RPC's Generation with Rational Function Model based on Line of Sight Model for IRS -1C/1D

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ABSTRACT
Day-by-day the satellite data utility is increasing in the fields of urban planning, forestry studies, climate studies, draught estimations and for effective management when natural disasters occurred and this utility may be from state , central government or private organisations. For urban, forestry, climate studies temporal data is more useful and consequently there is huge requirement of older Indian Remote Sensing Satellite (IRS) data from user community for the study of above applications. Also photogrammetric processing levels may be different depending on their application. But after development of physical sensor model satellite vendors are using(Rational polynomial coefficients(RPC) based generic software for any photogrammetric processing but in the data product generation chain rad-ortho kit(raw product with RPC's file and metadata file which gives the information about the data) products are not available for older Indian Remote Sensing Satellites (IRS) IRS-1C/1D.In this paper developed methodology for the generation of RPC's based on the physical sensor model namely line of sight (LOS)model and applied on IRS-1C/1D,LISS-III sensor data but in this paper only IRS-1C LISS-III sensor results are published due to similarity of results and sensors of IRS-1C/1D.

Keywords
LOS model, Rad-ortho kit, RPC, Raw product, RMS.

1. INTRODUCTION
Indian Remote Sensing Satellite-1C (IRS-1C) was launched on December 28,1995 and its applications such as crop acreage and yield estimation, drought monitoring and assessment, flood mapping, wasteland mapping, ocean/marine resources survey, urban mapping, mineral prospecting, forest resource survey etc have integral part of the resources management system in the country[6].The IRS-1C payload consists of Linear imaging self scanning sensors (LISS-III) of spatial resolution of 23.5 meters for B2,B3,B4 and 70.5 meters in SWIR-B5 ,PAN camera of spatial resolution 5.8 meters and WiFS camera with188.3 meters of spatial resolution. IRS – 1D was launched on September 27, 1997 by PSLV – C1. IRS – 1D, a follow on satellite to IRS – 1C. It has 3 payloads viz., PAN, LISS 3 &WiFS. It has similar capabilities as IRC – 1C in terms of spatial resolution, spectral bands, stereoscopic imaging, wide field coverage and revisit capability. The improvements carried out in the IRS – 1D satellite taking into account the IRS – 1C experiences have resulted in better quality imageries [6].

For IRS-1C/1D satellites in the NRSC data product generation chain RPC products are not available and after development of physical sensor model vendors are using rpc's based generic software for any photogrammetric processing and therefore there is a requirement for generation of RPC's for IRS-1C/1D data. In this paper developed methodology for RPC's generation based on the physical sensor model namely line of sight (LOS) model[7] and hence this methodology enables the user community to do the photogrammetric processing with generic software for these satellites also. Finally generated ortho rectified product using the above rpc's from GDAL-libraries and these products are validated with reference images.

Also LOS model is not straightforward as in[2,3,7] due to non-availability of some of inputs for LOS model from ADIF and OAT of IRS-1C/1D like sidereal angle etc and hence for calculation of sidereal angle is also presented in this paper as in[1].

1.1 LINE-OF-SIGHT MODEL (LOS)
The line-of-sight model (LOS) is optical sensor model which transforms image coordinates to ground coordinates with series of transformations using sensor look angles, payload alignment angles, satellite attitude angles, satellite position, satellite velocity , sidereal angle and earth model[2,3]. In this paper all the sub models which are used in LOS model are discussed [3]. The inputs for LOS model are taken from ADIF(scene information) and OAT(position, velocity, attitude and sidereal angle).But onlyResourcesat-1 onwards sidereal angle has been giving in OAT module but which is not available in OAT of older satellites like IRS-1C/1D and hence in this paper algorithm for sidereal angle calculation is also presented. Line of sight model algorithm based on [2,3,7] is mentioned below.

Sensor model-look vector computation:
The sensor model takes a pixel in the satellite image and computes its look vector in the sensor coordinate system. It also computes the time for the instance of this look. The sensor model is initialized with parameters specific for the sensor design. It can be a generic model for a push broom scanner, which will have parameters such as focal length, detector positions in the focal plane and scan-line time interval. The sensor model is the sub-model that is most often modified when implementing a new satellite system.
Formula for finding detector look angle (across track):

ys=(pixel number-center pixel number)*IFOV;

xs=-FL;

zs=0;

Image coordinate in CCD plane= [-xsyszs] T

Body model:

In this model payload alignment angles are used to rotate the look vector from the sensor coordinate system to the satellite body coordinate system.

Attitude model:

From OAT file roll, pitch, yaw angles are used to transform the look vector from the body coordinate system to the flight coordinate system.

Flight model:

In this models satellite position and velocity vector is used to transform the look vector from the flight coordinate system to the Earth Centered Inertial (ECI) coordinate system.

Astronomical model:

The astronomical model is used to transform the position and look vectors from the ECI system to the Earth Centered Rotating (ECR) coordinate system. In this model sidereal angle is to transform look vector from ECI to ECR conversion by definition of ECI and ECR coordinate systems.

Intersection model:

The intersection model calculates the intersection point between the look vector and an Earth model (ellipsoidal) centered in the ECR system. The ellipsoidal height is also input to get a unique position.

Geodesy model:

The geodesy model transforms the Earth intersection point, expressed in ECR coordinates, to a geographic coordinate (longitude, latitude, optometric height). It uses a geoids model to account for the irregularities in the Earth zero potential surfaces. The result is a coordinate in the WGS84 system.

Sidereal time/angle calculation:

Sidereal time is measured by the rotation of the earth relative to the fixed stars and Local sidereal time θ of a site is the time elapsed since the local meridian of the site passed through the vernal equinox [1]. To know the location of a point on the earth at any given instant relative to the geocentric equatorial frame requires knowing its local sidereal time. The local side-real time of a site is found by first determining the Greenwich sidereal time θ_G (the sidereal time of the Greenwich meridian), and then adding the east longitude (or subtracting the west longitude) of the site. The following procedure [1] followed to determine sidereal time.
The Julian day number at 0 hr UT is

$$J_0 = 367y - INT\left\{\frac{7\left[INT\left(\frac{m+9}{12}\right)\right]}{4}\right\} + INT\left(\frac{275m}{9}\right) + d + 1{,}721{,}013.5$$

Where y, m and d are integers lying in the following ranges.

$$1901 \le y \le 2099\ , 1 \le m \le 12\ ,1 \le d \le 31$$

INT(x) means to retain only the integer portion of x, without rounding (or, in other words, round towards zero);

The time T_0in Julian centuries between the Julian day J_0 and J2000 is

$$T_0 = \frac{J_0 - 2{,}451{,}545}{36{,}525}$$

The Greenwich sidereal time θ_{G0}in degrees is given by the series

$$\theta_{G0} = 100.4606184 + 36{,}000.77004T_0 + 0.000387933T_0^2 - 2.583(10^{-8})T_0^3$$

This formula can yield a value outside of the range $0 \le \theta_{G0} \le 360^0$.If so, then the appropriate integer multiple of 360 ° must be added or subtracted to bring θ_{G0}into that range.

Once θ_{G0}has been determined, the Greenwich sidereal time $\theta_G$at any other universal time is found using the relation

$$\theta_G = \theta_{G0} + 360.98564724\frac{UT}{24}$$

WhereUT is in hours. The coefficient of the second term on the right is the number of degrees the earth rotates in 24 hours (solar time).

Finally, the local sidereal time θ of a site is obtained by adding its east longitude Λ to the Greenwich sidereal time,

$$\theta = \theta_G + \Lambda$$

Here again it is possible for the computed value of θ to exceed 360°. If so, it must be reduced to within that limit by subtracting the appropriate integer multiple of 360 °.

2. RATIONAL FUNCTION MODEL

2.1 Introduction

A sensor model describes the geometric relationship of 3D object coordinates to 2D image coordinates. There are various models to provide the relation and each model has its own merits and demerits. There are two broad categories of sensor models, which are (a) generalized and (b) physical. The choice of a sensor model depends primarily on the performance and the accuracy required the camera and control information available. A physical sensor model represents the physical imaging process. The parameters involved describe the position and the orientation of a sensor with respect to an object space co-ordinate system. In physical models, parameters are normally uncorrelated because each parameter has a physical significance. All the rigorous physical sensor models are more accurate, the development of generalized sensor models independent of sensor platforms and sensor types. In a generalized sensor model, the transformation between the image and the object space is represented as some general function without modeling the physical imaging process. The function can be of different form such as a polynomials or rational functions.
A physical sensor model needs the physical parameters of the sensor such as the position and the orientation of the sensor with respect to an object space co-ordinate system and these parameters are not correlated and yield accurate results. LOS model developed in this project and is described in the previous chapter belongs to this category. But in the recent past the satellite data providers are not ready to part with the sensor model.
As an alternative, remote sensing data providers started rational polynomial coefficients (RPC) models where sensor model is provided in the form of RPCs, The generalized models represent the transformation of the co-ordinates from 3D to 2D by rational function models (RFM). This chapter describes implementation of RFM to establish relationship between object space and image space for all the optical sensors, taking the LOS model as the basis.

2.2 Mathematical Preliminaries

RFM is a mathematical transformation between object space and image space co-ordinates. The RFM is defined as ratio of two cubic polynomials separately for row and column coordinates of image and hence it is given[5,8,9] as:

$$\boldsymbol{r} = \frac{\boldsymbol{N_r(X,Y,Z)}}{\boldsymbol{D_r(X,Y,Z)}} \boldsymbol{c} = \frac{\boldsymbol{N_c(X,Y,Z)}}{\boldsymbol{D_c(X,Y,Z)}}$$

Where r, c are normalized row and column coordinates of image space and X, Y, Z are normalized coordinates of object space. The constant term in the denominator of r and c is taken as 1 to avoid singularity in equation (1). Also **Nr**(X,Y,Z), **Dr**(X,Y,Z), **Nc**(X,Y,Z), **Dc**(X,Y,Z) defined as:

$$N_r(X,Y,Z) = a_1 + a_2X + a_3Y + a_4Z + a_5XY + a_6XZ + a_7YZ + a_8X^2 + a_9Y^2 + a_{10}Z^2 + a_{11}XYZ + a_{12}X^3 + a_{13}XY^2 + a_{14}XZ^2 + a_{15}X^2Y + a_{16}Y^3 + a_{17}YZ^2 + a_{18}ZX^2 + a_{19}ZY^2 + a_{20}Z^3$$

$$D_r(X,Y,Z) = b_1 + b_2X + b_3Y + b_4Z + bXY + bXZ + bYZ + b_8X^2 + b_9Y^2 + b_{10}Z^2 + b_{11}XYZ + b_{12}X^3 + b_{13}XY^2 + b_{14}XZ^2 + b_{15}X^2Y + b_{16}Y^3 + b_{17}YZ^2 + b_{18}ZX^2 + b_{19}ZY^2 + b_{20}Z^3$$

$$b_1 = 1$$

$$N_c(X,Y,Z) = c_1 + c_2X + c_3Y + c_4Z + c_5XY + c_6XZ + c_7YZ + c_8X^2 + c_9Y^2 + c_{10}Z^2 + c_{11}XYZ + c_{12}X^3 + c_{13}XY^2 + c_{14}XZ^2 + c_{15}X^2Y + c_{16}Y^3 + c_{17}YZ^2 + c_{18}ZX^2 + c_{19}ZY^2 + c_{20}Z^3$$

$$D_c(X,Y,Z) = d_1 + d_2X + d_3Y + d_4Z + d_5XY + d_6XZ + d_7YZ + d_8X^2 + d_9Y^2 + d_{10}Z^2 + d_{11}XYZ + d_{12}X^3 + d_{13}XY^2 + d_{14}XZ^2 + d_{15}X^2Y + d_{16}Y^3 + d_{17}YZ^2 + d_{18}ZX^2 + d_{19}ZY^2 + d_{20}Z^3$$

$$d_1 = 1$$

X, Y, Z are the normalized object space coordinates i.e. normalized latitude, longitude and height respectively. x and y are the normalized scan line number and pixel number between (-1,+1). These $a_i's, b_i's, c_i's, d_i's$are polynomial coefficients called rational function coefficients (RFC's).

In RFM model, the distortions caused by optical projection can be expressed as 1st-order polynomial coefficients, and the error caused by the earth curvature, atmospheric refraction and lens distortion can be corrected by 2nd-order polynomial coefficients, and that caused by other unknown distortions can be simulated by 3rd-order polynomial coefficients.

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The methodology of developing the RFM is summarized in the figure1[1] as below:

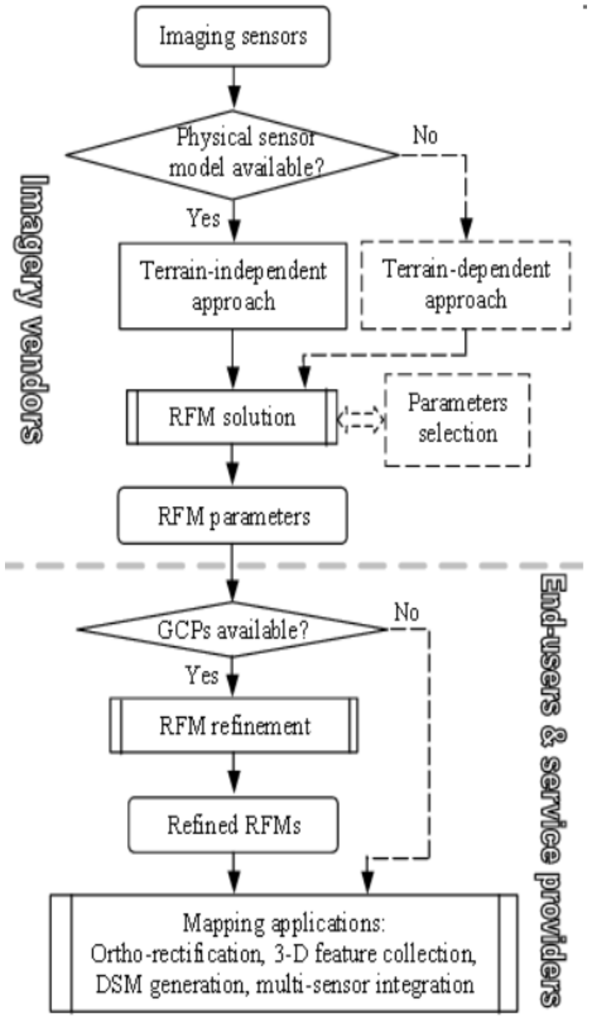

Fig. 1

The RFM uses ratios of polynomials to establish the relationship between the images coordinates and the object coordinates. The universal real-time model is in fact an extension to the RFM. It employs interpolation of high-order correction functions. Because the RFM is the most popular model in use, the emphasis in this study is placed on the investigation of the RFM.

2.3 RFC Generation Methodology

In order to avoid time consuming process and to improve the numerical stability of the equations in 1, both image and object space co-ordinates are normalized to the range of -1.0 to 1.0 .The normalization of the coordinates is computed as follows:

$$\boldsymbol{x_n} = \frac{\boldsymbol{x - x_o}}{\boldsymbol{x_s}} \quad , \quad \boldsymbol{y_n} = \frac{\boldsymbol{y - y_o}}{\boldsymbol{y_s}}$$

$$\boldsymbol{X_n} = \frac{\boldsymbol{X-X_o}}{\boldsymbol{X}} \quad , \quad \boldsymbol{Y_n} = \frac{\boldsymbol{Y-Y_o}}{\boldsymbol{Y_s}} \quad , \quad \boldsymbol{Z_n} = \frac{\boldsymbol{Z-Z_o}}{\boldsymbol{Z_s}}.$$

Where, $x_o, y_o, X_o, Y_o, Z_o$are the mean values for scan line number, pixel number, latitude, longitude and height respectively and $x_s, y_s, X_s, Y_s, Z_s$are the scale values for scanline number, pixel number, latitude, longitude and height respectively.

The maximum power of each ground co-ordinate is typically limited to 3; and the total power of all the ground co-ordinates is also limited to 3.In such a case, each polynomial is of 20-term cubic form.

So from the expression of scan line and pixel line functions, we have[5]

$$\boldsymbol{r} = \frac{(\boldsymbol{1XYZ} \ldots \boldsymbol{Y^3Z^3}) \, . \, (\boldsymbol{a_1a_2} \ldots \boldsymbol{a_{20}})^{\boldsymbol{T}}}{(\boldsymbol{1XYZ} \ldots \boldsymbol{Y^3Z^3}) \, . \, (\boldsymbol{1b_2} \ldots \boldsymbol{b_{20}})^{\boldsymbol{T}}}$$

$$\boldsymbol{c} = \frac{(\boldsymbol{1XYZ} \ldots \boldsymbol{Y^3Z^3}) \, . \, (\boldsymbol{c_1c_2} \ldots \boldsymbol{c_{20}})^{\boldsymbol{T}}}{(\boldsymbol{1XYZ} \ldots \boldsymbol{Y^3Z^3}) \, . \, (\boldsymbol{1d_2} \ldots \boldsymbol{d_{20}})^{\boldsymbol{T}}}$$

Where $a_i's, b_i's, c_i's, d_i's$are polynomial coefficients.

$$\text{Let } M = \begin{bmatrix} 1 & X_1 & \cdots & -r_1Z_1^3 \\ 1 & X_2 & & -r_nZ_2^3 \\ \vdots & & \ddots & \vdots \\ 1 & X_3 & \cdots & -r_nZ_n^3 \end{bmatrix}$$

$$J = (M^TM)^{-1}M^TR$$

$R = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$Is the set of‘n’ number of ground control points.

$J = (a_0a_1 \cdots a_{19}b_1b_2 \cdots b_{19})^T$Is the result matrix containing the RFCs for row of image coordinates? We get from equation (1) as

$$V = MJ - R \qquad - (2)$$

Where V is the error matrix.

Since the error to be minimum, first take the values of V to be zero for direct least square solution [1]. It is noticed that solving equation (2) normally is critical because the matrix size is large and hence to solve the above equation multiply both the sides of the equation by the inverse of M matrix in order to get square matrices which can be worked easily with in terms of taking inverse and hence the equation becomes

$$0 = (M^TM)J - M^TR$$

Simplifying it further gives:

$J = (M^TM)^{-1}M^TR.$

Inverse of M^TM is found by singular value decomposition method and then the rational polynomial coefficients are computed.

One common difficulty in fitting nonlinear models is finding adequate starting values. A major advantage of rational function models is the ability to compute starting values using a linear least squares fit. To do this, *p* points are chosen from the data set, with *p* denoting the number of parameters in the rational model. For example, given the linear/quadratic model

$$y = \frac{A_0 + A_1x}{1 + B_1x + B_2x^2}$$

The RPC model forms the co-ordinates of the image point as ratios of the cubic polynomials in the co-ordinates of the world or object space or ground point. A set of images is given to determine the set of polynomial coefficients in the RPC model to minimize the error.

2.4 Software Implementation

The RFM model relates the ground coordinate to its corresponding image pixel coordinate using a ratio polynomial. For one image, the following ratio polynomial is defined

$$\begin{cases} x = \dfrac{P_1(X,Y,Z)}{P_2(X,Y,Z)} \\ y = \dfrac{P_3(X,Y,Z)}{P_4(X,Y,Z)} \end{cases}$$

Where (X, Y, Z) is the regularized control point ground coordinates, and (x, y) is the regularized image pixel coordinates. The polynomial Pi (i = 1, 2, 3, 4) consists of a third degree polynomial containing X, Y, Z, and has the same independent form of parameter.

$P_1(y, x, z) = a_0 + a_1X + a_2Y + a_3Z + a_4XY + a_5XZ + a_6YZ + a_7X^2 + a_8Y^2 + a_9Z^2 + a_{10}Y^2Z + a_{11}X^3 + a_{12}XY^2 + a_{13}XZ^2 + a_{14}X^2Y + a_{15}Y^3 + a_{16}YZ^2 + a_{17}X^2Z + a_{18}YXZ + a_{19}Z^3$

$P_2(y, x, z) = b_0 + b_1X + b_2Y + b_3Z + b_4XY + b_5XZ + b_6YZ + b_7X^2 + b_8Y^2 + b_9Z^2 + b_{10}Y^2Z + b_{11}X^3 + b_{12}XY^2 + b_{13}XZ^2 + b_{14}X^2Y + b_{15}Y^3 + b_{16}YZ^2 + b_{17}X^2Z + b_{18}YXZ + b_{19}Z^3$

$P_3(y, x, z) = c_0 + c_1X + c_2Y + c_3Z + c_4XY + c_5XZ + c_6YZ + c_7X^2 + c_8Y^2 + c_9Z^2 + c_{10}Y^2Z + c_{11}X^3 + c_{12}XY^2 + c_{13}XZ^2 + c_{14}X^2Y + c_{15}Y^3 + c_{16}YZ^2 + c_{17}X^2Z + c_{18}YXZ + c_{19}Z^3$

$P_4(y, x, z) = d_0 + d_1X + d_2Y + d_3Z + d_4XY + d_5XZ + d_6YZ + d_7X^2 + d_8Y^2 + d_9Z^2 + d_{10}Y^2Z + d_{11}X^3 + d_{12}XY^2 + d_{13}XZ^2 + d_{14}X^2Y + d_{15}Y^3 + d_{16}YZ^2 + d_{17}X^2Z + d_{18}YXZ + d_{19}Z^3$

Where a*i*, b*i*, c*i*, d*i* (i= 0, ..., 19) are the coefficients of *Pi*(i=1,2,3,4)respectively. Where

b0 =1and d0=1.

In order to enhance the stability of the parameter solution, the regularized to between -1 and 1. Before running a linear algorithm to calculate parameters, it is important to normalize the coordinate data using scale factors and offsets.

$$\begin{cases} X_i^R = \dfrac{X_i - X_O}{X_S} \\ X_O = \dfrac{\sum_{i=1}^{n} X_i}{n} \\ X_S = \max(|X_{max} - X_O|, |X_{min} - X_O|) \end{cases}$$

Where *XI* is the original coordinate, X_i^R (i=1, 2, ton) is the normalized coordinate.

$$r = \frac{(1\,Z\,Y\,X \ldots Y^3X^3) \, . \, (a_0a_1 \ldots a_{19})^{T}}{(1\,Z\,Y\,X \ldots Y^3X^3) \, . \, (1\,b_1 \ldots b_{19})^{T}}$$

$$c = \frac{(1\ Z\ Y\ X \ldots Y^3X^3)\,.\,(c_0 c_1 \ldots c_{19})^T}{(1\ Z\ Y\ X \ldots Y^3X^3)\,.\,(1\ d_1 \ldots d_{19})^T}$$

The observation error equations can then be formed as

$$v_r = \left(\frac{1}{\mathbf{B}}\frac{Z}{\mathbf{B}}\frac{Y}{\mathbf{B}}\frac{X}{\mathbf{B}} \ldots \frac{Y^3}{\mathbf{B}}\frac{X^3}{\mathbf{B}} - \frac{rZ}{\mathbf{B}} - \frac{rY}{\mathbf{B}} \ldots \frac{rY^3}{\mathbf{B}} - \frac{rX^3}{\mathbf{B}}\right).\mathbf{J} - \frac{r}{\mathbf{B}}$$

$$v_c = \left(\frac{1}{\mathbf{D}}\frac{Z}{\mathbf{D}}\frac{Y}{\mathbf{D}}\frac{X}{\mathbf{D}} \ldots \frac{Y^3}{\mathbf{D}}\frac{X^3}{\mathbf{D}} - \frac{rZ}{\mathbf{D}} - \frac{rY}{\mathbf{D}} \ldots \frac{rY^3}{\mathbf{D}} - \frac{rX^3}{\mathbf{D}}\right).\mathbf{K} - \frac{c}{\mathbf{D}}$$

Or

$$v_r{}' = \mathbf{B}v_r = [\,1\ Z\ Y\ X \ldots Y^3\ X^3 - rZ - rY \ldots - rY^3 - rX^3\,].\mathbf{J} - r$$

$$v_c{}' = \mathbf{D}v_c = [\,1\ Z\ Y\ X \ldots Y^3\ X^3 - cZ - cY \ldots - cY^3 - cX^3\,].\mathbf{K} - c$$

Where $\mathbf{B} = (1\ Z\ Y\ X \ldots Y^3X^3)\,.\,(1\ b_1 \ldots b_{19})^T$

$\mathbf{J} = (a_0 a_1 \ldots a_{19}\ b_1\ b_2 \ldots b_{19})^T$

$\mathbf{D} = (1\ Z\ Y\ X \ldots Y^3X^3)\,.\,(1\ d_1 \ldots d_{19})^T$

$\mathbf{K} = (c_0 c_1 \ldots c_{19}\ d_1\ d_2 \ldots d_{19})^T$

Given n, the number of ground control points (GCPS) and the corresponding image points, the matrix form of Equation can be written as

$$\begin{bmatrix} v_{r1} \\ v_{r2} \\ \vdots \\ v_{rn} \end{bmatrix} = \begin{bmatrix} \frac{1}{\mathbf{B}_1} & 0 & \cdots & 0 \\ 0 & \frac{1}{\mathbf{B}_2} & 0 & \vdots \\ \vdots & 0 & \ddots & 0 \\ 0 & \cdots & 0 & \frac{1}{\mathbf{B}_n} \end{bmatrix} . \begin{bmatrix} 1 & Z_1 & \cdots & {X_1}^3 & -r_1Z_1 & \cdots & -r_1{X_1}^3 \\ 1 & Z_2 & \cdots & {X_2}^3 & -r_2Z_2 & \cdots & -r_2{X_2}^3 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 1 & Z_n & \cdots & {X_n}^3 & -r_nZ_n & \cdots & -r_n{X_n}^3 \end{bmatrix} . \mathbf{J} - \begin{bmatrix} \frac{1}{\mathbf{B}_1} & 0 & \cdots & 0 \\ 0 & \frac{1}{\mathbf{B}_2} & 0 & \vdots \\ \vdots & 0 & \ddots & 0 \\ 0 & \cdots & 0 & \frac{1}{\mathbf{B}_n} \end{bmatrix} . \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_n \end{bmatrix}$$

Or

$$\mathbf{V_r} = \mathbf{W_r MJ - WrR}$$

Where

$$\mathbf{M} = \begin{bmatrix} 1 & Z_1 & \cdots & {X_1}^3 & -r_1Z_1 & \cdots & -r_1{X_1}^3 \\ 1 & Z_2 & \cdots & {X_2}^3 & -r_2Z_2 & \cdots & -r_2{X_2}^3 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 1 & Z_n & \cdots & {X_n}^3 & -r_nZ_n & \cdots & -r_n{X_n}^3 \end{bmatrix},$$

$$\mathbf{R} = \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_n \end{bmatrix}, \quad \mathbf{W_r} = \begin{bmatrix} \frac{1}{\mathbf{B}_1} & 0 & \cdots & 0 \\ 0 & \frac{1}{\mathbf{B}_2} & 0 & \vdots \\ \vdots & 0 & \ddots & 0 \\ 0 & \cdots & 0 & \frac{1}{\mathbf{B}_n} \end{bmatrix}$$

$\mathbf{W_r}$ can be considered as the weight matrix for the residuals and consequently, the obtained normal equation is

$$\mathbf{M^T W_r{}^2 MJ - MTWr^2R} = 0$$

If $\mathbf{W_r}$ is set to be identity matrix, the direct solution of RFCs can be represented as

$$\mathbf{J = (M^T M)^{-1} M^T R}$$

Similarly, for pixel $\mathbf{K = (M^T M)^{-1} M^T C}$

The inverse of the matrix is calculated using singular value decomposition in the software implementation

2.5 Results and data sets used

Features in Google reference image and generated product were identified manually and following results are obtained. The four points at the corner and one point in the centre of the scene were selected in both Google reference image and in the IRS-1C satellite LISS-III scenes 96_49,96_55,96_62 as in figure 2 figure 3 and figure 4 respectively shown after correction with RPC and same procedure is applied on IRS-1D data also analysed but in this paper only IRS-1C results are published due to similarity of results and sensor.

The following points with Lat/Lon and the offset/error from the same point in Google reference image. The distance was measured with the accuracies in kilometres and shown in the table. Similarly for other scenes, same procedure was adopted and the results are shown in the table 2.1

Table 2.1 RPC accuracy results of IRS-1C LISS-III sensor

S. No	**Sensor**	**Satellite**	**DOP**	**Req. ID**	**Path /Row**	**Latitude**	**Longitude**	**Radial error offset in kilometers**
1	LISS-III	IRS-1C		19250131 1	96/49	30.4593	77.8977	0.77
						30.2729 8	78.42433	0.89
						31.6524 3	77.32006	0.95
						30.9335	77.6240	0.76
2	LISS-III	IRS-1C	041020	19250132 1	96/55	24.6544 5	75.60262	0.76
						23.9699 2	76.74979	0.32

						24.51447	76.53457	0.08
						24.5061	76.3277	0.16
3	LISS-III	IRS-1C		192501331	96/62	14.84104	74.11629	0.51
						15.12938	73.93159	0.57
						15.85161	73.61255	0.52
						16.15596	74.62691	0.79
						16.13546	74.64037	0.93

The reverse RPC were also computed for the above scenes and the Mean and RMSE for line and sample are shown in this table

Table 2.2 Mean and RMSE for (Lat, Long, Hei) to (line, Sample) Conversion

S. No	Sensor	Satellite	DOP	Req.ID	Path/Row	Line Mean	Line RMSE	Sample Mean	Sample RMSE
1	LISS-III	IRS-1C	041020	192501311	96/49	0.26080043	0.510686234	0.174554644	0.417797372
2	LISS-III	IRS-1C	041020	192501321	96/55	0.215108638	0.463798058	0.189464047	0.43527468
3	LISS-III	IRS-1C	041020	192501331	96/62	0.432253294	0.656601971	0.456359999	0.675544224

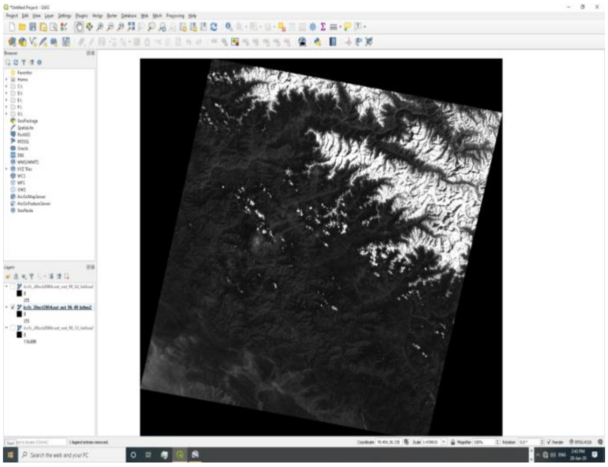

Fig. 2 RPC was applied to raw 1C_96_49 scene and result visualized in QGIS viewer

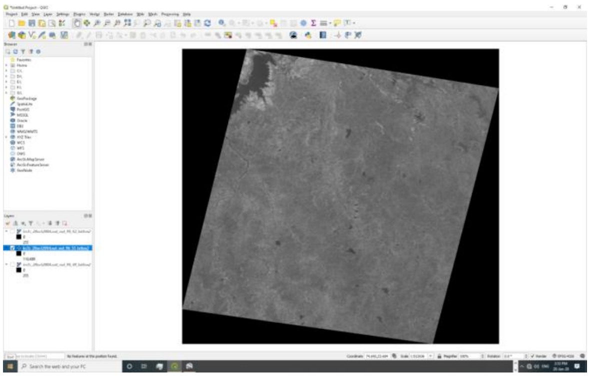

Fig. 3 RPC was applied to raw 1C_96_55 scene and result visualized in QGIS viewer

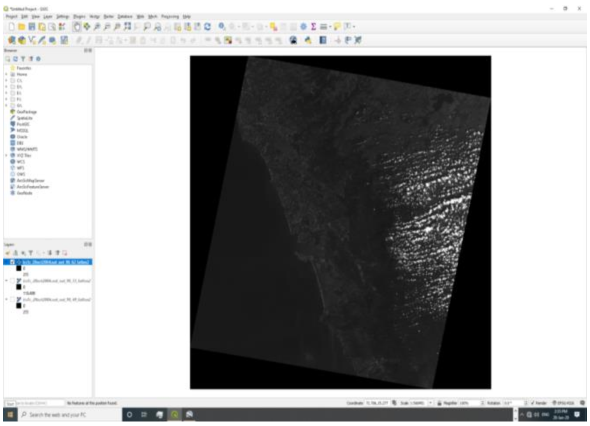

Fig .4 RPC was applied to raw 1C_96_62 scene and result visualized in QGIS viewer

2.5 CONCLUSION

Through RFM generated RPCs based on LOS model and validated with known GCP's on IRS-1C/1D data and future scope is same methodology can be implemented for other sensors of IRS-1C/1D.

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