Visco-Elastic Oscillatory Flow in a Porous Channel with Heat Transfer in Presence of Magnetic Field

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ABSTRACT
In this invstigation the oscillatory flow of visco-elastic fluid through a porous channel is considered. The fluid is subjected to a transverse magnetic field also slip velocity at the lower plate is taken into consideration. The vertical channel is maintained at non-uniform temperature The perturbation scheme has been used to solve the equations governing the flow. The expressions for the velocity, temperature, skin-friction have been obtained. The results are illustrated graphically, for various values of flow parameters such as Darcy parameter, suction/injection parameter, magnetic parameter, Grashof number, Prandtl number, thermal radiation parameter, Navier-slip parameter and visco-elastic parameter. It is observed that the visco-elastic parameter plays a significant in role in flow field. The acquired knowledge in this study can be used in blood flow in arteries, oil industry.

Keywords
visco-elastic, porous medium, oscillatory, slip effects, skin-friction.

1. INTRODUCTION
In different areas of science and engineering technology MHD oscillatory flow with heat transfer plays an important role in physiological and engineering applications. It is revealed from theoretical or experimental investigation that MHD flow of electrically conducting fluid assumes considerable importance because of various natural phenomena. These natural phenomena are generated by the action of Coriolis force and magnetic force.

Also, the flow of an electrically conducting fluid through a porous channel saturated with porous medium has many engineering applications such as MHD generators, arterial blood flow, petroleum engineering and many more. A number of researchers [1-4] have studied the MHD flow with various perspectives. Adesanya[5], studied the free convective flow of heat generating fluid through a porous vertical channel with velocity slip and temperature jump.

The flow of visco-elastic fluid through has gained importance because of their increasing application in industry for certain special flows. Sivaraj et al.[6] have studied the unsteady MHD dusty visco-elastic fluid Couette flow in an irregular channel with varying mass diffusion. Adesanya[7] have decomposition approach to steady visco-elastic fluid flow with slip through a planer channel.

The specific aim of this analysis is to extend the work done in [4] to the non-Newtonian case characterized by Second order fluid [Coleman and Noll [9]] and [Coleman and Markovitz [10]]. Oscillatory flows of second grade fluid in a porous space have been considered by Hussain et al.[8]. A constant magnetic field is applied across the normal to the channel. Also, we assume that fluid has a very low electrical conductivity and the electromagnetic force produced is very small. Also, due to presence of suction /injection the flow of fluid is subjected to suction at the cold wall and injection at the heated wall. The velocity, skin friction coefficient have been presented graphically for various values of the non-Newtonian parameter along with other flow parameters.

2. MATHEMATICAL ANALYSIS
Consider the steady laminar flow of a non-Newtonian electrically conducting fluid through a channel with slip at the cold plate. An external magnetic field is placed across the normal to the channel. It is assumed that the fluid has small electrical conductivity and the electro-magnetic force produced is very small. The flow is subjected to suction at the cold wall and injection at the heated wall. We choose a Cartesian coordinate system (x', y') where x' lies along the centre of the channel and y' is the distance measured in the normal section such that $y' = a$ is the half channel width.

Under the usual Bousinesq approximation the equations governing the flow are as follows

$$\frac{\partial u'}{\partial t'} - v_0 \frac{\partial u'}{\partial y'} = -\frac{1}{\rho}\frac{dP'}{dx'} + v_1 \frac{\partial^2 u'}{\partial y'^2} x + v_2 \left(\frac{\partial^3 u'}{\partial y'^2 \partial t'} - v_0 \frac{\partial^3 u'}{\partial y'^3}\right) - \frac{v_1 u'}{K} - \frac{\sigma_e B_0^2 u'}{\rho} + g\beta (T' - T_0) \quad (2)$$

$$\frac{\partial T'}{\partial t'} - v_0 \frac{\partial T'}{\partial y'} = \frac{k_f}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2} + \frac{4\alpha^2}{\rho C_p}(T - T_0) \quad (3)$$

With the boundary conditions

$$\left.\begin{aligned} u' &= \frac{\sqrt{K}}{\alpha_s}\frac{du'}{dy'}, T = T_0 \quad on \;\; y' = 0 \\ u' &= 0, \; T' = T_1 \qquad on \;\; y' = a \end{aligned}\right\} \quad (4)$$

Where t' -time, u' -axial velocity, v_0-constant horizontal velocity, ρ-fluid density, P' -fluid pressure, $v_i = \frac{\mu_i}{\rho}$, i=1,2where ρ is the density of the fluid, μ_1-viscosity of the fluid μ_2-elasticity of the fluid, K-porous permeability, σ_e -electrical conductivity, B_0-magnetic field intensity, g-gravitational acceleration, β-volumetric expansion, C_p-the specific heat at constant pressure, α-the term due to thermal radiation, k-thermal conductivity, T' -fluid temperature and T_0- referenced fluid temperature.

Introducing the dimensionless parameters and variables

$$x = \frac{x'}{h_1}, y = \frac{y'}{h_1}, u = \frac{h_1 u'}{v_1}, t = \frac{vt'}{{h_1}^2}, p = \frac{{h_1}^2 p'}{\rho v^2},$$

$$Gr = \frac{g\beta(T_1 - T_0){h_1}^3}{v_1^2}, Grashofnumber$$
$$Pr = \frac{\rho C_p v_1}{k}, Pr\,andtlnumber,$$
$$R = \frac{4\alpha^2 {h_1}^2}{\rho C_p v_1}, thermalradiationparameter$$
$$h = \frac{\sqrt{K}}{\alpha_s h_1}, Navierslipparameter,$$
$$M^2 = \frac{\sigma_e B_0^2 {h_1}^2}{\rho v_1}, Hartmann'snumber$$
$$Da = \frac{K}{{h_1}^2}, Darcyparameter,$$
$$s = \frac{v_0 h_1}{v_1}, suction/injectionparameter,$$
$$\theta = \frac{T - T_0}{T_1 - T_0}.$$

The following dimensionless equations are obtained:

$$\frac{\partial v}{\partial y} = 0 (5)$$
$$\frac{\partial u}{\partial t} - s\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} + D_0(\frac{\partial^3 u}{\partial y^2\,\partial t} - s\frac{\partial^3 u}{\partial y^3}) - \frac{u}{Da} - Mu + Gr\theta (6)$$

$\frac{\partial \theta}{\partial t} - s\frac{\partial \theta}{\partial y} = \frac{1}{Pr\frac{\partial^2\theta}{\partial y^2}}$With the appropriate boundary condition

$$\left.\begin{array}{l} u = h\frac{du}{dy}, \theta = 0 on y = 0 \\ u = h\frac{du}{dy}, \theta = 0 on y = 0 \end{array}\right\} \quad (8)$$

3. METHOD OF SOLUTION

It is assumed that an oscillatory pressure gradient, such that solutions of the dimensionless equations [6]-[7] is in the following form:

$$-\frac{\partial p}{\partial x} = Le^{i\omega t}, u(y,t) = u_0(y)e^{i\omega t},$$
$$\theta(y,t) = \theta_0(y)e^{i\omega t} \quad (9)$$

Where L is any positive constant and ω is the frequency of oscillation.

In view of [9], equations [6] to [7] reduced to a boundary – valued problem in the following form:

$$dsu_0'' - u_0(1+d) - su_0'$$
$$-u_0(M + \frac{1}{Da} + i\omega) = -L - Gr\theta_0 \quad (10)$$
$$\theta_0'' + s\Pr\theta_0' + (R - i\omega)\Pr\theta_0 = 0 \quad (11)$$

The corresponding boundary conditions are:

$$u_0(0) = hu_0'(0),\ u(1) = 0 \quad (12)$$
$$\theta_0(0) = 0, \theta_0(1) = 1 \quad (13)$$

Consider $u_0 = u_{00} + du_{01}$ then the equation(10) reduces to,

$$u_{00}'' + su_{00}' + u_{00}\left(i\omega + \frac{1}{Da} + M\right) = L + Gr\theta_0 \quad (14)$$
$$u_{01}'' + su_{01}' + \left(i\omega + \frac{1}{Da} + M\right)u_{01} = su_{00}''' - u_{00}'' \quad (15)$$

The relevant boundary conditions are:

$$\left.\begin{array}{ll} u_{00} = hu_{00}',\ u_{01} = hu_{01}' & at\ y = 0 \\ u_{00} = 0\ ,\quad u_{01} = 0 & at\ y = 1 \end{array}\right\} \quad (16)$$

The coefficient of skin friction at y=0 is

$$c_f = \frac{\partial^2 u}{\partial y^2} + d(\frac{\partial^3 u}{\partial y^2 \partial t} - s\frac{\partial^3 u}{\partial y^3})$$

4. RESULTS AND DISCUSSIONS

In this analysis, convective visco-elastic fluid flow through a saturated porous medium with slip effect is studied. The flow through a vertical channel takes place due to increase in pressure gradient and free convection. The non- zero value of d represents the visco-elastic parameter.

In figure 1, exhibit the effect of visco-elastic parameter on fluid flow. It is observed from the figure that the velocity of visco-elastic fluid increases as compared to Newtonian fluid.

In figure 2, the effect of magnetic parameter(M) on velocity profile is represented. The velocity of visco-elastic fluid deaccelerating with the rise of magnetic parameter. This is because the effect of Lorentz force which is the combination of electric and magnetic force on the moving fluid particles which deaccelates the flow of fluid.

In figure 3, represents the effect of Grashof number(Gr) on fluid velocity. Grashof number is the ratio of buoyancy force to the viscous. So, magnification in Grashof number leads to the increase in buoyancy force hence the velocity of the fluid is also accelerating .

The effects of Navier slip-parameter(h) on velocity profile are represented in figure 4. It is revealed from the figure that velocity of fluid increases with the rising values Navier slip-parameter(γ).

Figure 5, depict the velocity profile of fluid with the variation of permeability of porous medium parameter(Da). Since the obstacles on the flow of fluid reduces due to increase in permeability of the medium. Hence the velocity of fluid enhanced with the magnification of porous medium parameter(Da).

Figure 6, display the velocity profile of fluid flow with the variation radiation parameter(R). When the thermal radiation parameter increases than internal heat generation capacity also increases and hence heat gained by the fluid particles also gets more energy. This is lead to increase in velocity of fluid.

Figure 7 and 8 represents the profile of skin friction against time.

It is revealed from the figures that the skin friction coefficient of fluid diminishing with time. Also, it is noticed that the magnitude of skin coefficient for visco-elastic fluid less as compared to Newtonian flow in figure 8. Again, from figure 9 it is observed that the with rise of magnetic parameter the coefficient of skin friction decreases for visco-elastic fluid as compared to Newtonian.

Figure9, display the coefficient of skin friction against Grashof number(Gr). It is observed that the skin friction coefficient enhanced with the rise in Grashof number (Gr). Also, skin friction coefficient increases in case of non-Newtonian fluid as compared to Newtonian fluid.

5. CONCLUSIONS

From the above analysis, following conclusions have been drawn:

1. The velocity of fluid is significantly effected by visco-elastic parameter.

2. The coefficient of skin friction of fluid is significantly effected by visco-elastic parameter.

3. The velocity of fluid increases with the rise of Grashof number(Gr), Navier slip parameter(h), radiation parameter(R) , permeability of porous medium(Da) and reverse is the phenomena for the rise of Magnetic parameter(M).

4. The coefficient of skin friction of fluid increases with the rise of Grashof number(Gr) but shows the reverse trend for the rise of magnetic parameter(M).

6. REFERENCES

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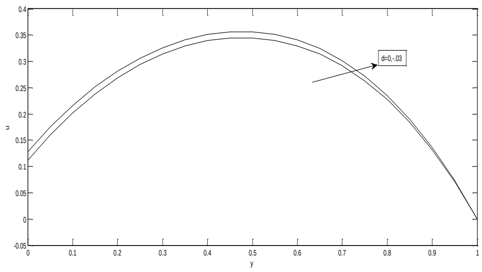

Fig. 1:Velocity profile for variation of visco-elastic parameter (d) against the displacement variable y for Gr=4, M=3,Pr=.71,ω=π, t=1,s=.2,Da=.1,R=.3,h=.5,,L=1.

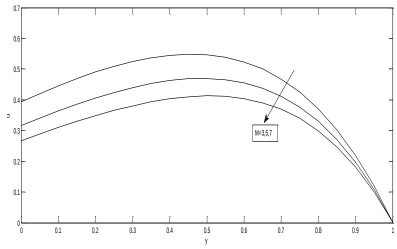

Fig. 2:Velocity profile for variation of Magnetic Parameter(M) against the displacement variable y for for Gr=4,Pr=.71,ω=π, t=1,s=.2,Da=.1,R=.3,h=.5,,L=1,d= -.03

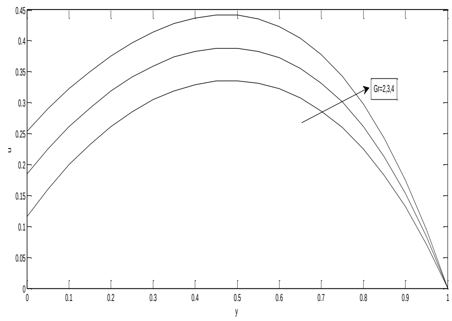

Fig. 3:Velocity profile for variation of Grashof number(Gr) against the displacement variable y for for M=3,Pr=.71,ω=π, t=1,s=.2,Da=.1,R=.3,h=.5,,L=1,d= -.03

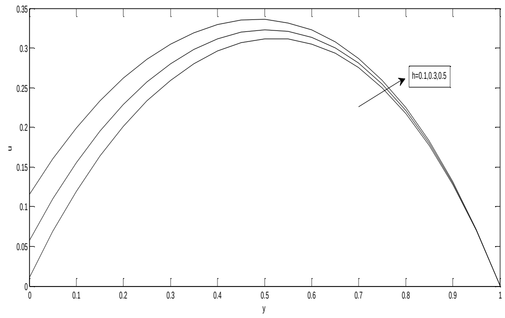

Fig. 4:Velocity profile for variation of Navier slip-parameter(h) against the displacement variable y for for Gr=4, M=3,Pr=.71,ω=π, t=1,s=.2,Da=.1,R=.3,L=1,d= -.03

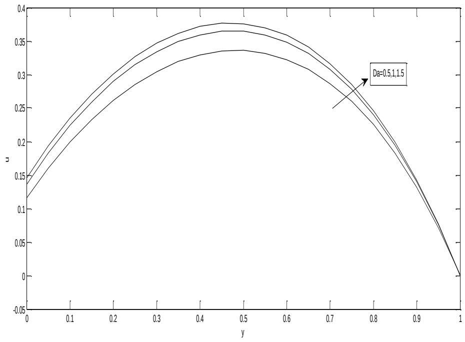

Fig. 5:Velocity profile of for variation of permeability of porous medium parameter(Da) against the displacement variable y for for Gr=4, M=3,Pr=.71,ω=π, t=1,s=.2,R=.3,h=.5,,L=1,d= -.03

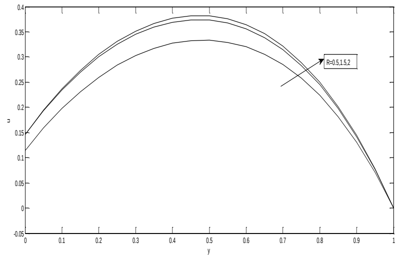

Fig.6:Velocity profile of dust particle for variation of radiation parameter(R) against the displacement variable y for for Gr=4, M=3,Pr=.71,ω=π, t=1,s=.2,Da=.1,h=.5,L=1,d= -.03

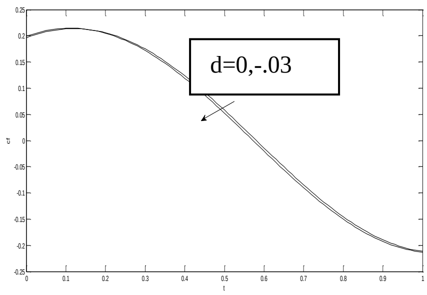

Fig.7:Variation of Shearing stress (Cf) against Time(t) for for Gr=4, M=3,Pr=.71,ω=π, t=1,s=.2,Da=.1,R=.3,h=.5,L=1.

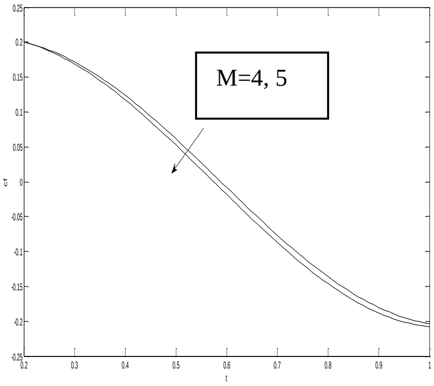

Fig.8: Variation of Shearing stress (Cf) against Time(t) for for Gr=4,Pr=.71,ω=π, t=1,s=.2,Da=.1,R=.3,h=.5,,L=1,d= -.03

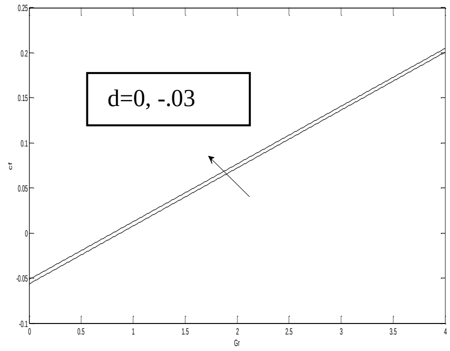

Fig.9: Variation of Shearing stress (Cf) against Grashof number(Gr) for M=3,Pr=.71,ω=π, t=1,s=.2,Da=.1,R=.3,h=.5,,L=1,d= -.03