Frequency Response Analysis for T-S Fuzzy Systems

Shinq-Jen Wu
Department of Electrical Engineering Da-Yeh University
168 University Rd., Dacun
Changhua 51591, Taiwan, R.O.C

ABSTRACT
Frequency response is an important physical quantity in audio systems for minimizing audible distortion and in control systems for assessing system relative stability. For linear time-invariant systems (LTI systems), frequency response has one-to-one relationship to system impulse response. However, the LTI systems-based analysis cannot describe nonlinear frequency response well. T-S fuzzy systems are based on fuzzily blending several linear subsystems and are widely used to describe nonlinear systemsbehavior in various fields. Recently, researcherstried to obtain frequency response of T-S fuzzy systems through the assumption thatthe same fuzzy relationship exists in both time domain and frequency domain. In this paper, we focus on deriving fuzzy frequency response from both *basic* definitions of frequency response and previously proposed neural-network-based fuzzy blending feature to find out the limitations, the range of frequency response for T-S fuzzy systems. Steady-state system response is additionally discussed and tested with two active magnetic bearing systems.

Keywords
Fuzzy systems, frequency response, relative stability

1. INTRODUCTION
Fuzzy logic (an extension of multivalued logic) is always used to describe the classes of objects or data which are vague and lack certainty.Recently, T-S type fuzzy systems were widely used to describe the dynamic behavior of various *physical systems* and have demonstrated to be universal approximations of any smooth nonlinear systems [1-3]. Various fuzzy controllers for physical systems have been developed since 2000. In 2016 Wiktorowic further used frequency domain methods to design output-feedback-based adaptive fuzzy controllers [4]. Kluska and Zabinski recently proposed PID-like adaptive fuzzy controllers [5]. Zhao and coworkers developed observer-based H_∞ controllers for spacing type-II T-S fuzzy systems [6]. We demonstrated that affine T-S fuzzy systems possess more advantages than T-S fuzzy systems in nonlinear modelling [7].Wang and Yang introduced dilated linear matrix inequality to develop piecewise controllers for affine T-S fuzzy systems [8, 9], and proposedpiecewise-Lyapunov-functions-based H_∞ controllers for affine T-S fuzzy systems in finite frequency domain [10]. Qiu and coworkers used quantized measurements to develop observer-based piecewise output-feedback controllers for affine T-S fuzzy systems [11]. Recently, research focus on the development of filter design of T-S fuzzy models of physical systems. Liu and Yang discussed sensor fault detection for systems in finite domain [12]. Hellani and coworkers developed finite-frequency-basedH_∞filter design [13]. Li and coworkers proposed finite-frequency-based $L_2 - L_\infty$ filters [14].

Fuzzy set theory showsgreat potential in dealing with biological data and modelling*biological systems* because of the use of linguistic variables and fuzzy relationship. Luo and An took a review of fuzzy set theory in device control, biological control, classification and pattern recognition, and prediction and association [15]. Komlyama and coworkers emphasized that fuzzy interactions always exist in cell macromolecularnanoarchitectonics[16].Abyad and coworkers used T-S fuzzy models to describe biomass growth processes and then optimal fuzzy control was used to control the process [17]. Bordon et al. used fuzzy logic to achieve the quantitatively modelling of repressilators withunknown kinetic data [18].Liu and coworkers introduced fuzzy Petri nets for biological system modellingand discussed the capacities and applications [19].They further proposed a hybrid of continuous Petri nets and fuzzy inference systems to achieve integrated modelling of biological systems with uncertainties [20].Zhu and coworkers used fuzzy neural networksas inverse systems to achieve decoupling control ofmarine biological enzyme fermentation processes [21].An adaptive T–S type neural-fuzzy scheme was proposed to achieve the fuzzy modeling of multi-inputs multi-outputs *biological systems* (small-scale genetic networks, branchpathways and cascadenetworks systems) [22]. The number of generated rules depends on the number of input variables of underlying systems and the division of the input space: There are 3^n rule numbers for an n-dimensional biological system with each input variable being divided into three intervals. To reduce the number of rules, researchers tried to construct biological fuzzy systems with a fixed number of rules. However, to determine the number of rules which are sufficient to ensure the accuracy researchers should fully understand underlying biological systems. This will lose the essence of adaptive T–S type neural-fuzzy modeling.

Frequency response is one of important analysis methods in audio, control and statistic fields. For liner time-invariant system, sinusoidal input signals generate sinusoidal output signals with differentamplitudes and phasesdetermined by system frequency response. However, such good characteristics do not exist in nonlinear systems. T-S fuzzy systems, at a time instant, are the fuzzy blending of their linear subsystems. This feature prompted researchers to explore the feasibility of frequency analysis of T-S fuzzy systems. Kumar and coworkers proposed a probabilistic data-driven geospatial fuzzy frequency ratio for avalanche susceptibility mapping [23]. Ali and coworkers used frequency domain analysis (power spectral density graphs) to get both primary and average frequencies as fuzzy inputs for rotating machinery vibration analysis [24]. Ferreira and Serra tried to propose a definition on fuzzy frequency response [25].They used the proposed estimation methods of frequency response to deal with experimental data of mechanical structures of aircraft and aerospace vehicles[26], and data from flexible robot arms [27]. They also do a case study for pH neutralization process [28]. However, an important issue was ignored that membership functions of fuzzy systems are time-varying functions, and the corresponding normalized firing strength cannot be represented by fixed functions. In this study, we shall stem on the principle of frequency response to define the frequency response of T-S fuzzy systems.

2. FUZZY-NEURAL-NETWORKS-BASED FREQUENCY RESPONSE

Frequency response methods serves as the backbone of the classical control methods and still give shed light to such an importantcharacteristic as robustness for modern control techniques. Musical notes generated by a guitar are related to its frequency response [29]. For linearly time-invariant systems, system outputs $y(t)$ are the convolution of system impulse response $h(t)$ and system inputs $u(t)$; $y(t) = h(t) * u(t)$, where the notation $*$ denotes the convolution operation. We then have $Y(jw) = H(jw)U(jw)$, where $Y(jw), H(jw)$ and $U(jw)$ are, respectively, the Fourier transforms of $y(t)$, $h(t)$ and $u(t)$, and $H(jw)$ denotes system frequency response. However, there does not exist such a relationship in nonlinear systems although system inputs and outputs also have their corresponding Fourier transforms.

Wehave previously developed neural–fuzzy inference networks to capture the dynamic behavior of current/voltage-controlled 1/4-vehicle MagLev systems [7], car model systems [30], radial active magnetic bearing systems [31], half-car active suspension systems [32].In this study, we shall derive the frequency response of T-S fuzzy systems based onpreviously proposed self-constructing neural–fuzzy inference networks in Fig. 1 which were used to realize T–S fuzzy modelling of various physical systems [7, 30-32].

The neural-fuzzy inference network in Fig. 1 possesses sixlayers [7]. Layers 1 and 6 are the input layer and output layer, respectively. There are four hidden layers which are corresponding to fuzzification, fuzzy blending, normalization anddefuzzification of fuzzy logic inference systems. Each node has finite weighted fan-in connections to the last-layer nodes and fan-out connectionsto the next-layer nodes.

Layer 1: The nodesdenote input variables and directly transmit input variables to the next layer.
Layer 2: This layer performs fuzzification and each node in this layer denotes a linguistic label. In this study, we choose Gaussian distributions as membership functions to achieve smooth and general fuzzification.
Layer 3: This layer performs fuzzily blending operation.Each node describes one fuzzy logic rule.
Layer 4: This layer performs normalization to let the summation of firing strength of rules be one.
Layer 5: This layer is the consequence layer which performs Sugeno-type defuzzification.
Layer 6: This layer performs defuzzification operations and each node corresponds to one output variable of underlying systems.

The numeric input variable x_l is directly transmitted into the network in Layer 1, and then fuzzified to fuzzy term sets T_{lj} which possess Guassian membership functions with mean m_{lj} and standard deviation σ_{lj} in Layer 2. The firing strength of each fuzzy rule are obtained in Layer 3 and the corresponding normalized firing strength are then estimated in Layer 4.The linear-system-type rule consequences are generated in Layer 5 and system outputs are obtained through Sugeno-type defuzzification in Layer 6.

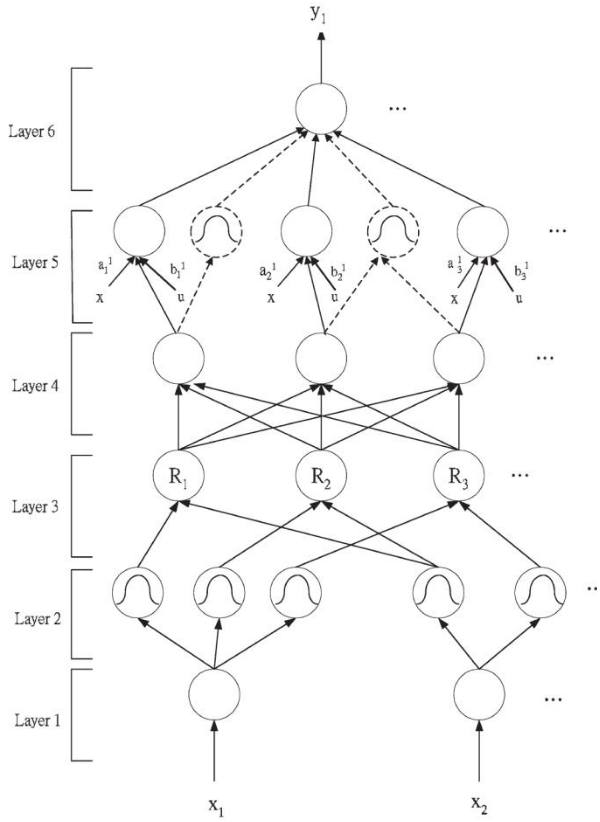

Fig. 1: Neural–fuzzy inference network for T–S fuzzy systems [7].

According to fuzzy set theory and the scheme of neural fuzzy inference systems in Fig. 1 [7], we know at any time instance the overall dynamic behavior of T-S fuzzy systems can be described as fuzzily blending the dynamic behavior of their linear subsystems. Therefore, the unit impulse response of the entire fuzzy system is $\mathrm{h(t)} = \sum_i^n \mu_i(x(t))h_i(t)$, where$h_i(t), \mu_i(x(t))$are the unit impulse response of the i-th subsystem and the corresponding firing strength, respectively. Then, the following theorems are derived.

Theorem 1: The frequency response of the entire fuzzy system is

$$\mathrm{H(jw)} = \sum_i^n \mathrm{M}_i(\mathrm{x(jw)}) * H_i(jw), \quad (1)$$

where * denotes the convolution operator; $H_i(jw)$ and $\mathrm{M}_i(\mathrm{x(jw)})$ are, respectively, the corresponding Fourier transform of the unit impulse response $h_i(t)$ and the normalized firing strength$\mu_i(x(t))$, i.e.,$H_i(jw) = \mathcal{F}\{h_i(x(t))$ and $\mathrm{M}_i(\mathrm{x(jw)}) = \mathcal{F}\{\mu_i(x(t))\}$.

Proof:At any time-instant,the unit impulse response of the entire fuzzy system is $h(t) = \sum_i^n \mu_i(x(t))h_i(t)$.The system frequency response is then obtained through Fourier transform,

$$\mathrm{H(jw)} = \mathcal{F}\{h(t)\} = \mathcal{F}\left\{\sum_i^n \mu_i(x(t))h_i(t)\right\}$$

$$= \sum_i^n \mathcal{F}\{\mu_i(x(t)) * H_i(jw)$$

$$= \sum_i^n \mathrm{M}_i(\mathrm{x(jw)}) * H_i(jw) \qquad (2) \qquad \blacksquare$$

Therefore, the frequency response of the entire fuzzy systems $\mathrm{H(jw)} \neq \sum_i^n \mu_i(x(t))H_i(jw)$. Ferreira and Serra proposed a series of researches in fuzzy frequency response [25, 27, 33] which are all based on an assumption that the normalized firing strength $\mu_i(x(t))$ is a constant value μ_i and assume that $\mathrm{H(jw)} = \sum_i^n \mu_i \cdot H_i(jw)$.

Theorem 2: In the frequency domain there does not exist a simple relationship between system outputs Y(jw)and system inputs U(jw):

$\mathrm{Y(jw)} \neq \mathrm{H(jw)U(jw)}$,(3)

where Y(jw) and U(jw) are the Fourier transform of system output y(t) and system input u(t).

Proof: At any time-instant, system response is the fuzzily blending of all of subsystem response. We have

$y(t) = \sum_i^n[\mu_i(x(t)) \cdot (h_i\ (t) * u(t))]$, (4)

where * denotes convolution operators. We know Fourier transform is a linear operator. So, $Y(jw) = \mathcal{F}\{y(t)\}$becomes

$$Y(jw) = \sum_i^n \mathcal{F}\{\mu_i(x(t)) \cdot (h_i\ (t) * u(t))\}$$

$$= \sum_i^n \mathcal{F}\{\mu_i(x(t))\} * (H_i\ (jw)U(jw))$$

$$= \sum_i^n \mathrm{M}_i(\mathrm{x(jw)}) * H_i(jw)\mathrm{U(jw)}$$

$$\neq \mathrm{H(jw)U(jw)}. \qquad (5) \quad \blacksquare$$

We know that the normalized firing strength$\mu_i(x(t))$not only varies by time but also depends on subsystems. We cannot use a fixed function to describe the normalized firing strength even the Gaussian membership function is used. Therefore, there are no analytical solution for the corresponding Fourier transform, $\mathrm{M}_i(\mathrm{x(jw)})$. In the future we shall integrate block-based graphical methods and numerical methods to estimate $\mathrm{M}_i(\mathrm{x(jw)})$ and then to get a numericalapproximation ofH(jw).

Theorem 3: $2\pi\delta(\mathrm{w}) \min_i \inf_t h_i\ (t) \leq \mathrm{H(jw)} \leq 2\pi\delta(\mathrm{w})\max_i\|h_i(t)\|_\infty$, where $\delta(\mathrm{w})$ is the impulse function.

Proof: $h(t) = \sum_i^n \mu_i(x(t))h_i\ (t)$. The corresponding Fourier transform is

$\mathrm{H(jw)} = \int_{-\infty}^{\infty} \sum_{i=1}^n \mu_i(x(t))h_i\ (t)e^{-jwt}\ dt.$ (6)

According to $\sum_{i=1}^n \mu_i(x(t)) = 1$,we obtain

$$\sum_{i=1}^n \mu_i(x(t))h_i\ (t) \leq \sum_{i=1}^n \mu_i(x(t)) \cdot \sup_t h_i\ (t)$$

$$= \sum_{i=1}^n \mu_i(x(t)) \cdot \|h_i(t)\|_\infty$$

$$\leq \sum_{i=1}^n \mu_i(x(t)) \cdot \max_i\|h_i(t)\|_\infty$$

$$= \max_i\|h_i(t)\|_\infty. \qquad (7)$$

$$\sum_{i=1}^n \mu_i(x(t))h_i\ (t) \geq \sum_{i=1}^n \mu_i(x(t)) \cdot \inf_t h_i\ (t)$$

$$\geq \sum_{i=1}^n \mu_i(x(t)) \cdot \min_i \inf_t h_i\ (t)$$

$$= \min_i \inf_t h_i\ (t). \quad (8)$$

Therefore, we have

$$\int_{-\infty}^{\infty} \min_i \inf_t h_i\ (t)e^{-jwt}\ dt \leq \mathrm{H(jw)}$$

$$\leq \int_{-\infty}^{\infty} \max_i\|h_i(t)\|_\infty e^{-jwt}\ dt. \quad (9)$$

We then obtain the following inequality because both $\max_i\|h_i(t)\|_\infty$ and$\min_i \inf_t h_i\ (t)$ are constants for the integral, and $\mathcal{F}\{e^{-jw_0t}\} = 2\pi\delta(w - w_0)$.

$2\pi\delta(\mathrm{w}) \min_i \inf_t h_i\ (t) \leq \mathrm{H(jw)} \leq 2\pi\delta(\mathrm{w})\max_i\|h_i(t)\|_\infty$. (10) ■

3. STEADY-STATE SYSTEMS RESPONSE

Steward mentioned in [29] that frequency response is related to the steady state of a system when a harmonic function is applied as the input.In this section we demonstrate that the steady states responseof T-S fuzzy systems are predicable when input signals are exponential functions, sine functions and cosine functions, even frequency response of underlying systems cannot obtain through analytical methods. However, there does not exist such a relationship that the responses to sine input signals and cosine input signals are exactly the real part and imaginary part of the response to exponential input signals, respectively.

Theorem 4: The steady state response $y_{ss}(\mathrm{t})$of T-S fuzzy systems to harmonic input signals u(t) is $y_{ss}(\mathrm{t}) = u_0\overline{H}(\mathrm{jw})e^{jwt}$ for$\mathrm{u(t)} = u_0e^{jwt}$,(11)

$y_{ss}^s(\mathrm{t}) = u_0\overline{H}(\mathrm{jw})\sin(\mathrm{wt})$ for$\mathrm{u(t)} = u_0\sin(\mathrm{wt})$,(12)

$y_{ss}^c(\mathrm{t}) = u_0\overline{H}(\mathrm{jw})\cos(\mathrm{wt})$ for$\mathrm{u(t)} = u_0\cos(\mathrm{wt})$,(13)

where $\overline{H}(\mathrm{jw}) = \sum_i^n \mu_i(x_s)\,H_i(\mathrm{jw})$ and $\mu_i(x_s)$ is the normalized firing strength of the i-th subsystem at the steady state x_s . However, $y_{ss}^s(\mathrm{t}) \neq I_m[y_{ss}(\mathrm{t})]$ and $y_{ss}^c(\mathrm{t}) \neq R_e[y_{ss}(\mathrm{t})]$.

Proof:The steady state response is defined as $y_{ss}(t) \triangleq \lim_{t\to\infty} y(t)$which is the fuzzily blending of the response of subsystems at the steady state, i.e., $y_{ss}(t) = \lim_{t\to\infty} \sum_i^n \mu_i(x(t))[h_i\ (t) * u(t)] = \sum_i^n \mu_i(x_s)\lim_{t\to\infty}[h_i\ (t) * u(t)]$. For each subsystem which is linear time-invariant system, we have the following responses to harmonic inputs at the steady state,

$$h_i(t) * u(t) = u_0H_i(\mathrm{jw})e^{jwt}\ ,\mathrm{for u(t)} = u_0e^{jwt},$$

$$= u_0H_i(\mathrm{jw})\sin(\mathrm{wt}), \mathrm{for u(t)} = u_0\sin(\mathrm{wt}),$$

$$= u_0H_i(\mathrm{jw})\cos(\mathrm{wt}), \mathrm{for u(t)} = u_0\cos(\mathrm{wt}). \ (14)$$

At the steady state x_s, the normalized firing strength $\mu_i(x_s)$is a constant value. So, we obtain Eqs. (11)~(13) through setting $\overline{H}(\mathrm{jw}) = \sum_i^n \mu_i(x_s)\,H_i(\mathrm{jw})$. ■

Theorem 4 is a predictionofthe dynamic behavior of T-S fuzzy systems. The steady states of biological systems always depend on experimental environments. Therefore, we here use two physical systems to examine the prediction.

- ***horizontal radial active magnetic bearings***

We first consider anradial magnetic bearing system, as shown in Fig. 2 [34],wherethe roll mass $m = 0.2Kg$,the nominal air gap $e = 0.5mm$ and the force constants $\lambda_1 = \lambda_1 = 0.000005\left[\frac{Nm^2}{A^2}\right]$; $x$denotes the horizon deviation of the center of the ball from its nominal position; $F_1, F_2$are the magnetic forces; and $i_{1,}i_2$are the associated currents. Thedynamics of this active magnetic bearing system is described as Eq. (15) [34].

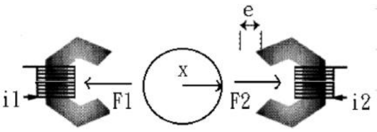

Fig.2: horizontal active magnetic bearing systems [34].

$$m\ddot{x} = F_1 - F_2 = \frac{\lambda_1 i_1^2}{(e-x)^2} - \frac{\lambda_2 i_2^2}{(e+x)^2}. \quad (15)$$

We choose the rotor position and control current as the traininginputs and the rotor acceleration as the trainingoutput.2300 training patterns generated from Eq. (15) are fed into the neural–fuzzy inference network in Fig. 1.Thelearning rate is set at 0.005. After training we have the following T-S fuzzy system [31],

$$\begin{aligned} &R^1: If\ x(t)\ is\ T_1(-0.0025, 0.005), \\ &\qquad then\ \dot{X}(t) = A_1X(t) + B_1u(t). \\ &R^2: If\ x(t)\ is\ T_2(-0.003536, 0.07497), \\ &\qquad then\ \dot{X}(t) = A_2X(t) + B_2u(t), \quad (16) \end{aligned}$$

where the system states $X(t) = [\dot{x}(t)\ x(t)]^T$ and the system inputs $u(t) = [i_1(t) i_2(t)]^T$;the fuzzy term set$T_i(m_{i,}\sigma_i)$,$i = 1{,}2$,has Gaussian membership function with mean $m_{i,}$ and standard deviation σ_i ;the system parameters $A_1 = \begin{bmatrix} 0 & 55000 \\ 1 & 0 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 43000 \\ 1 & 0 \end{bmatrix}$, $B_1 = \begin{bmatrix} 63 & -59.6 \\ 0 & 0 \end{bmatrix}, B_2 = \begin{bmatrix} 49.9525 & -46.251 \\ 0 & 0 \end{bmatrix}$. According to Theorem 4, we are able to predict the steady state response of T-S fuzzy system in Eq. (13) to two sinusoidal inputs, $u_1(t) = \sin(0.3907t)$ and $u_2(t) = \sin(0.1931t)$. The sinusoidal input$u_1(t)$ and the correspondingsteady state response are shown in blue color, and those to $u_2(t)$are shown in red color. We observe that the steady state response y_{ss}(t)oscillates at $\pm 0.001105$for $u_1(t)$and at $\pm 0.001079$for $u_2(t)$.

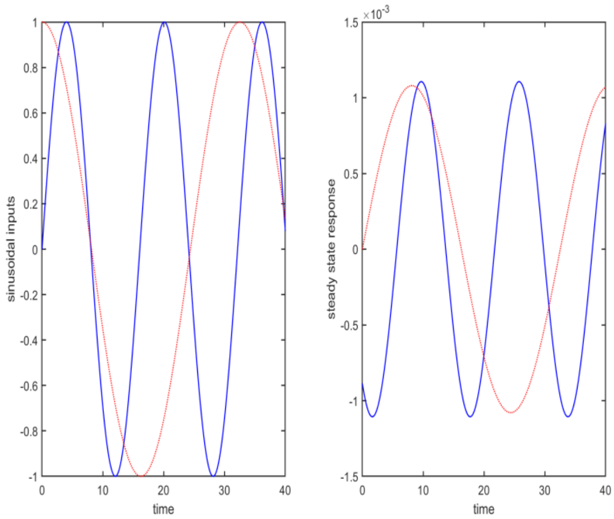

Fig. 3: steady state response y_{ss}(t) for fuzzy system in Eq. (16).

- ***horizontal differential-driving-mode magnetic bearings***

We thenconsider a differential-driving-mode bearing systems in Fig. 4 [35]. One magnet is driven by the sum of bias-current and control-current and the other by the differencebetween these two currents. We have the dynamic equation of rotor motion in Eq. (17) [35].

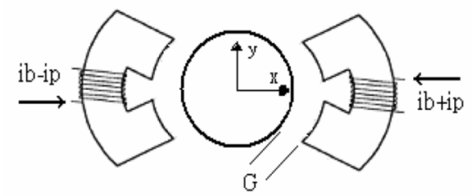

Fig.4: horizontal differential-driving-mode magnetic bearing system[35].

$$m\ddot{x} = \lambda\frac{(i_b+i_p)^2}{(G-\beta x)^2} - \lambda\frac{(i_b-i_p)^2}{(G+\beta x)^2}, (17)$$

The$i_p$is the control current and $i_b = 0.3$is the bias current; the rotor mass $m = 0.0126\ lb \cdot \frac{sec^2}{in}$, the nominal air gap $G = 0.02\ in$, the force constant $\lambda = 0.0186\ lb \cdot \frac{in^2}{A^2}$ and the sensitivity of the air gap to shaft displacement $\beta = 0.974$.

We also choose the rotor position and the control current as the training inputs, and the rotor acceleration as the training output. After feeding 2300 training patterns generated from Eq. (17) into the neural-fuzzy inference network in Fig. 1 with a learning rate at 0.005. We have the following T-S fuzzy system [31],

$$\begin{aligned} &R^1: If\ x(t)\ is\ T_1(0, 0.2), \\ &\qquad then\ \dot{X}(t) = A_1X(t) + B_1u(t). \\ &R^2: If\ x(t)\ is\ T_2(-0.4389, 0.08826), \\ &\qquad then\ \dot{X}(t) = A_2X(t) + B_2u(t), \quad (18) \end{aligned}$$

where the system states$X(t) = [\dot{x}(t)\ x(t)]^T$and the system input $u(t) = i_p$; the fuzzy term set $T_i(m_{i,}\sigma_i)$, $i = 1{,}2$, has Gaussian membership function with mean $m_{i,}$and standard deviation σ_i ;the parameters $A_1 = \begin{bmatrix} 0 & 14000 \\ 1 & 0 \end{bmatrix}, A_2 =$

$\begin{bmatrix}0 & 16880\\ 1 & 0\end{bmatrix}$, $B_1 = \begin{bmatrix}300\\ 0\end{bmatrix}$, $B_2 = \begin{bmatrix}560\\ 0\end{bmatrix}$.Based on Theorem 4, we have the steady state response of the T-S fuzzy system in Eq. (14) to a sinusoidal input signal$u_1(t) = \sin(0.3907t)$and to a cosine input signal $u_2(t) = \cos(0.1931 + 0.1)$. The sinusoidal input $u_1(t)$ and the corresponding steady state response are shown in blue color, and those to $u_2(t)$ are shown in red color. We observe that the steady state response y_{ss}(t)oscillates at ± 0.2143 for both signals.

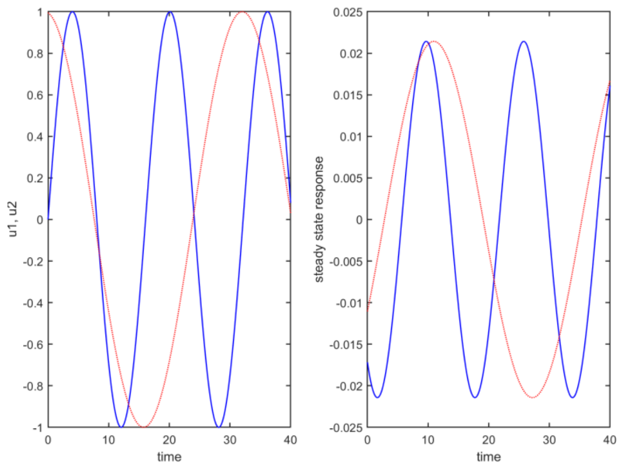

Fig. 5: steady state response y_{ss}(t)for fuzzy system in Eq. (18).

4. CONCLUSION
Frequency response is used to minimize audible distortion of an audio system and to assess system stabilityof control systems, for example, vehicle cruise control systems. However, linear frequency domain analysis cannot apply to nonlinear systems. T-S fuzzy systems possess linear subsystems and at any time instant the dynamic behavior of entire fuzzy systems is a fuzzily blending effect of all linear subsystems. Based on this characteristic, we introduce previously proposed neural fuzzy inference network to describe system response to input signals in time domain, and then to demonstrate the feasibilities and limitationsof frequency domain. In the future we shall develop module-based numerical approaches to overcome the limitations of nonlinear T-S fuzzy frequency response.

5. ACKNOWLEDGMENTS
This research was supported by grant number MOST 107-2221-E-212-013 from the Ministry of Science and Technology of Taiwan, R.O.C.

6. REFERENCES
[1] Tanaka, K and Wang, H.O. 2001. Fuzzy control systems design and analysis. New York: Wiley.

[2] Zeng, X. J. and Singh, M. G. 1995. Approximation theory of fuzzy systems—MIMO case. IEEE Trans. Fuzzy Syst, 3(2): 219–235.

[3] Wang, H. O., Li, J., Niemann,D. and Tanaka, K. 2000. T–S fuzzy model with linear rule consequence and PDC controller: A universal framework for nonlinear control system. In Proceedings of FUZZ-IEEE.

[4] Wiktorowicz, K. 2016. Output feedback direct adaptive fuzzy controller based on frequency-domain methods. EEE Trans. Fuzzy Syst, 24(3):622-634.

[5] Kluska, J. and Żabiński, T. 2020. PID-like adaptive fuzzy controller design based on absolute stability criterion. IEEE Trans Fuzzy Syst, 28(3): 523-533.

[6] Zhao, T., Wei, Z., Dian, S. and Xiao, J. 2016. Observer-based H_∞ controller design for interval type-2 T–S fuzzy systems. Neurocomputing, 177:9-25.

[7] Wu, S. J., Wu, C. T. and Chang, Y. C. Chang, . 2008. Neural–fuzzy gap control for a current/voltage-controlled 1/4-vehicle MagLev system.IEEE trans Intell Transp Syst, 9(1):122-136.

[8] Wang, H. and Yang, G. H. 2012. Piecewise controller design for affine fuzzy systems via dilated linear matrix inequality characterizations. ISA Transactions, 51(6):771-777.

[9] Wang, H. and Yang, G. H. 2013. Controller design for affine fuzzy systems via characterization of dilated linear matrix inequalities. Fuzzy Sets Syst, 217:96-109.

[10] Wang, H. and Yang, G. H. 2016.H_∞controller design for affine fuzzy systems based on piecewise Lyapunov functions in finite-frequency domain. Fuzzy Sets Syst, 290:22–38.

[11] Qiu, J., Feng, G. and Gao, H. 2012. Observer-based piecewise affine output feedback controller synthesis of continuous-time T-S Fuzzy affine dynamic systems using quantized measurements. IEEE Trans. Fuzzy Syst, 20(6):1046-1062.

[12] Li, X. J. and Yang, G. H. 2014. Fault detection in finite domain for Takagi-Sugeno fuzzy systems with sensor faults. IEEE Trans. Man Cybern, 44(8):1446-1458.

[13] Hellani, D.E., Hajjaji, A.E. and Ceschi, R. 2018. Finite frequency H_∞ filter design for T-S fuzzy systems: new approach. Signal Processing, 143:191–199.

[14] Li, X.J. and Yang, G. H. 2017. Finite frequency L_2-L_∞ filtering of T-S fuzzy systems with unknown membership functions. IIEEE Trans Syst Man Cybern Syst, 47(8):1884-1897.

[15] Luo, Z. P., An, K. N. 2001.Fuzzy systems in biomedical science.Int J Gen Syst,30(2):209–217.

[16] Komiyama, M., Yoshimoto, K., Sisido, M., Ariga, K. 2017. Chemistry can make strict and fuzzy controls for bio-Systems: DNA nanoarchitectonics and cell-macromolecular nanoarchitectonics. Bull Chem Soc Jpn, 90(9):967–1004.

[17] Abyad, M., Karama, A., and Khallouq, A. 2017. Modelling and control of a biological process using the fuzzy logic Takagi-Sugeno. In Proceeding of the 2017 International Renewable and Sustainable Energy Conference (IRSEC).

[18] Bordon, J., Moskon, M., Zimic, N., and Mraz, M. 2015. Fuzzy logic as a computational tool for quantitative modelling of biological systems with uncertain kinetic data. IEEE/ACM Trans Comput Biol Bioinform, 12(5):1199–1205.

[19] Liu, F., Heiner, M., and Gilbert, D.2020. Fuzzy Petri nets for modelling of uncertain biological systems.BriefBioinformatics, 21(1):198–210.

[20] Liu, F., Sun, W., Heiner, H., and Gilbert, G. 2021. Hybrid modelling of biological systems using fuzzy continuous Petri nets.BriefBioinformatics, 22(1):438–450.

[21] Zhu, X.L., Jiang, Z.Y., Wang, B., and He Y.J. 2018. Decoupling control based on fuzzy neural-network

inverse system in marine biological enzyme fermentation process. IEEE Access, 6:36168–36175.

[22] Wu, S. J., Wu, C. T.,and Chang, J. Y. 2013. Adaptive neural-based fuzzy modeling for biological systems. Math Biosci,242(2):153-60.

[23] Kumar, S., Snehmani, Srivastava, P. K., Gore, A.and Singh, M. K. 2016. Fuzzy–frequency ratio model for avalanche susceptibility mapping, Int J Digit Earth, 9:12.

[24] Ali, A., El-Serafi, K., A. K. Mostafa, S. and El-Sheimy, N. 2016. Frequency features based fuzzy system for rotating machinery vibration analysis using smartphones low-cost MEMS sensors. J Sens Technol, 6:56-74.

[25] Ferreira, C. C. T.and Serra, G. L. d. O. 2010.Fuzzy frequency response: definition and analysis for complex dynamic systems.In the International Conference on Fuzzy Systems.

[26] Ferreira, C. C. T.and Serra, G. L. d. O. 2011. Fuzzy frequency response estimation from experimental data: definition and application in mechanical structures of aircraft and aerospace vehicles. In Proceedings of 9th IEEE ICCA.

[27] Ferreira, C. C. T. and Serra, G. L. d. O. 2012.An approach for fuzzy frequency response estimation of flexible robot arm from experimental data.In Proceedings ofIEEE International Conference on Industrial Technology.

[28] Ferreira, C. C. T. and Serra, G. L. d. O. 2012.Fuzzy frequency response estimation: a case study for the pH neutralization process.In Proceedings of IEEE International Conference on Fuzzy Systems.

[29] Tewari, A. 2003. Modern control design with matlab and simulink. John Wiley & Sons, Ltd.

[30] Wu, S. J., Chiang, H. H., Lin, H.T. and Lee, T.T. 2005. Neural-network-based optimal fuzzy controller design for nonlinear systems. Fuzzy Sets Syst, 154:182-207.

[31] Yu, S. S., Wu, S. J. and Lee, T. T. 2003. Application of neural- fuzzy modeling and optimal fuzzy controller for nonlinear magnetic bearing systems. In Proceedings ofIEEE/ASME International Conference on AIM.

[32] Wu, S. J., Wu, C. T. and Lee, T. T. 2005. Neural-network-based optimal fuzzy control design for half-car active suspension system. In Proceedings ofIEEE-IV.

[33] Ferreira, C. C. T. and Serra, G. L. O. 2011. Fuzzy frequency response: proposal and application for uncertain dynamic systems. Eng Appl Artif Intell, 24:1186–1194.

[34] Levine, J, Lottin, J. and Ponsart, J.C. 1996. A nonlinear approach to the control of magnetic bearings.IEEE Trans.on Con Syst Tec, 4(5):524-544.

[35] Hong, S.K., Langari, R. and Joh, J. 1997. Fuzzy modeling and control of a nonlinear magnetic bearing system. In Proceeding of the 1997 IEEE International Conference on Control Applications.