Multi-Attribute Decision Making Method for Car Selection by Individuals

Zikrullah Khan
Department of Economics
Ganesh Dutt College, Begusai
Bihar, India, 851101

Amal Kumar Adak, PhD
Department of Mathematics
Ganesh Dutt College, Begusarai
Bihar, India, 851101

ABSTRACT
Purchasing management is most essential in todays competitive world, especially the most useful and essential purchase of the car by individuals. With the development of management and economics, real-world decision-making problems are becoming diversified and complicated to an increasing extent, especially within a changeable and unpredictable environment. Multi-attribute decision making is a decision-making technique that explicitly evaluates numerous contradictory criteria. There are many characteristics possessed by car like engine displacement, mileage in city and highway, max power, comfort, attractiveness, cost etc. and a customer has to choose the best car among several brand cars. In this paper, TOPSIS are used to find best car to be purchased. TOPSIS is a well-known multi-criteria decision-making process. The distance between two Pythagorean fuzzy numbers are utilised to create the model using the spherical distance measure. To construct a ranking order of alternatives and determine the best one, the revised index approach is utilised. Finally, demonstrate proposed method to solve a car selection problems from different brands with different attributes for customer. In addition, it shows comparative data from the relative closeness and updated index methods.

Keywords
Multiple attribute decision making (MADM); TOPSIS; score function; spherical distance measurement; revised index method.

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1. INTRODUCTION
Multi-criteria decision making plays a vital roles in real life situations. Zadeh [1] introduced fuzzy set theory which provides a convenient and efficient tool for characterizing imprecision by membership functions in [0, 1] and managing MCDA problems with vagueness and uncertainty. Nonetheless, in real decision situations, sometimes the membership function of an ordinary fuzzy set is not enough to depict the characters of assessment information because of the complexity of evaluation values and the ambiguity of human subjective judgments. Adak et.al.,[2,3,4,5] have extend the MCDA in generalized form.

However in reality, it may not always be true that the degree of non-membership of an element in a fuzzy set is equal to 1 minus the membership degree because there may be some hesitation degree. To overcome this situation, Atanassov [6] introduced the concept of Instuitionistic fuzzy sets, which is a generalization of fuzzy sets and incorporate with the membership degree (α), non-membership degree (β) and hesitation degree (γ) (defined as 1 minus the sum of membership and non-membership degrees). The notion of intuitionistic fuzzy set is quite interesting and useful in many application areas. The knowledge and semantic representation of intuitionistic fuzzy set become more meaningful, resourceful and applicable since it includes the degree of belongingness, degree of non-belongingness and the hesitation margin.

In IFSs[7], the pair of membership grades and non-membership grades are denoted by (α, β) satisfying the condition of $0 \leq \alpha + \beta \leq 1$ but Pythagorean fuzzy sets (PFSs) whose membership values are ordered pairs (α, β) that fulfills the required condition of $\alpha^2 + \beta^2 \leq 1$. For instance, consider the situation when $\alpha = 0.7$ and $\beta = 0.4$, we can use PFSs, but IFSs cannot be used since$\alpha + \beta \leq 1$, but $\alpha^2 + \beta^2 \leq 1$PFSs are wider than IFSs so that they can tackle more daily life problems under imprecision and uncertainty cases. Garg [8,9] introduced some new operators on generalized pythagorean fuzzy sets. There lots of research work done in this field in [10,11,12]

How to measure the distance between two pythagorean fuzzy sets is still an open issue. Many kinds of methods have been proposed to present the of the question in former researches. However, not all of existing methods can accurately manifest differences among pythagorean fuzzy sets and satisfy the property of similarity. And some other kinds of methods neglect the relationship among three variables of pythagorean fuzzy set.

Zhang and Xu [13] considered three parameters of PFSs, namely, the membership degree, the nonmembership degree, and the hesitation degree, while ignoring the direction of commitment, the strength of commitment, and the radian. Li and Zeng [14] considered four basic parameters (the membership degree, the non-membership degree, the strength of commitment, and the direction of commitment) of PF sets in the distance measure equation. Zeng et al. [15] incorporated a parameter, namely, the hesitation degree, both approaches ignore the angle and the procedure is directly extended from the IF sets but does not consider the greater space of the PF sets. Yu et al. [16] proposed a new distance formula that employs induced ordered weighted averaging (IOWA) with PF information; however, this basic distance formula considers only three parameters, which are the same as the parameters that are considered in the method of Zhang and Xu [13]. Peng and Li [17] proposed a new distance measure for IVPF sets that has two parameters (the membership degree and the non-membership degree) for resolving the counter-intuitive situation.

To address the problem, a new method of measuring distance is proposed which meets the requirements of axiom of distance measurement and is able to indicate the degree of distinction of PFSs well. For a Pythagorean fuzzy number, membership (α) and non-membership (β) degree satisfying the condition $0 \leq \alpha^2 + \beta^2 \leq 1$ and hesitation degree is $\gamma = \sqrt{1 - \alpha^2 - \beta^2}$, i.e., $\alpha^2 + \beta^2 + \gamma^2 = 1$. From, this relation we may assume that the triplet (α, β, γ) lies on the spherical surface of unit radius and centre at the origin. This interpretation encourage to define the spherical distance between two Pythagorean fuzzy numbers on restricted spherical surface.

The purpose of this paper is to find the distance between two pythagorean fuzzy numbers and applied in TOPSIS method. This measurement is essential to determine distances for both the positive ideal solution and negative ideal solution. Score function of pythagorean fuzzy is used to determine PIS and NIS in this approach. Revised index and relative closeness are used to rank the alternatives.

The remainder of this paper is organized as follows. Section 2 briefly introduces some basic concepts. Section 3 formulates spherical distance measurement method for pythagoren fuzzy numbers. Moreover, some comparative discussions with other measurement method are conducted to demonstrate the effectiveness and advantages of the developed method. Section 4 develops TOPSIS for solving MCDM problems. Section 5 applies the proposed methodology to a real-life problem to demonstrate its feasibility and practicality. Finally, Section 6 presents the conclusion.

2. PRELIMINARIES AND DEFINITIONS

In this section, we recall some basic notions such as the instuitionistic fuzzy sets and the Pythagorean fuzzy sets. Also, we include some elementary aspects that are necessary for this paper.

Definition 2.1[18] ***(Pythagorean Fuzzy set (PFS))*** *A pythagorean fuzzy set P in a finite universe o discourse X is given by*

$$P = \{\langle x, \alpha_P(x), \beta_P(x)\rangle | x \in X\},$$

where $\alpha_P(x): X \to [0,1]$ denotes the degree of membership and $\beta_P(x): X \to [0,1]$ denotes the degree of non-membership of the element $x \in X$ to the set A respectively with the condition that $0 \le (\alpha_P(x))^2 + (\beta_P(x))^2 \le 1$.

The degree of indeterminacy $\gamma_P(x) = \sqrt{1 - (\alpha_P(x))^2 - (\beta_P(x))^2}$.

Definition 2.2 *[19] Let $p = \langle \alpha, \beta \rangle$ be a pythagorean fuzzy number. The score function of p is defined as*

$$s(p) = (\alpha)^2 - (\beta)^2,$$

where $s(p) \in [-1,1]$.

Example 1 *Let* $p_1 = \langle 0.7, 0.3 \rangle$ *and* $p_2 = \langle 0.4, 0.6 \rangle$, *then* $s(p_1) = 0.40$ *and* $s(p_1) = -0.20$.

In some situation, score function is not sufficient for magnitude comparison of Pythagorean fuzzy numbers. Using the concept of score function Peng and Yang [14] developed accuracy function for magnitude comparison of Pythagorean fuzzy numbers.

Definition 2.3 *Let $p = \langle \alpha, \beta \rangle$ be a pythagorean fuzzy number. The accuracy function of p is defined as*

$$h(p) = (\alpha)^2 + (\beta)^2,$$

where $h(p) \in [0,1]$.

Let $p_1 = \langle \alpha_1, \beta_1 \rangle$ and $p_2 = \langle \alpha_2, \beta_2 \rangle$ be two PFNs; $s(p_1) = \alpha_1^2 - \beta_1^2$ and $s(p_2) = \alpha_2^2 - \beta_2^2$ be their score functions; $h(p_1) = \alpha_1^2 + \beta_1^2$ and $h(p_2) = \alpha_2^2 + \beta_2^2$ be the accuracy functions of p_1 and p_2, defined the following:

1) *If* $s(p_1) < s(p_2)$, *then* p_1 *is smaller than* p_2, *that is* $p_1 < p_2$;

2) *If* $s(p_1) > s(p_2)$, *then* $p_1 > p_2$,

3) *If* $s(p_1) = s(p_2)$, *then*

if $h(p_1) < h(p_2)$, *then* $p_1 < p_2$

if $h(p_1) > h(p_2)$, *then* $p_1 > p_2$

if $h(p_1) = h(p_2)$, *then* p_1 *and* p_2 *represent the same information, that is* $p_1 = p_2$.

3. SPHERICAL DISTANCE MEASUREMENT METHOD FOR PFNS

Let $p =< \alpha, \beta >$ be a Pythagorean fuzzy number satisfying the condition $0 \le \alpha^2 + \beta^2 \le 1$ and hesitation function is $\gamma = \sqrt{1 - \alpha^2 - \beta^2}$, i.e., $\alpha^2 + \beta^2 + \gamma^2 = 1$.

From, this relation we may assume that the triplet (α, β, γ) lies on the spherical surface of unit radius and centre at the origin. This interpretation encourage to define the spherical distance between two Pythagorean fuzzy numbers on restricted spherical surface.

On spherical surface the shortest distance is the length arc of the great circle passing through both points.

Definition 3.1 *Let A and C be two points on the spherical surface with co-ordinate (x_1, y_1, z_1) and (x_2, y_2, z_2), then the spherical distance between these two points is defined as*

$$D_{SP}(A, C) = arccos\left\{1 - \frac{1}{2}\begin{bmatrix}(x_1 - x_2)^2 \\ +(y_1 - y_2)^2 + (z_1 - z_2)^2\end{bmatrix}\right\} \quad (1)$$

Incorporated this expression, the spherical distance between two Pythagorean fuzzy numbers are defined as follows:

Definition 3.2 *Let $p_1 =< \alpha_1, \beta_1 >$ and $p_1 =< \alpha_2, \beta_2 >$ be two Pythagorean fuzzy numbers with hesitation function γ_1 and γ_2 respectively. Then the spherical distance between these two Pythagorean fuzzy numbers is*

$$D_S(p_1, p_2) = \frac{2}{\pi} arccos\left\{1 - \frac{1}{2}[(\alpha_1 - \alpha_2)^2 + (\beta_1 - \beta_2)^2 + (\gamma_1 - \gamma_2)^2]\right\} \quad (2)$$

To get the distance value in between $[0,1]$ the factor $\frac{2}{\pi}$ is introduced.

Since, $\alpha_1^2+\beta_1^2+\gamma_1^2=1$ and $\alpha_2^2+\beta_2^2+\gamma_2^2=1$, so after simplifying the equation (2), we have

$$D_S(p_1,p_2)=\frac{2}{\pi}arccos[\alpha_1\alpha_2+\beta_1\beta_2+\gamma_1\gamma_2] \quad (3)$$

Now, we define the spherical and normalized distances between two Pythagorean fuzzy sets.

Definition 3.3 *Let* $P=\{x_i,<\alpha_P(x_i),\beta_P(x_i)>: x_i\in X\}$ *and* $Q=\{x_i,<\alpha_Q(x_i),\beta_Q(x_i)>: x_i\in X\}$ *of the universe of discourse* $X=\{x_1,x_2,\ldots,x_n\}$ *, then their spherical and normalized spherical distances defined as follows:*

Spherical Distance:

$$D_S(P,Q) = \frac{2}{\pi}\sum_{i=1}^{n}\arccos\begin{bmatrix}\alpha_P(x_i)\alpha_Q(x_i)\\ +\beta_P(x_i)\beta_Q(x_i)+\gamma_P(x_i)\gamma_Q(x_i)\end{bmatrix} \quad (9)$$

where $0\le D_{SP}(P,Q)\le n$. (4)

Normalized Spherical Distance:

$$D_{NS}(P,Q) = \frac{2}{n\pi}\sum_{i=1}^{n}\arccos\begin{bmatrix}\alpha_P(x_i)\alpha_Q(x_i)\\ +\beta_P(x_i)\beta_Q(x_i)+\gamma_P(x_i)\gamma_Q(x_i)\end{bmatrix} \quad (10)$$

where $0\le D_{NSP}(P,Q)\le 1$.

Example 2 *Let* $p_1=\langle 0.9,0.2\rangle$ *and* $p_2=\langle 0.7,0.3\rangle$ *be two pythagorean fuzzy numbers. Then the spherical distance between* p_1 *and* p_2 *is*

$$D_S(p_1,p_2)=0.2198$$

Definition 3.4 *Let* $P_1=(\alpha_{1j},\beta_{1j})$, $P_2=(\alpha_{2j},\beta_{2j})$, $j=1,2,\ldots,n$*, be two sets of Pythagorean fuzzy numbers.* w_j *is the weight of* j *, i.e.,* $w=(w_1,w_2,\ldots,w_n)^T$ *, where* $0\le w_i\le 1$ *and* $\sum_{i=1}^{n}w_i=1$*. Then the weighted normalized spherical distance between* P_1 *and* P_2 *is defined s*

$$D_{NS}^{'}(P_1,P_2) = \frac{2}{n\pi}\sum_{j=1}^{n}w_j\arccos\left[\alpha_{1j}\alpha_{2j}+\beta_{1j}\beta_{2j}+\gamma_{1j}\gamma_{2j}\right] \quad (6)$$

Example 3 *Let* $P_1=\{\langle 0.6,0.3\rangle,\langle 0.8,0.2\rangle,\langle 0.5,0.4\rangle\}$ *and* $P_2=\{\langle 0.7,0.2\rangle,\langle 0.7,0.3\rangle,\langle 0.9,0.1\rangle\}$ *be two pythagorean fuzzy sets with weights* $w=\{0.2,0.5,0.3\}$*. Then, the weighted spherical distance between* P_1 *and* P_2 *is calculated as*

$$D_{NS}^{'}(P_1,P_2)=0.0499$$

4. 4 Proposed TOPSIS method for MCDM Problems

In this section, we introduce multi-criteria decision making problem where the information has been taken in the form of the Pythagorean fuzzy numbers and apply spherical distance measurement method to solve this problems.

Let $A=\{A_1,A_2,\cdots,A_m\}$, $(m\ge 2)$ be a set of alternatives and $C=\{C_1,C_2,\cdots,C_n\}$, $(n\ge 2)$ be a set of criterion, $w=(w_1,w_2,\ldots,w_n)^T$, where $0\le w_i\le 1$ and $\sum_{i=1}^{n} w_i=1$, be the weight vector for each criteria.

Let the pythagorean fuzzy numbers $\langle\alpha_{ij},\beta_{ij}\rangle$ denotes the assessment value of the i-th alternative for the j-th criteria, viz, $C_j(v_i)=\langle\alpha_{ij},\beta_{ij}\rangle$ and $R=(C_j(v_i))_{m\times n}$ denotes Pythagorean fuzzy decision matrix, where

$$R=\begin{bmatrix}\langle\alpha_{11},\beta_{11}\rangle & \langle\alpha_{12},\beta_{12}\rangle & \cdots & \langle\alpha_{1n},\beta_{1n}\rangle\\ \langle\alpha_{21},\beta_{21}\rangle & \langle\alpha_{22},\beta_{22}\rangle & \cdots & \langle\alpha_{2n},\beta_{2n}\rangle\\ \cdots & \cdots & \cdots & \cdots\\ \langle\alpha_{m1},\beta_{m1}\rangle & \langle\alpha_{m2},\beta_{m2}\rangle & \cdots & \langle\alpha_{mn},\beta_{mn}\rangle\end{bmatrix}$$

4.1 Process of the Proposed Method

To solve MCDM problems in Pythagorean fuzzy environment, we present Pythagorean fuzzy TOPSIS system. The primary concept of the TOPSIS approach is that the most preferred alternative should not only have shortest distance from the positive ideal solution but also have the furthest distance from the negative ideal solution.

We start this method by computing PFPIS and PFNIS. Let J_1 be the set of benefit criteria and J_2 be the set of cost criteria. PFPIS and PFNIS were determined by using score function. Let v^+ and v^- denote PFPIS and PFNIS respectively. These values calculated using the following formula

$$v^+=\{C_j,max(S(C_j(v_i)))|j=1,2,\ldots,n\} \quad (7)$$

$$v^-=\{C_j,min(S(C_j(v_i)))|j=1,2,\ldots,n\} \quad (8)$$

Next, we calculate normalized spherical distance from each alternative to the PFPIS $D_{NS}(v_i,v^+)$ and PFNIS $D_{NS}(v_i,v^-)$.

Now, we obtain weighted normalized spherical distance of alternative v_i from PFPIS v^+ based on (6), which can be defined as follows

$$D_{NS}(v_i,v^+)=\sum_{j=1}^{n} D_{NS}(C_j(v_i),C_j(v^+))$$

$$=\frac{2}{n\pi}\sum_{j=1}^{n}w_j\arccos(\alpha_{ij}\alpha_j^+ +\beta_{ij}\beta_j^+ +\gamma_{ij}\gamma_j^+) \quad (9)$$

where $i = 1,2,\dots,n$.

According to the principle of TOPSIS, the smaller $D_{NS}(v_i, v^+)$ is the better alternative x_i.

Let

$$D_{\min}(x_i, x^+) = \min_i D_{NS}(v_i, v^+), i = 1,2,\dots,n$$

Similarly, the weighted normalized spherical distance of alternative v_i from PFNIS v^- calculated as follows

$$D_{NS}(v_i, v^-) = \sum_{j=1}^{n} D_{NS}(C_j(v_i), C_j(v^-))$$

$$= \frac{2}{n\pi}\sum_{j=1}^{n} w_j \arccos(\alpha_{ij}\alpha_j^- + \beta_{ij}\beta_j^- + \gamma_{ij}\gamma_j^-) \quad (10)$$

where $i = 1,2,\dots,n$.

According to the principle of TOPSIS, the greater $D_{NS}(v_i, v^-)$ is the better alternative v_i. Let

$$D_{\max}(v_i, v^+) = \max_i D_{NS}(v_i, v^+), i = 1,2,\dots,n$$

Now, we calculate relative closeness co-efficient of the alternative x_i with respect to PFPIS (x^+) and PFNIS (x^-) with the help of basic principle of classical TOPSIS method.

The formula for $RC(x_i)$ is as follows

$$RC(v_i) = \frac{D_{NS}(v_i, v^-)}{D_{NS}(v_i, v^+) + D_{NS}(v_i, v^-)} \quad (11)$$

According to the Hadi Venecheh [9], the optimal solution is the shortest distance from positive ideal solution and farthest distance from negative ideal solution. Consequently, Zhang and Xu [19] utilized revised index, which is denoted by $\xi(v_i)$ to determine the ranking order. The index formula is expressed as follows

$$\xi(v_i) = \frac{D_{NS}(v_i, v^-)}{D_{\max}(v_i, v^-)} - \frac{D_{NS}(v_i, v^+)}{D_{\min}(v_i, v^-)} \quad (12)$$

According to $RC(v_i)$ or $\xi(v_i)$, we obtain the rank the alternatives x_i, which is used to determine the optimal solution according to the maximum value of $RC(v_i)$ or $\xi(v_i)$.

4.2 Algorithm for proposed method

The traditional TOPSIS introduced by Hwang and Yoon [20] is a classic and useful method to solve the MCDM problems with crisp numbers. Zhang and Xu[13] developed a revised TOPSIS method to deal effectively with the MCDM problems with Pythagorean fuzzy information. The algorithm involves the following steps:

Step 1. For a MCDM problem with PFNs, we construct the decision matrix $R = (C_j(v_i))_{m\times n}$, where the elements $C_j(v_i)$, $i = 1,2,\dots,m$, $j = 1,2,\dots,n$ are the assessments of alternative v_i with respect to the criterion C_j.

Step 2. Utilize the score function to determine the Pythagorean fuzzy positive ideal solution (v^+) and the Pythagorean fuzzy negative ideal solution (v^-).

Step 3. Use Eq. (9) and (10) to calculate the weighted spherical distances of each alternative v_i from the Pythagorean fuzzy PIS (v^+) and the Pythagorean fuzzy NIS (v^-).

Step 4. Utilize equation (11) and (12) to calculate relative closeness $RC(v_i)$ and the revised closeness $\xi(v_i)$ of the alternative v_i.

Step 5. Rank the alternatives and select the best one(s) according to the decreasing relative closeness $RC(v_i)$ and revised closeness $\xi(v_i)$ obtained from Step 4.

The bigger the $RC(v_i)$ the more desirable the v_i, $(i = 1,2,\dots,m)$ will be.

5. ILLUSTRATIVE EXAMPLE

In this section, we consider a decision-making problem that concerns with daily life problems to illustrate the proposed approach.

A decision maker want to buy a car. There are more than one branded cars with their criterion. Decision maker consider only five banded cars v_1, v_2, v_3, v_4 and v_5 among these he/she want to buy a particular with his/her availability. In order to buy the cars four criterion viz., cost (C_1), fuel consumption (C_2), comfort (C_3) and attractiveness (C_4) are considered as evaluation factor. According to the assessment of attributes and criterion, Pythagorean fuzzy decision matrix are considered as follows:

	C_1	C_2	C_3	C_4
v_1	$\langle 0.7,0.3\rangle$	$\langle 0.5,0.4\rangle$	$\langle 0.7,0.6\rangle$	$\langle 0.9,0.2\rangle$
v_2	$\langle 0.6,0.4\rangle$	$\langle 0.6,0.3\rangle$	$\langle 0.5,0.3\rangle$	$\langle 0.7,0.4\rangle$
v_3	$\langle 0.5,0.6\rangle$	$\langle 0.7,0.4\rangle$	$\langle 0.5,0.4\rangle$	$\langle 0.6,0.4\rangle$
v_4	$\langle 0.4,0.7\rangle$	$\langle 0.5,0.8\rangle$	$\langle 0.6,0.2\rangle$	$\langle 0.5,0.3\rangle$
v_5	$\langle 0.5,0.8\rangle$	$\langle 0.7,0.2\rangle$	$\langle 0.6,0.5\rangle$	$\langle 0.6,0.2\rangle$

where $C_1(v_1) = \langle 0.7,0.3\rangle$ represents that the degree to which alternative v_1 satisfies criteria C_1 is 0.7 and degree to which satisfies alternative v_1 dissatisfies criterion C_1 is 0.3.

Considering that fuel consumption, comfort and attractiveness of the cars as benefit criteria, $J_1 = \{C_2, C_3, C_4\}$ and cost of the car is the cost criterion $J_2 = \{C_1\}$.

To calculate score type Pythagorean fuzzy positive ideal solutions (v^+) and Pythagorean fuzzy negative ideal solutions (v^-), we utilize the formula (7) and (8). We get the results as follows

$$v^+ = \{\langle 0.5,0.8\rangle, \langle 0.7,0.2\rangle, \langle 0.6,0.2\rangle, \langle 0.9,0.2\rangle\}$$
$$v^- = \{\langle 0.7,0.3\rangle, \langle 0.5,0.8\rangle, \langle 0.5,0.4\rangle, \langle 0.5,0.3\rangle\}$$

Next, utilize equation (9) and (10) to calculate the weighted spherical distances of each alternatives v_i from Pythagorean fuzzy positive ideal solution and Pythagorean fuzzy negative ideal solutions.

	$D_{NS}(v_i, v^+)$	$D_{NS}(v_i, v^-)$
v_1	0.0590	0.0554
v_2	0.0593	0.0475
v_3	0.0561	0.0475
v_4	0.0712	0.0359
v_5	0.0290	0.0742

We utilize Equation (11) and (12) to compute the $RC(v_i)$ and $\xi(v_i)$ for each alternative v_i and results are listed bellow:

	$RC(v_i)$ $(Rank)$	$\xi(v_i)(Rank)$
v_1	0.4842(2)	−1.2878(3)
v_2	0.4447(4)	−1.4046(4)
v_3	0.4642(3)	−1.2794(2)
v_4	0.3352(5)	−1.9713(5)
v_5	0.7189(1)	0(1)

According to $RC(v_i)$ rank of the alternatives are $v_5 \succ v_1 \succ v_3 \succ v_2 \succ v_4$ among which v_5 is the best alternative. However, according to the revised index $\xi(v_i)$ the ranking of the alternatives are $v_5 \succ v_3 \succ v_1 \succ v_2 \succ v_4$. Here also, the best alternative is v_5.

6. CONCLUSION AND FUTURE WORK

In this paper, the spherical distance measurements method has been introduced and apply in TOPSIS method to solve MCDM problem. The main advantage of this method is that it is able to reflect the importance of the degrees of membership, non-membership and hesitancy of decision maker. Moreover, it provides a more complete representation of the decision process because the decision makers can consider many different scenarios depending on his. The spherical distance measurement method combined with the TOPSIS method with pythagorean fuzzy data has enormous chance of success for MCDM problems. Ordering of the alternatives by utilizing relative closeness and revised index method.

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8. CONFLICTS OF INTEREST

The authors declare that there is no competing of interests.

9. ETHICAL APPROVAL

This article does not contain any studies with human participants or animals performed by any of the authors.

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