a closing. The operators α_S and β_S are called the opening and closing by A, respectively. The opening has the property that it is increasing that is, suppose f and $g \in$ Fun($\mathbf{Z}^d$, $\mathbb{T}$) and $(f \leq g)$ implies that $\alpha_A(f) \leq \alpha_A(g)$, anti-extensive $\alpha_A(f) \leq f$ and idempotent $(\alpha_A \alpha_A(f) \leq (f))$. Similar properties hold for the closing, where the closing is extensive implies($\phi_A(f) \geq f$).The opening eliminates peaks and the closing eliminates valleys [24].

3.2 Multiresolution Signal Decomposition

In this section the basic concept of multiresolution signal decomposition is outlined [16, 24, 25]. The concept encompasses both linear as well as non-linear pyramids as follows:
Let $f \in V_0$ be an initial signal in the signal space V_0 that will be decompose into approximation signals f_j with $j = 0, 1, 2...$ and j is called the level of the decomposition. The set $f_0, f_1, f_2, ..., f_k$ is referred to as an approximation pyramid.

Definition *An Analysis or decomposition operator* $\psi_j^{\uparrow} : V_j \rightarrow V_{j+1}$ *which maps a signal to a higher level in the pyramid, leading to a reduced information and the Synthesis operator or reconstruction operator* $\psi_j^{\downarrow} : V_j \rightarrow V_{j+1}$ *which maps a signal to the level lower in the pyramid leading to lost of some information.* To ensure that the information lost during the *Analysis* process can be recovered during the *Synthesis*, the so called *pyramid condition* defined as follows, is used:

Definition *The analysis and synthesis operators* $\psi_j^{\uparrow}$ *and* $\psi_j^{\downarrow}$ *respectively satisfy the pyramid condition that is:*

$$\psi_j^{\uparrow}\psi_j^{\downarrow}(f) = f \textit{ for all } f \in V_{j+1}$$

where $\psi_j^{\uparrow}\psi_j^{\downarrow}$ *is known as the identity operator. The decomposition and reconstruction of a signal* $f \in V_0$ *is as follows:*

$$f = f_0$$
$$f_{j+1} = \psi_j^{\uparrow}(f_j),\ j \geq 0$$

results in a pyramid with sequence of detail signals $d_0, d_1, d_2, ..., d_{L-1}$ *and* f_L *the signal at the highest level*

then a perfect reconstruction is achieved, i.e. f_0 can be exactly reconstructed as:

$$f_j = \psi_j^{\downarrow}(f_{j+1}) + d_j, \quad j = L-1, L-2, ..., 0.$$

On the other hand moving from any level i in the pyramid to a higher level j is effected by successively composing analysis operators to avoid the detail operators. This gives an operator called the multilevel analysis operator

$$\psi_{ij}^{\uparrow} = \psi_{j-1}^{\uparrow}\psi_{j-2}^{\uparrow}...\psi_i^{\uparrow}, \quad j > i$$

Similarly, another map that can travel from any level j back to the level i is obtained by successively composing synthesis operators called the multilevel synthesis operator

$$\psi_{ij}^{\downarrow} = \psi_{j-1}^{\downarrow}\psi_{j-2}^{\downarrow}...\psi_i^{\downarrow}, \quad j > i.$$

Now we define the composition:

$$\hat{\psi_{i,j}} = \psi_{j,i}^{\downarrow}\psi_{i,j}^{\uparrow} \quad j > i$$

which takes a signal from level i to the level j and back to i again. The operator $\hat{\psi_{i,j}}$ can be regarded as an *approximation* operator. Finally define a level j approximation $f_{0,j}$ of $f \in V_0$ by $\hat{f_{0,j}}$=$\hat{\psi_{i,j}}(f)$=$\psi_{j,0}^{\downarrow}\psi_{0,j}^{\uparrow}(f)$=$\psi_{j,0}^{\downarrow}(f_j)$.

4. EXPERIMENTS ON CHEST X-RAYS

Multiscale images are generally of poor contrast [25]. In particular, medical images obtained by X-Rays, are filled with noise due to interference from capturing devices and anatomical structures [23]. The morphological pyramid approach discussed above was implemented on a set of chest X-ray images from a standard Public Database; the Japanese Society of Radiological Technology. This database is endowed with different cases which makes it the appropriate choice. These images were collected over a three year period from 14 medical institutions and are made up of anterior and posterior films of measure 14 by 14 inches. There are 154 images which have lung nodules, out of which 100 are malignant and 54 are benign. Ninety-three of the images are without lung nodules. Nodules are confirmed by scans and their locations are confirmed by three radiologists [29]. The python libraries Matplotlib and Mahotas [7] were imported and used in the implementation. The images were loaded and the morphological closing operation was applied. The structuring element used was the *1-cross*, which is the default structuring element for the *morph.close* function in the Mahotas module. The images were shown and saved using Python's Matplotlib module.

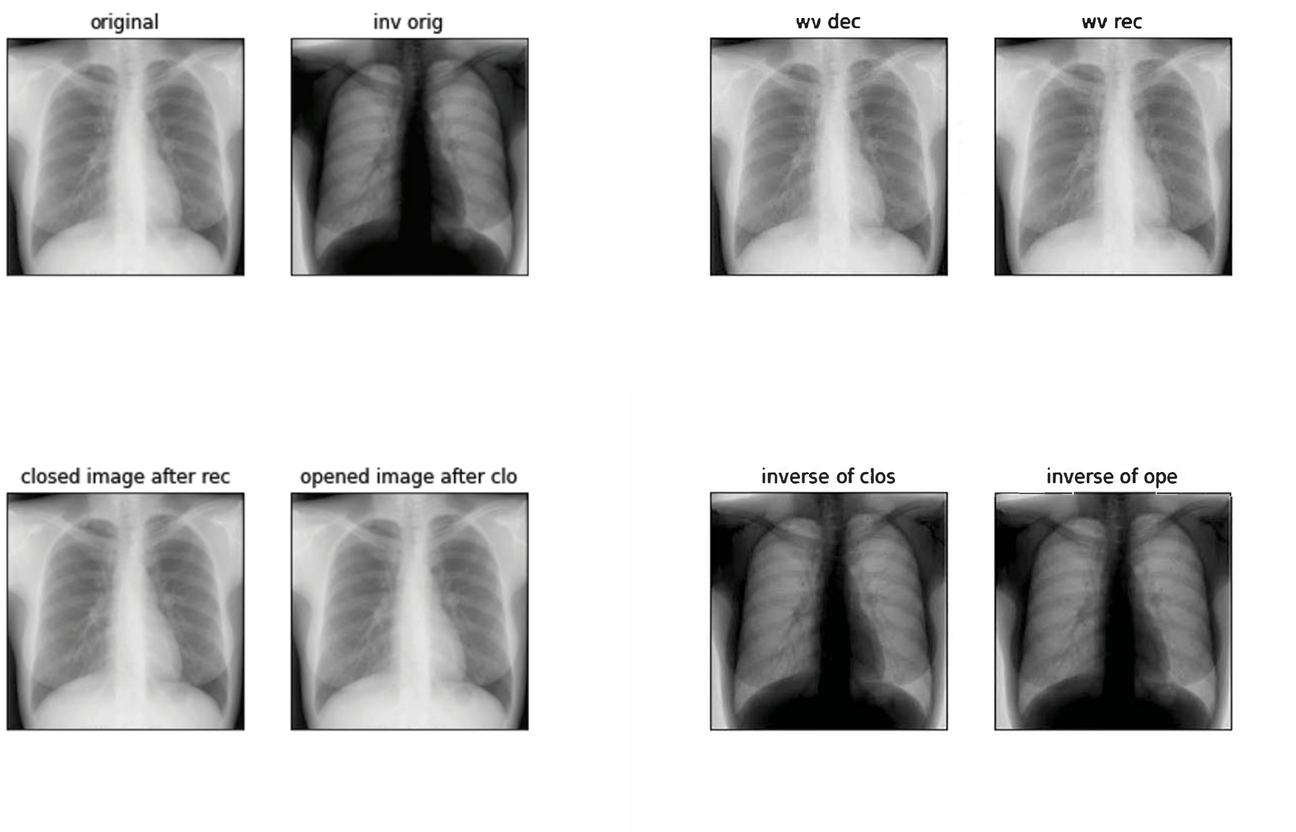

Fig. 4. Morphological operations applied to X-ray image without nodules: (a) Original image and its inverse (b) Wavelet decomposition and reconstructed image (c) Sequence of closed and open image after reconstruction (d) Inverse of closure and opened image

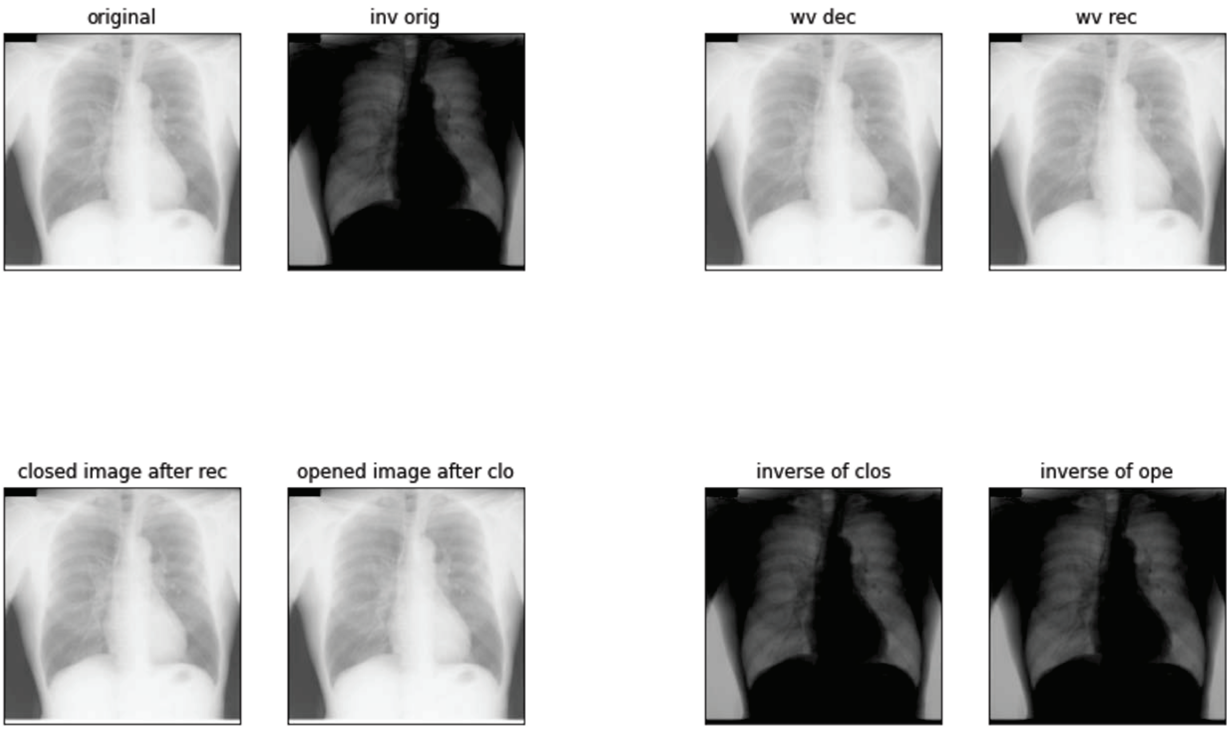

Fig. 5. Morphological operations applied to X-ray image with nodules: (a) Original image and its inverse (b) Wavelet decomposition and reconstructed image (c) Sequence of closed and open image after reconstruction (d) Inverse of closure and opened image

5. PERFORMANCE EVALUATION AND IMAGE QUALITY ANALYSIS

The X-Rays were classified with the WEKA tool, and the Support Vector Machine (SVM) was used and 80% of all the nodules and normal cases of 247 CRs were used in the training set and 20% as the testing set which yielded an average of 82.7% in sensitivity. The CRs which contain the nodules were grouped according to its degree of dificulty for detection. Compared with the Total Variation (TV) approach and previous studies by Wei et al [32], Coppini et al [8], Schilham et al., [28], Hardie et al, and [5, 6], the MMP method performed creditably. The accuracy of this method is demonstrated in the form of sensitivity, specificity, and accuracy.
The peak signal to noise ratio (PSNR) is used to evaluate the performance of our algorithm, where

$$PSNR = 10log_{10}\left(\frac{I^2_{(m,n)}peak}{MSE}\right) \tag{4}$$

where $I_{(m,n)}peak$ is the peak pixel value in the image $I_{(m,n)}$ and is usually 255 for pixels represented using 8 bits per sample, and MSE is the mean square error. The average PSNR we obtained is 58.0dB. The results shown in Table 1 confirms the validity and efficiency of our method.

Table 1. Performance Metrics

Method	Sensitivity	Specificity	Accuracy	PSNR (dB)
TV	79.0%	80.0%	91.7%	40.2
TV and UDWT	82.5%	93.3%	97.0	42.8
Coppini et al.	97.3%	73.8%	96.9%	-
MMP	82.7%	93.3%	97.1%	58.0

6. DISCUSSION AND CONCLUSION

We developed a technique for image denoising and enhancement based on a combination of wavelets and morphological erosion and dilation which is presented and applied to a large sample of chest x-ray images, some of which contained cancer nodules in order to enhance the quality and contrast of the x-ray images. Our approach is tested on a number of publicly available chest radiograph images. The combined wavelet based and mathematical morphology technique retains and elucidate more detail image information of interest on both cancer nodules and anatomical structures captured in chest radiographs. The technique not only suppresses unwanted noise, it also preserves the edges of the nodules to enable accurate detection. From the results obtained, we conclude that our technique is efficient and compares favorably with nonmorphological based techniques for chest radiograph image enhancement.

The morphological pyramid scheme is a powerful tool in several image processing applications. The inherrent nonlinearity of the scheme comes with a more complicated computational framework. However, as shown by the results, this is clearly offset by the gains in image quality owing to the properties of morphological adjunction pyramids which enable progressive image refinement and have the additional property of perfect reconstruction. The morphological filtering technique operates on the specific shapes in an image. The major effects are perceived in the changes in certain geometric details of the image, leaving the remaining image structure intact. When the resolution is reduced, no overall blurring is introduced in the image as the conventional linear filtering method does. A major disadvantage of the pyramid transform is that the output data size is far greater than the size of the input data. This suggests the composition with the wavelet transform which, on the other hand, cuts up data in such a way that this drawback is eliminated [**?**].

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