International Journal of Computer Applications (0975 – 8887)
Volume 184– No.39, December 2022

Performance Assessment of a Multi-state Standby Series System using Copula Distribution and Catastrophic Failure

Praveen Kumar Poonia
University of Technology and Applied Sciences
College of Applied Sciences Campus, Ibri-516
Oman

ABSTRACT
In this paper, the authors study the reliability measures of a complex engineering system consisting of three subsystems namely 1, 2 and 3 in series configuration. The subsystem-1 has three units working under 1-out-of-3: G; policy, the subsystem-2 has two units working under 1-ot-of-2: G policy and the subsystem-3 has one unit working under 1-out-of-1: G; policy. Moreover, the system may face catastrophic failure at any time t. The failure rates of units of all the subsystems are constant and assumed to follow exponential distribution, but their repair supports two types of distribution namely general distribution and Gumbel-Hougaard family copula distribution. The system is analyzed by using the supplementary variable technique, Laplace transformation and Gumbel-Hougaard family of copula to derive differential equations and obtain important reliability characteristics such as availability of the system, reliability of the system and profit analysis. It gives a new aspect to scientific community to adopt multi-dimension repair in form of copula. Furthermore, the results of the model are beneficial for system engineers and designers, reliability and maintenance managers.

Keywords
Availability, Reliability, k-out-of-n, Goumbel-Hougard copula distribution, catastrophic failure

1. INTRODUCTION
A variety of standby systems have been designed and analyzed during the last few decades. The main objective of these studies has been to develop methods and tools for evaluation and to demonstrate the reliability, availability, and cost analysis. Redundant systems, which have been widely used in practice, such as space shuttles, communication satellites, Nuclear reactors, or a fighter plane, are frequently discussed in research literature. Initially, redundant parts are design to improve the reliability of the system, meaning that some additional paths are developed or identical components connected in such a way that when one component fails the others will keep the system functioning. It is a technique called redundancy commonly used to improve system reliability and availability. It can be defined as cold standby, hot standby and warm standby. Moreover, redundancy is highly cost effective in achieving reliability. Therefore, in order to enhance reliability k-out-of-n system structure in which at least k components out of n must be functioning for the system to be operational play a vital role. In order to improve the reliability of k-out-of-n systems, numerous researches have presented their works and contributions by constructing different types of complex repairable systems under the different types of failure and repair distributions. For instance, authors consider warm standby system by She and Pecht [21], generalized multi state system by Huang et al. [3], repairable consecutive systems with r repairman by Wu and Guan [20], two-stage weighted systems with components in common by Chen and Yang [2], main unit with helping unit by Kumar and Gupta [5], Markov repairable system with neglected or delayed failures by Bao and Cui [1], evaluated exact reliability formula for consecutive repairable systems by Liang et al. [9], general system with non-identical components considering shut-off rules using quasi-birth-death process by Moghaddass et al. [10], generalized block replacement policy with respect to a threshold number of failed components and risk costs by Park and Pham [11], and repairable multi-state system when repair time can be neglected by Jia et al. [4].

The k-out-of-n effective policy plays a crucial role into maintaining the reliability of repairable systems. The researchers have paid their attention in evaluating reliability and availability of redundant repairable systems like k-out-of-n in series configuration. In particular, Kumar and Sirohi [6, 7, 8] analyzed a two-unit cold standby system with delayed repair of partially failed unit and better utilization of units. Sirohi et al. [19] studied an engineering system, which consists of two subsystems, viz. subsystem-1 and subsystem-2 with switching device in series. Subsystem-1 and 2 works under the k-out-of- n: good policy consists of three and two identical units in parallel configuration respectively. Authors evaluated reliability characteristics using supplementary variable technique. Poonia et al. [16] and Singh and Poonia [17] provided exact reliability formula for a warm standby repairable k-out-of-n computer lab network with similar computers and all the computers are connected in parallel to a data server and a router. The author modelled the problem as a finite series using supplementary variable technique, Laplace transform and copula repair. Singh et al. [18] and Sirohi and Poonia [15] studied k-out-of-n: G type of subsystems in series configuration for various values of n and k. The authors used copula repair for completely failed units with switching device in one or both the subsystems under catastrophic failure. They compared cost analysis under copula and general repair and proved that system performance is better if copula repair is being used for repairing. Poonia [13] analyzed a computer network system comprising of two load balancers, five web servers, and three database replica servers as a series parallel system with four subsystems. In this model the author developed the first order partial differential equations and solved using supplementary variable techniques and copula modus-operandi. The analysis of results indicates that copula repair is more effective in availability and expected profit analysis. Lastly Poonia [12, 14] considered a computer network and studied reliability physiognomies using supplementary variable technique.

Many researchers around the world have presented their research works, but no one paid attention to the study of the system which having three subsystems connected in series configuration with catastrophic failure.Treating the above realities in the present study, the model consisting three subsystems in series configuration considering catastrophic failure. The subsystem-1 has three identical units, subsystem-2 has two identical units and subsystem-3 has one unit only. The subsystem-1 is working under 1-out-of-3: G; scheme, the subsystem-2 is working under 1-out-of-2: G; scheme, however, the subsystem-3 works under 1-out-of-1: G; scheme. The catastrophic failure is treated as complete failed state. The failure rates of units of subsystems are constant and assumed to follow exponential distribution, but their repair supports two types of distribution namely general distribution and Gumbel-Hougaard family copula distribution. This present study in this paper completed two objectives using supplementary variable technique. First is to obtain expressions for the reliability of the system, availability of the system, mean time to failure and profit function. Second is to perform numerical simulation using MAPLE 2017 with respect to profit function. The transition state diagram of the designed model is shown in Figure-1.

2. ASSUMPTION

The following assumptions are made through this paper:

1. Initially the system is in state S_0, and all the units of subsystem-1, 2 and 3 are in good working conditions.
2. The subsystem-1 works successfully if minimum one unit is in good working condition i.e. 1-out-of-3: G policy, the subsystem-2 works successfully if minimum one unit is in good working condition i.e. 1-out-of-2: G policy, and the subsystem-3 works successfully if the lonely unit is in good working condition i.e. 1-out-of-1: G policy.
3. There may be unpredictable catastrophic failure to the system at any time (t).
4. One repairperson is available full time with the system.
5. All failure rates are constant and follows the exponential distribution.
6. The failure rate and repair rate in all the three subsystems is same unit wise, while different subsystem wise.
7. The complete failed system needs repair immediately. For this Gumbel-Hougaard, family of copula can be employed to restore the system.

3. NOTATIONS

The following symbols are made through this paper:

s , t	Laplace transform / Time scale variable
$\lambda_1 / \phi_1(x)$	Failure rate / Repair rate of each unit in subsystem-L.
$\lambda_2 / \phi_2(x)$	Failure rate / Repair rate of each unit in subsystem-M.
λ_3	Failure rate of the unit in subsystem-N.
λ_E	Deliberate failure rate when two units in subsystem-L and one unit in subsystem-M failed.
λ_C	Failure rate related to catastrophic failure mode.
$P_0(t)$	The state transition probability that the system is in $S_i$state at an instant for $i=0$.
$\bar{P}(s)$	Laplace transformation of the state transition probability $P(t)$.
$P_i(x,t)$	The Probability that the system is in state S_i for $i=1$ to 9, E, C and the system is under repair with elapsed repair time is x,t.
$E_p(t)$	Expected profit in the interval $[0,t)$.
K_1, K_2	Revenue generated and service cost per unit time respectively.
$\mu_0(x)$	An expression of the joint probability from failed state S_i to good state S_0 according to Gumbel-Hougaard family copula

4. STATE DESCRIPTION

In transition diagram, S_0 is perfect state, S_1, S_2, S_3, S_4 and S_5 partial failed/degraded and S_6, S_7, S_8, S_9, S_E and S_C are complete failed states. Due to failure of unit (s) in the subsystem-1, 2 or/and 3, the transitions approach to partially failed states S_1, S_2, S_3 S_4 and S_5 and the general repair is employed. The state S_6, S_7, S_8 and S_9 are complete failed states due to failure of units in all the subsystems, while S_E is completely failed state due to deliberate failure. The states S_C is complete failed state due to catastrophic failure. Repair is being applied using Gumbel-Hougaard family copula distribution.

Table 1 State Description

State	Description
S_0	This is a perfect state and all units of subsystem-1, 2 and 3 are in good working condition.
S_1	The indicated state is degraded but is in operational mode after the failure of the one unit in subsystem-1. All units of subsystem-2 and 3 are in the good operational state. The system is under general repair.
S_2	The indicated state is degraded but is in operational mode after the failure of two units in subsystem-1. All units of subsystem-2 and 3 are in the good operational state. The system is under general repair.
S_3	The indicated state is degraded but is in operational mode after the failure of the one unit in subsystem-2. All units of subsystem-1 and 2 are in the good operational state. The system is under general repair.
S_4	The indicated state is degraded but is in operational mode after the failure of the one unit in subsystem-1 and one unit in subsystem-2. All units of subsystem-3 are in the good operational state. The system is under general repair.
S_5	The indicated state is degraded but is in operational mode after the failure of two units in subsystem-1 and one unit in subsystem-2. All units of subsystem-3 are in the good operational state. The system is under general repair.
S_6, S_7 S_8, S_9 S_E, S_C	The states represent that the system is in completely failure mode and the system is under repair using Gumbel-Hougaard family copula distribution.

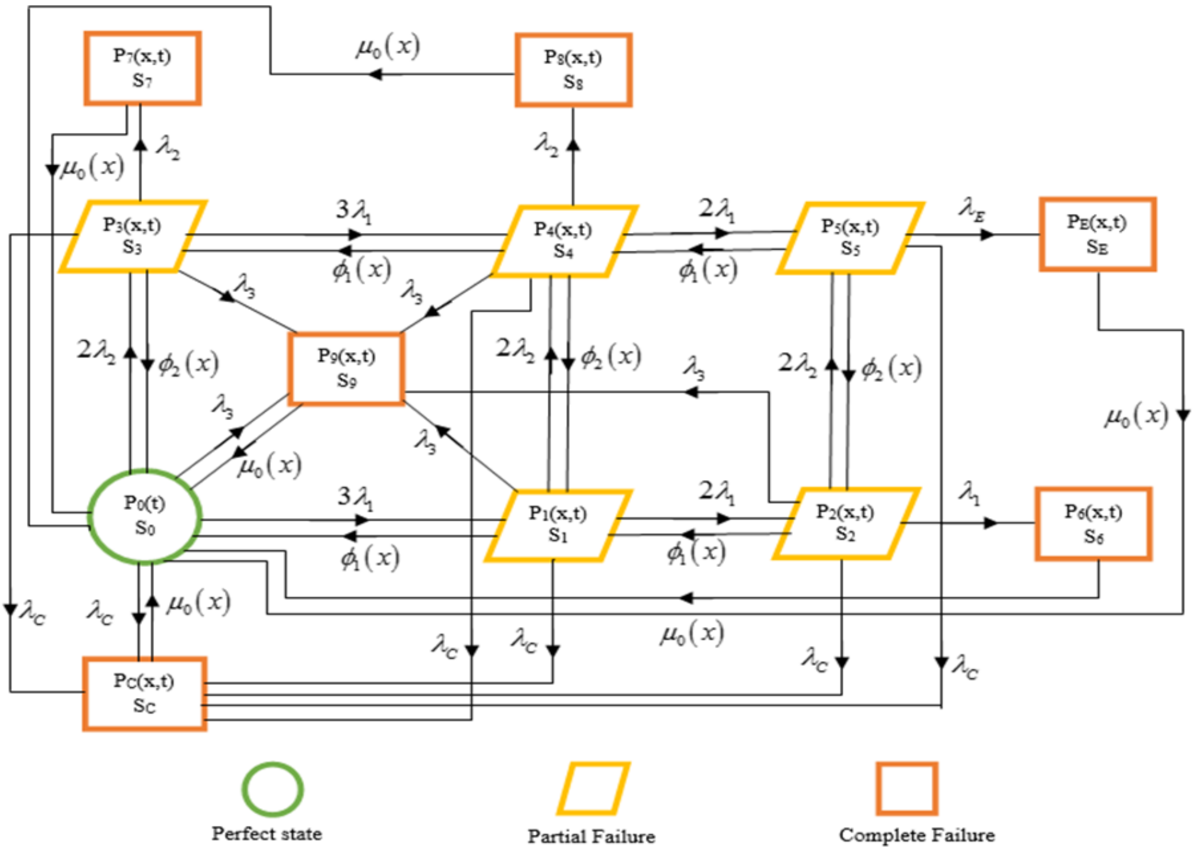

Fig 1: State transition diagram of the model

5. FORMULATION OF MATHEMATICAL MODEL

By probability of considerations and continuity arguments, we can obtain the following set of difference-differential equations associated with the present mathematical model:

$$\left[\frac{\partial}{\partial t}+3\lambda_1+2\lambda_2+\lambda_3+\lambda_C\right]P_0(t)=\int_0^\infty \phi_1(x)P_1(x,t)dx \quad (1)$$
$$+\int_0^\infty \phi_2(x)P_3(x,t)dx+\sum_k\int_0^\infty \mu_0(x)P_k(x,t)dx$$

where $k=6,7,8,9,E,C$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+2\lambda_1+2\lambda_2+\lambda_3+\lambda_C+\phi_1(x)\right]P_1(x,t)=0 \quad (2)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\lambda_1+2\lambda_2+\lambda_3+\lambda_C+\phi_1(x)\right]P_2(x,t)=0 \quad (3)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+3\lambda_1+\lambda_2+\lambda_3+\lambda_C+\phi_2(x)\right]P_3(x,t)=0 \quad (4)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+2\lambda_1+\lambda_2+\lambda_3+\lambda_C+\phi_1(x)+\phi_2(x)\right]P_4(x,t)=0 \quad (5)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\lambda_E+\lambda_C+\phi_1(x)+\phi_2(x)\right]P_5(x,t)=0 \quad (6)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\exp\left[x^\theta+\{\log\phi(x)\}^\theta\right]^{1/\theta}\right]P_k(x,t)=0 \quad (7)$$

$k=6,7,8,9,E,C$

Boundary Conditions,

$$P_1(0,t)=3\lambda_1P_0(t) \quad (8)$$

$$P_2(0,t)=2\lambda_1P_1(0,t)=6\lambda_1^2P_0(t) \quad (9)$$

$$P_3(0,t)=2\lambda_2P_0(t) \quad (10)$$

$$P_4(0,t)=3\lambda_1P_3(0,t)+2\lambda_2P_1(0,t)=12\lambda_1\lambda_2P_0(t) \quad (11)$$

$$P_5(0,t)=2\lambda_2P_2(0,t)+2\lambda_1P_4(0,t)=36\lambda_1^2\lambda_2P_0(t) \quad (12)$$

$$P_6(0,t)=\lambda_1P_2(0,t)=6\lambda_1^3P_0(t) \quad (13)$$

$$P_7(0,t)=\lambda_2P_3(0,t)=2\lambda_2^2P_0(t) \quad (14)$$

$$P_8(0,t)=\lambda_2P_4(0,t)=12\lambda_1\lambda_2^2P_0(t) \quad (15)$$

$$P_9(0,t)=\lambda_3\left(1+3\lambda_1+6\lambda_1^2+2\lambda_2+12\lambda_1\lambda_2\right)P_0(t) \quad (16)$$

$$P_E(0,t)=\lambda_EP_3(0,t)=2\lambda_2\lambda_EP_0(t) \quad (17)$$

$$P_C(0,t) = \lambda_C\left(1+3\lambda_1+6\lambda_1^2+2\lambda_2+12\lambda_1\lambda_2+36\lambda_1^2\lambda_2\right)P_0(t)$$
(18)

Initials conditions

$P_0(0)=1$, and other state probabilities are zero at $t=0$ (19)

Taking Laplace transformation of equations (1) to (18) and using equation (19), we obtain

$$\left[s+3\lambda_1+2\lambda_2+\lambda_3+\lambda_C\right]\bar{P}_0(s) = 1+\int_0^\infty \phi_1(x)\bar{P}_1(x,s)dx+\int_0^\infty \phi_2(x)\bar{P}_3(x,s)dx+\sum_k\int_0^\infty \mu_0(x)\bar{P}_k(x,s)dx$$

where $k=6,7,8,9,E,C$ and $\bar{P}_i(x,s)=\int_0^\infty e^{-st}P_i(x,t)dt$ (20)

$$\left[s+\frac{\partial}{\partial x}+2\lambda_1+2\lambda_2+\lambda_3+\lambda_C+\phi_1(x)\right]\bar{P}_1(x,s)=0 \quad (21)$$

$$\left[s+\frac{\partial}{\partial x}+\lambda_1+2\lambda_2+\lambda_3+\lambda_C+\phi_1(x)\right]\bar{P}_2(x,s)=0 \quad (22)$$

$$\left[s+\frac{\partial}{\partial x}+3\lambda_1+\lambda_2+\lambda_3+\lambda_C+\phi_2(x)\right]\bar{P}_3(x,s)=0 \quad (23)$$

$$\left[s+\frac{\partial}{\partial x}+2\lambda_1+\lambda_2+\lambda_3+\lambda_C+\phi_1(x)+\phi_2(x)\right]\bar{P}_4(x,s)=0 \quad (24)$$

$$\left[s+\frac{\partial}{\partial x}+\lambda_E+\lambda_C+\phi_1(x)+\phi_2(x)\right]\bar{P}_5(x,s)=0 \quad (25)$$

$$\left[s+\frac{\partial}{\partial x}+\exp\left[x^\theta+\left\{\log\phi(x)\right\}^\theta\right]^{1/\theta}\right]\bar{P}_k(x,s)=0$$
(26)

$$k=6,7,8,9,E,C$$

Boundary Conditions,

$$\bar{P}_1(0,s)=3\lambda_1\bar{P}_0(s) \quad (27)$$

$$\bar{P}_2(0,s)=2\lambda_1\bar{P}_1(0,s)=6\lambda_1^2\bar{P}_0(s) \quad (28)$$

$$\bar{P}_3(0,s)=2\lambda_2\bar{P}_0(s) \quad (29)$$

$$\bar{P}_4(0,s)=3\lambda_1\bar{P}_3(0,s)+2\lambda_2\bar{P}_1(0,s)=12\lambda_1\lambda_2\bar{P}_0(s) \quad (30)$$

$$\bar{P}_5(0,s)=2\lambda_2\bar{P}_2(0,s)+2\lambda_1\bar{P}_4(0,s)=36\lambda_1^2\lambda_2\bar{P}_0(s) \quad (31)$$

$$\bar{P}_6(0,s)=\lambda_1\bar{P}_2(0,s)=6\lambda_1^3\bar{P}_0(s) \quad (32)$$

$$\bar{P}_7(0,s)=\lambda_2\bar{P}_3(0,s)=2\lambda_2^2\bar{P}_0(s) \quad (33)$$

$$\bar{P}_8(0,s)=\lambda_2\bar{P}_4(0,s)=12\lambda_1\lambda_2^2\bar{P}_0(s) \quad (34)$$

$$\bar{P}_9(0,s)=\lambda_3\left(1+3\lambda_1+6\lambda_1^2+2\lambda_2+12\lambda_1\lambda_2\right)\bar{P}_0(s) \quad (35)$$

$$\bar{P}_E(0,s)=\lambda_E\bar{P}_3(0,s)=2\lambda_2\lambda_E\bar{P}_0(s) \quad (36)$$

$$\bar{P}_C(0,s)=\lambda_C\left(1+3\lambda_1+6\lambda_1^2+2\lambda_2+12\lambda_1\lambda_2+36\lambda_1^2\lambda_2\right)\bar{P}_0(s) \quad (37)$$

Change in Laplace transformation of boundary conditions after repair, if any

$$\bar{P}_1(0,s)=3\lambda_1\bar{P}_0(s)+\int_0^\infty\phi_1(x)\bar{P}_2(x,s)dx \quad (38)$$

$$\bar{P}_2(0,s)=2\lambda_1\bar{P}_1(0,s)+\int_0^\infty\phi_2(x)\bar{P}_5(x,s)dx \quad (39)$$

$$\bar{P}_3(0,s)=2\lambda_2\bar{P}_0(s)+\int_0^\infty\phi_1(x)\bar{P}_4(x,s)dx \quad (40)$$

$$\bar{P}_4(0,s)=3\lambda_1\bar{P}_3(0,s)+2\lambda_2\bar{P}_1(0,s)+\int_0^\infty\phi_1(x)\bar{P}_5(x,s)dx \quad (41)$$

Now solving all the equations with the boundary conditions, one may get

$$\bar{P}_0(s)=\frac{1}{D(s)} \quad (42)$$

$$\bar{P}_1(s)=\frac{3\lambda_1}{D(s)}\frac{1-P}{s+2\lambda_1+2\lambda_2+\lambda_3+\lambda_C} \quad (43)$$

$$\bar{P}_2(s)=\frac{6\lambda_1^2}{D(s)}\frac{1-Q}{s+\lambda_1+2\lambda_2+\lambda_3+\lambda_C} \quad (44)$$

$$\bar{P}_3(s)=\frac{2\lambda_2}{D(s)}\frac{1-R}{s+3\lambda_1+\lambda_2+\lambda_3+\lambda_C} \quad (45)$$

$$\bar{P}_4(s)=\frac{12\lambda_1\lambda_2}{D(s)}\frac{1-S}{s+2\lambda_1+\lambda_2+\lambda_3+\lambda_C} \quad (46)$$

$$\bar{P}_5(s)=\frac{36\lambda_1^2\lambda_2}{D(s)}\frac{1-T}{s+\lambda_E+\lambda_C} \quad (47)$$

$$\bar{P}_6(s)=\frac{6\lambda_1^3}{D(s)}\frac{1-\bar{S}_{\mu_0}(s)}{s}=\frac{6\lambda_1^3}{D(s)}\frac{1-U}{s} \quad (48)$$

$$\bar{P}_7(s)=\frac{2\lambda_2^2}{D(s)}\frac{1-\bar{S}_{\mu_0}(s)}{s}=\frac{2\lambda_2^2}{D(s)}\frac{1-U}{s} \quad (49)$$

$$\bar{P}_8(s)=\frac{12\lambda_1\lambda_2^2}{D(s)}\frac{1-\bar{S}_{\mu_0}(s)}{s}=\frac{12\lambda_1\lambda_2^2}{D(s)}\frac{1-U}{s} \quad (50)$$

$$\bar{P}_9(s)=\frac{\lambda_3 V}{D(s)}\frac{1-U}{s} \quad (51)$$

$$\bar{P}_E(s)=\frac{2\lambda_2\lambda_E}{D(s)}\frac{1-\bar{S}_{\mu_0}(s)}{s}=\frac{2\lambda_2\lambda_E}{D(s)}\frac{1-U}{s} \quad (52)$$

$$\bar{P}_C(s)=\frac{\lambda_C\left(V+36\lambda_1^2\lambda_2\right)}{D(s)}\frac{1-U}{s} \quad (53)$$

Where,

$$D(s)=s+3\lambda_1+2\lambda_2+\lambda_3+\lambda_C-3\lambda_1P-2\lambda_2R-UW$$

$$P=\frac{\phi_1}{s+2\lambda_1+2\lambda_2+\lambda_3+\lambda_C+\phi_1}$$

$$Q=\frac{\phi_1}{s+\lambda_1+2\lambda_2+\lambda_3+\lambda_C+\phi_1}$$

$$R=\frac{\phi_2}{s+3\lambda_1+\lambda_2+\lambda_3+\lambda_C+\phi_2}$$

$$S=\frac{\phi_3}{s+2\lambda_1+\lambda_2+\lambda_3+\lambda_C+\phi_3}$$

$$T=\bar{S}_{\phi_3}\left(s+\lambda_E+\lambda_C\right)=\frac{\phi_3}{s+\lambda_E+\lambda_C+\phi_3},$$

$$U=\bar{S}_{\mu_0}\left(s\right)=\frac{\mu_0}{s+\mu_0},$$

$$V=1+3\lambda_1+6\lambda_1^2+2\lambda_2+12\lambda_1\lambda_2$$

and

$$W=6\lambda_1^3+2\lambda_2^2+12\lambda_1\lambda_2^2+2\lambda_2\lambda_E+V\left(\lambda_3+\lambda_C\right)+36\lambda_1^2\lambda_2\lambda_C$$

Sum of Laplace transformations of the state transitions, where the system is in operational mode and failed state at any time, is as follows

$$\bar{P}_{up}\left(s\right)=\bar{P}_0\left(s\right)+\bar{P}_1\left(s\right)+\bar{P}_2\left(s\right)+\bar{P}_3\left(s\right)+\bar{P}_4\left(s\right)+\bar{P}_5\left(s\right) \quad (54)$$

$$\bar{P}_{down}\left(s\right)=1-\bar{P}_{up}\left(s\right) \quad (55)$$

6. ANALYTICAL STUDY

6.1 Availability Analysis

When repair follows general and Gumbel-Hougaard family copula distribution, we have

$$\bar{S}_{\mu_0}\left(s\right)=\frac{\exp\left[x^\theta+\left\{\log\phi\left(x\right)\right\}^\theta\right]^{1/\theta}}{s+\exp\left[x^\theta+\left\{\log\phi\left(x\right)\right\}^\theta\right]^{1/\theta}}$$

setting $\bar{S}_{\alpha_i}\left(s\right)=\frac{\alpha_i}{s+\alpha_i}, i=1,2,3$. Following cases have been considered:

(a) Taking the values of different parameters as $\lambda_1=0.020$, $\lambda_2=0.025$, $\lambda_3=0.030$, $\lambda_E=0.040$, $\lambda_C=0.015, \phi_i=1, x=1\left(i=1,2,3\right)$ in (54) and then taking inverse Laplace transform, we obtain the availability of the system:

$$\begin{aligned}P_{up}(t)=&-0.000324e^{-1.1150t}-0.001009e^{-1.1100t}\\&-0.000149e^{-1.0550t}+0.020391e^{-2.7672t}\\&-0.018120e^{-1.2268t}-0.000026e^{-1.1322t}\\&+0.999239e^{-0.0037t}\end{aligned} \quad (56)$$

(b) Taking the values of different parameters as $\lambda=0.025$, $\phi_i=1, x=1\left(i=1,2,3\right)$ in (54) and then taking inverse Laplace transform, we obtain the availability of the system:

$$\begin{aligned}P_{up}(t)=&-0.000253e^{-1.0500t}-0.001858e^{-1.1250t}\\&+0.022656e^{-2.7740t}-0.022560e^{-1.2549t}\\&+1.002016e^{-0.00599t}\end{aligned} \quad (57)$$

(c) Repair follows general distribution by taking $\mu_0\left(x\right)=\phi_i\left(x\right)$ and same values of failure rates as in case (a) in (54) and then taking inverse Laplace transform, we obtain the availability of the system:

$$\begin{aligned}P_{up}(t)=&-0.000351e^{-1.1150t}+0.005406e^{-1.2561t}\\&+8.8603\ 10^{-7}e^{-1.1322t}+0.027113e^{-1.0279t}\\&+0.969217e^{-0.0036t}-0.000268e^{-1.0550t}\\&-0.001118e^{-1.1100t}\end{aligned} \quad (58)$$

For different values of time variable $t=0,10,20,30,40,50,$ $60,70,80,90$ and 100 units of time, one may get different values of $P_{up}(t)$ with the help of (51) as shown in Figure-2.

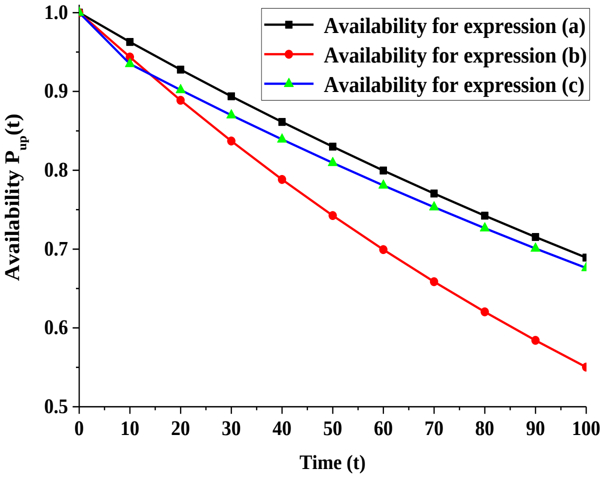

Figure-2 Availability as a function of time

6.2 Reliability Analysis

Taking all repair rates equal to zero and obtain inverse Laplace transform, we get an expression for the reliability of the system after taking the failure rates as $\lambda_1=0.020,\ \lambda_2=0.025,\ \lambda_3=0.030,\ \lambda_E=0.040$ and $\lambda_C=0.015$:

$$\begin{aligned}R_l(t)=&\,3.000000e^{-0.1350t}+0.060000e^{-0.1150t}\\&+2.000000e^{-0.1300t}+0.133333e^{-0.110000t}\\&-4.196933e^{-0.1550t}+0.003600e^{-0.0550t}\end{aligned} \quad (59)$$

For different values of time variable $t=0,10,20,30,$ $40,50,60,70,80,90$ and 100 units of time, one may get different values of reliability $R_l(t)$ with the help of (59) as shown in Figure-3.

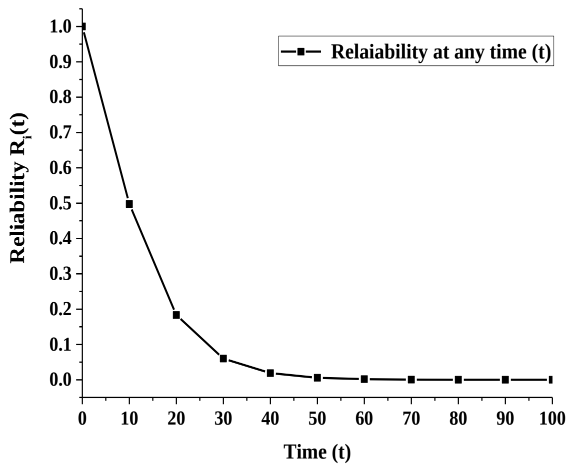

Figure 3 Reliability as a function of time

6.3 Cost Analysis

For the assumed failure and repair rates in section 5.1 and corresponding to the state transition diagram, we have computed the incurred profit for two cases when the system follows copula repair and general repair in (61) & (62). Let the service facility be always available, then expected profit during the interval $\left[0,t\right)$ is

$$E_p(t) = K_1 \int_0^t P_{up}\ (t)dt - K_2(t) \tag{60}$$

Where K_1 and K_2 are the revenue generation and service cost in unit time. For same set of parameters defined in (50), one can obtain expression for incurred profit as a function of time as:

$$\begin{aligned} E_p(t) = K_1\{&0.014770e^{-1.2268t} + 0.000024e^{-1.1322t} \\ &-0.007368e^{-2.7672t} + 0.000909e^{-1.1100t} \\ &+0.000291e^{-1.1150t} + 0.000142e^{-1.0550t} \\ &-269.039471e^{-0.0037t} + 269.030703\} - K_2 t \end{aligned} \tag{61}$$

$$\begin{aligned} E_p(t) = K_1\{&-7.8255\ 10^{-7}e^{-1.1322t} - 0.0264e^{-1.0279t} \\ &-0.001007e^{-1.1100t} - 0.004304e^{-1.2562t} \\ &+0.000315e^{-1.1150t} + 0.000254e^{-1.0550t} \\ &-269.001600e^{-0.0036t} + 269.030703\} - K_2 t \end{aligned} \tag{62}$$

Setting $K_1 = 1$ and $K_2 = 0.6, 0.5, 0.4, 0.3, 0.2$ and 0.1 respectively and varying $t = 0, 10, 20, 30, 40, 50, 60, 70,$ $80, 90$ and 100 units of time, the results for expected profit can be obtain as per table-5 and 6 and figure-5 and 6.

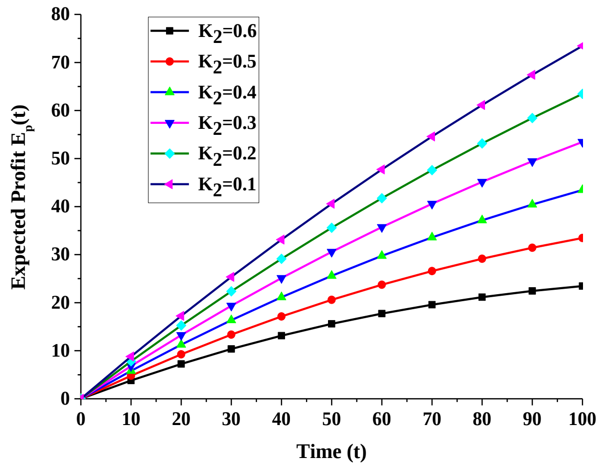

Figure-4 Expected profit as a function of time for Copula repair

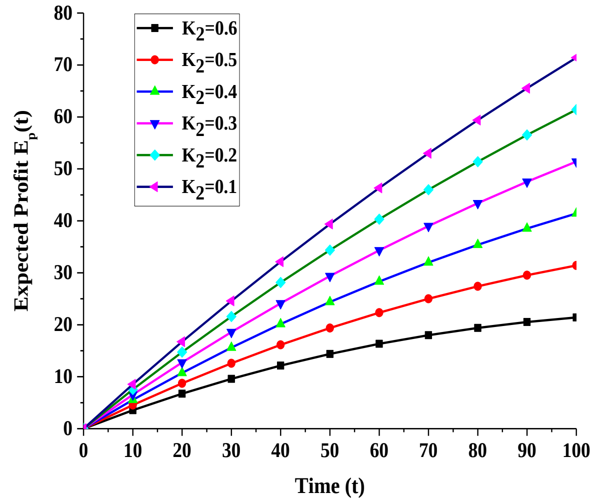

Figure-5 Expected profit as a function of time for general repair

7. RESULTS

This paper studies the reliability characteristics of standby system consisting of three subsystems in series configuration under catastrophic failure. Explicit expressions have been derived using supplementary variable technique.Figure-2 gives the analysis of availability of the system in three different possibilities. One can clearly observe that availability of the system initially decreases with respect to time. Figure-3 gives information for reliability and it shows a steep fall in reliability from top to lowermost in a very short period based on failure rate of units. We can observe that corresponding values of availability are greater than the values of reliability, which highlights the requirement of systematic repair for any complex systems for healthier performance.

An acute examination from Figure-6 and Figure-7 reveals that expected profit increases as service cost K_2 decreases, while the revenue cost per unit time is fixed at K_1=1 in case of both copula and general repair. The calculated expected profit is maximum for K_2= 0.1 and minimum for K_2=0.6. We observe that as service cost decreases, profit increase with variation of time. In general, for low service cost, the expected profit is high in comparison to high service cost. Conclusively, copula repair is more effective repair policy for better performance of repairable systems. It gives a new aspect to scientific community to adopt multi-dimension repair in form of copula. Furthermore, the model developed in this chapter has been found to be very useful for power generation and transmission systems, as we have six equipment groups and are helpful in proper maintenance analysis, decision-making and performance assessment. We may expand our work to a number of methods, such as the Kolmogorov method and the fuzzy reliability method, by considering repair rates as constant. Furthermore, the system can be analyzed by taking k-out-of-(m+n): G/F policy.

8. ACKNOWLEDGMENTS

Our thanks to the experts who have contributed towards development of the template.

9. REFERENCES

[1] Bao, X. and Cui, L., “An analysis of availability for series Markov repairable system with neglected or delayed failures,” IEEE Transactions on Reliability, 59(4): pp. 734–743, 2010.

[2] Chen Y. and Yang Q.Y., “Reliability of two-stage weighted k-out-of-n systems with components in common,” IEEE Transactions on Reliability, 54(3): pp. 431-440, 2005.

[3] Huang, J., Zuo M.J. and Wu, Y. 2000. Generalized multi-state k-out-of-n: G systems. IEEE Trans. Reliab. 49(1): pp. 105–111.

[4] Jia, X., Shen, J. and Xing, R., “Reliability analysis for repairable multistate two-unit series systems when repair time can be neglected,” IEEE Transactions on Reliability, 65(1): pp. 208-216, 2016.

[5] Kumar, P and Gupta, R. (2007). Reliability analysis of a single unit M|G|1 system model with helping unit. Journal of Combinatorics, Information & System Sciences, J. of Comb. Info. & System Sciences, Vol. 32, No. 1-4, pp. 209-219.

[6] Kumar, P and Sirohi, A. (2011). Transient analysis of reliability with and without repair for k-out-of-n: G systems with three failure modes, J. of mathematics and computing science, Vol-2 (1), pp. 151-162.

[7] Kumar, P and Sirohi, A. (2015). Profit analysis of a two-unit cold standby system with delayed repair of partially failed unit and better utilization of units. International Journal of Computer Applications. 117(1), pp. 41-46.

[8] Kumar, P and Sharma, S. K. (2006). A two non-identical unit parallel system with inspection and correlated lifetimes. J. Rajasthan Academy Physical Sciences, 5 (3), pp. 301-318.

[9] Liang, X., Xiong, Y. and Li, Z., “Exact reliability formula for consecutive k-out-of-n repairable systems,” IEEE Transactions on Reliability, 59(2): pp. 313–318, 2010.

[10] Moghaddass R., Zuo M.J. and Wang W., “Availability of a general k-out-of-n: G system with non-identical components considering shut-off rules using quasi-birth-death process,” Reliability Engineering and System Safety, 96: pp. 489–496, 2011.

[11] Park, M. and Pham, H., “A generalized block replacement policy for a k-out-of-n system with respect to a threshold number of failed components and risk costs,” IEEE Trans. System Man, Cybernet. A Syst., Humans, 42(2): pp. 453–463, 2012.

[12] Poonia, P.K. (2021) Performance assessment of a multi-state computer network system in series configuration using copula repair. Int. J. Reliability and Safety, Vol. 15, No. 1/2, pp. 68-88. doi: 10.1504/IJRS.2021.10043291

[13] Poonia, P.K. 2022. Exact reliability formula for n-clients computer network with catastrophic failure and copula repair. *Int. J. of Computing Science and Mathematics*, In press.

[14] Poonia, P.K. (2022) Performance assessment and sensitivity analysis of a computer lab network through copula repair with catastrophic failure. *Journal of Reliability and Statistical Studies*, Vol. 15, No. 1, pp. 105-128.

[15] Poonia, P.K. and Sirohi, A. (2022). Stochastic analysis of a priority based warm standby system working under k-out-of-n: G policy using multi-dimensional repair. Application of advanced optimization Techniques in Industrial Engineering. doi: 10.1201/9781003089636-7, pp. 101-119, CRC Press.

[16] Poonia, P.K., Sirohi, A. and Kumar, A. (2021) Cost analysis of a repairable warm standby *k*-out-of-*n*: G and 2-out-of-4: G systems in series configuration under catastrophic failure using copula repair. Life Cycle ReliabSaf Eng., Vol. 10, No. 2, pp. 121-133.

[17] Singh, V.V. and Poonia, P. K. (2022) Stochastic analysis of k-out-of-n: G type of repairable system in combination of subsystems with controllers and multi repair approach.*Journal of Optimization in Industrial Engineering*, Vol. 15, No. 1, pp. 121-130.

[18] Singh, V.V., Poonia, P.K., Labaran, J.U. and Abdullahi, I. (2021) Probabilistic analysis of a multi-state warm standby k-out-of-n: G system in a series configuration using copula linguists. Reliability theory and applications, Vol. 16, No. 2(62), pp. 22-39.

[19] Sirohi, A., Poonia, P.K. and Raghav, D. (2021) Reliability analysis of a complex repairable system in series configuration with switch and catastrophic failure using copula repair, J. Math. Comput. Sci., 11 (2), pp. 2403-2425.

[20] Wu, Y.Q. and Guan, J.C., “Repairable consecutive-k-out-of-n: G systems with r repairman,” IEEE Transactions on Reliability, 54(2): pp. 328-337, 2005.

[21] She, J. and Pecht, M.G. Reliability of a k-out-of-n warm-standby system,” IEEE Trans. Reliab., 41: pp. 72–