Pythagorean Fuzzy Semi-Prime Ideals of Ordered Semi-Groups

Amal Kumar Adak, PhD
Department of Mathematics
Ganesh Dutt College, Begusai
Bihar, India

Gaurikant Kumar
Department of Mathematics
Lalit Narayan Mithila University
Darbhanga, Bihar, India

Monoranjan Bhowmik
Department of Mathematics
V.T.T.College, Midnapore
Westbengal, India

ABSTRACT
Pythagorean fuzzy sets are expanded to include intuitionistic fuzzy sets, with the extra advantage of avoiding underlying limitations. Pythagorean fuzzy standards are defined in the literature using the concepts of Pythagorean fuzzy sets. The concepts of ordered semigroup semi-prime ideals and Pythagorean fuzzy prime aspirations are explained. Also illustrate how to construct Pythagorean fuzzy regular and intra-regular ideals using Pythagorean fuzzy regular and intra-regular ideals. Using the conception of the characteristic function of a non-empty subset of ordered semigroups, investigate certain fundamental facts. Several relations are given for the family of Pythagorean fuzzy ideals of ordered semigroups.

Keywords
Intuitionistic fuzzy set, Pythagorean fuzzy set, Pythagorean fuzzy ideals, Pythagorean fuzzy semi-prime ideals , Pythagorean fuzzy regular ideals.

2010 AMS Classification: 16Y30; 03E72; 16Y99.

1. INTRODUCTION
Zadeh [27] developed the fuzzy set methodology, that assigns a number from the unit range [0, 1] to each element of the discursive multiverse to indicate the degree of sense of belonging to the set under consideration using a degree of membership, μ. Fuzzy sets are a subset of set theory that allows for states halfway between entire and nothing. A membership function is employed in a fuzzy set to represent the extent to which an element belongs to a class. The membership value can be anything between 0 and 1, with 0 indicating that the element is not a member of a class, 1 indicating that it is, and other values indicating the degree of membership. The membership function in fuzzy sets replaced the characteristic function in crisp sets. Fuzzy set theory has been applied to a variety of domains since Zadeh's seminal work, including artificial intelligence, management sciences, engineering, mathematics, statistics, signal processing, automata theory, social sciences, medical sciences, and biological sciences.

Because of the absence of nonmembership functions and the disregard for the potential of hesitation margin, the idea of fuzzy sets theory appears to be inconclusive. Atanassov [8] examined these flaws and created the concept of intuitionistic fuzzy sets (IFSs) to address them. The construct (that is, IFSs) combines the membership function, μ, with the nonmembership function, ν, and the hesitation margin, π (that is, neither membership nor nonmembership functions), resulting in $\mu+\nu \le 1$ and $\mu+\nu+\pi=1$. IFSs give a versatile framework for elaborating uncertainty and ambiguity. There has been a lot of study done in the domain of IFSs in [1, 2, 3, 5].

Unlike with the instances in IFSs, there are situations where $\mu+\nu \ge 1$ exists. Pythagorean fuzzy sets were created as a result of this requirement in IFS (PFSs). The Pythagorean fuzzy set (PFS), suggested in [24, 23] is an unique tool for dealing with ambiguous when evaluating membership grade μ and nonmembership grade ν satisfying the constraints $0 \le \mu \le 1$ or $0 \le \nu \le 1$, with the result that $\mu^2+\nu^2+\pi^2 \le 1$, where π is the hesitation degree. PFSs can be used to characterise uncertain data more thoroughly and correctly than IFS.. PFS clearly outperforms IFS when it comes to simulating confusion in the real world.

Algebraic structures are important in mathematics, with applications in a variety of fields including theoretical physics, computer science, control engineering, and information science. This gives scholars with adequate motivation to revisit many concepts and discoveries from real abstract algebra in the context of a broader fuzzy setting. The concept of intuitionistic fuzzification of various semigroup ideals was introduced by Kim and Jun [12, 13, 14]. In [15], Kim and Lee gave the notion of intuitionistic fuzzy bi-ideals of semigroups. Sen and Saha [19, 20] defined the Γ-semigroup and established a relation between regular Γ-semigroup and Γ-group. In 2007, Sardar et al. [21] gave the concept of intuitionistic fuzzy prime ideals, semi-prime ideals and also intuitionistic fuzzy ideal extension in a Γ-semigroup in [17, 18, 22].

The remainder of the paper is laid out as follows. Preliminaries and definitions such as ordered set, ordered subgroups, intuitionistic fuzzy sets, and Pythagorean fuzzy sets are given in Section 2. In Section 3, we looked at how Pythagorean fuzzy prime ideals and semi-prime ideals of ordered semigroups are defined, as well as some of the important aspects of Pythagorean fuzzy regular and intra-regular ordered semigroup ideals. Section 4 concludes with a conclusion.

2. PRELIMINARIES AND DEFINITION
We will review the related concepts of fuzzy sets, intuitionistic fuzzy sets, and Pythagorean fuzzy sets in this section. The definition of oredered set, ordered semigroup, prime ideal, semi-prime ideal are represented.

Definition 2.1 Ordered Semigroup: *A non empty set M is called an ordered semigroup if it is both an ordered set and a semigroup that meets the following criteria:*

$$a \le b \Rightarrow xa \le xb \text{ and } ax \le bx \ \ \forall a,b,x \in M.$$

Definition 2.2 *Consider* $(M,.,\leq)$ *be an ordered semigroup. A non-empty subset* G *of* M *is called a subsemigroup of* M *if* $G^2 \subseteq G$.

Definition 2.3 *Let* P *be a subset of an ordered semigroup* M *, that isn't empty. Then* P *is called a left (resp. right) ideal of* M *if it satisfies:*

(i) $MP \subseteq P (resp. PM \subseteq P)$,

(ii) $(\forall p \in P)\ (\forall q \in M), (q \leq p \Rightarrow q \in P)$.

P will be ideal of M if both left and right ideal of M.

Definition 2.4 *Let* $(M,.,\leq)$ *be an ordered semigroup and* N *be a non-empty subset of* M.

Then N is called prime if $pq \in N \Rightarrow p \in N$ or $q \in N$ for all $p, q \in M$.

Let N be an ideal of M, if N is prime subset of M, then N is called prime ideal.

Definition 2.5 *Let* $(M,.,\leq)$ *be an ordered semigroup and* N *be a non-empty subset of* M.

Then N is called semi-prime if $p^2 \in N \Rightarrow p \in N$ for all $p \in M$. Let N be an ideal of M. If N is a semi-prime subset of M, then N is called semi-prime ideal.

Definition 2.6 *A fuzzy set* F *in a universal set* X *is defined as* $F = \{\langle x, \mu_F(x)\rangle : x \in X\}$,

where $\mu_F : X \to [0,1]$ is a mapping that is known as the fuzzy set's membership function.

The complement of μ is defined by $\overline{\mu}(x) = 1 - \mu(x)$ for all $x \in X$ and denoted by $\overline{\mu}$.

Definition 2.7 *Let* $(M,.,\leq)$ *be an ordered semigroup. A fuzzy subset* μ *of* M *is called a fuzzy ideal of* M *, if the following axioms are satisfied:*

(i) if $p \leq q$, then $\mu(p) \geq \mu(q)$,

(ii) $\mu(pq) \geq \max\{\mu(p), \mu(q)\}$, $\forall (p,q) \in M$.

Definition 2.8 *Let* X *be a fixed set. An intuitionistic fuzzy set (IFS)* A *in* X *is an expression having the form*

$$A = \{\langle x, \alpha_A(x), \beta_A(x)\rangle : x \in X\},$$

where the $\alpha_A(x)$ is tha memebership grade and $\beta_A(x)$ is the non-membership grade of the element $x \in X$ respectively.

Also $\alpha_A : X \to [0,1], \beta_A : X \to [0,1]$ and satisfy the condition

$0 \leq \alpha_A(x) + \beta_A(x) \leq 1,$ for all $x \in X$.

The degree of indeterminacy $h_A(x) = 1 - \alpha_A(x) - \beta_A(x)$.

In practise, the condition $0 \leq \mu(x) + \nu(x) \leq 1$ may not be true for any reason. $0.6 + 0.5 = 1.1 > 1$, but $0.6^2 + 0.7^2 < 1$, or $0.4 + 0.7 = 1.1 > 1$, but $0.4^2 + 0.7^2 < 1$. To address this issue, Yager [24, 23] proposed the concept of the Pythagorean fuzzy set in 2013.

Definition 2.9 *A Pythagorean fuzzy set* P *in a finite universe of discourse* X *is given by*

$$P = \{\langle x, \alpha_P(x), \beta_P(x)\rangle \mid x \in X\},$$

where $\alpha_P(x) : X \to [0,1]$ denotes the degree of membership and $\beta_P(x) : X \to [0,1]$ represents the degree to which the element $x \in X$ is not a member of the set P, with the condition that

$$0 \leq (\alpha_P(x))^2 + (\beta_P(x))^2 \leq 1,$$

for all $x \in X$.

The degree of indeterminacy $h_P(x) = \sqrt{1 - (\alpha_P(x))^2 - (\beta_P(x))^2}$.

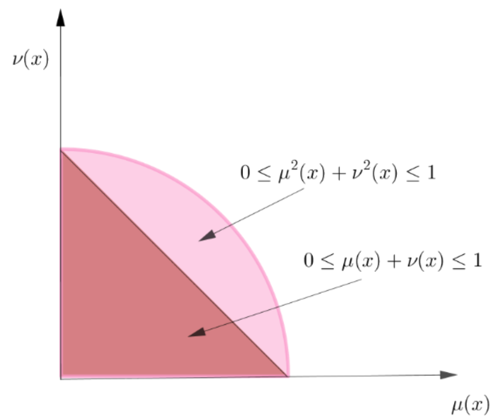

Figure 1: Intuitionistic fuzzy set vs Pythagorean fuzzy set

Definition 2.10 *Let* $P = (\alpha_P, \beta_P)$ *be a Pythagorean fuzzy set in* M *. Then* $P = (\alpha_P, \beta_P)$ *is called Pythagorean fuzzy subsemigroup of* M *if it satisfies the following axioms:*

(i) $\alpha_P(pq) \geq \min\{\alpha_P(p), \alpha_P(q)\}$,

(ii) $\beta_P(pq) \leq \max\{\beta_P(p), \beta_P(q)\}, \forall p, q \in M$.

Definition 2.11 *A Pythagorean fuzzy set* $P = (\alpha_P, \beta_P)$ *in* M *is said to be Pythagorean fuzzy left ideal of* M *if following axioms are satisfied:*

(i) $p \le q$ implies $\alpha_P(p) \ge \alpha_P(q)$ and $\alpha_P(pq) \ge \alpha_P(q)$,

(ii) $p \le q$ implies $\beta_P(p) \le \beta_P(q)$ and $\beta_P(pq) \le \beta_P(q), \forall p, q \in M$.

Definition2.12 *A Pythagorean fuzzy set* $P = (\alpha_P, \beta_P)$ *in* M *is said to be Pythagorean fuzzy right ideal of* M *if following axioms are satisfied:*

(i) $p \le q$ implies $\alpha_P(p) \ge \alpha_P(q)$ and $\alpha_P(pq) \ge \alpha_P(p)$,

(ii) $p \le q$ implies $\beta_P(p) \le \beta_P(q)$ and $\beta_P(pq) \le \beta_P(p), \forall p, q \in M$.

A Pythagorean fuzzy set $P = (\alpha_P, \beta_P)$ is called a Pythagorean fuzzy ideal of M if it is left ideal as well as right ideal.

3. PYTHAGOREAN FUZZY SEMI-PRIME IDEALS OF ORDERED SEMIGROUPS

This section introduces the notion of Pythagorean fuzzy prime ideal, Pythagorean fuzzy semi-prime ideal, Pythagorean fuzzy regular ideals and Pythagorean fuzzy intra-regular ideals of ordered semigroups. Also, prove some important results utilizing characteristic function of a non-empty subset of ordered semigroups.

Definition 3.1 *A fuzzy subset* μ *of* M *is called prime, if* $\mu(pq) = \max\{\mu(p), \mu(q)\}, \forall p, q \in M$,

where $(M, ., \le)$ be an ordered semigroup. A fuzzy ideal μ of M is called a fuzzy prime ideal of M if μ is a prime fuzzy subset of M.

Definition 3.2 *Let* $P = (\alpha_P, \beta_P)$ *be a Pythagorean fuzzy set in* M. *Then* $P = (\alpha_P, \beta_P)$ *is called Pythagorean fuzzy prime of* M *if it satisfies the following axioms:*

(i) $\alpha_P(pq) = \max\{\alpha_P(p), \alpha_P(q)\}$,

(ii) $\beta_P(pq) = \min\{\beta_P(p), \beta_P(q)\}, \forall p, q \in M$.

We denote the characteristic function of a nonempty subset G of an ordered semigroup by χ_G

Theorem 3.1 *If* G *is a prime ideal, then* $G = (\chi_G, \overline{\chi}_G)$ *is a Pythagorean fuzzy prime of* M.

Proof. Let us consider $p, q \in M$, if $p, q \in G$, then $p \in G$ or $q \in G$. Thus $\chi_G(p) = 1$ or $\chi_G(q) = 1$.

Thus we have

$$\chi_G(pq) = 1 = \max\{\chi_G(p), \chi_G(q)\}$$

and $\overline{\chi}_G(pq) = 1 - \chi_G(pq) = 0$

$$= \min \overline{\chi}_G(p), \overline{\chi}_U(q).$$

If $pq \notin G$, then $p \notin G$, and $q \notin G$.

Thus $\chi_G(p) = 0$ and $\chi_G(q) = 0$.

Thus we have

$$\chi_U(pq) = 0 = \max\{\chi_G(p), \chi_G(q)\}$$

and

$$\overline{\chi}_G(pq) = 1 - \chi_G(pq) = 1$$

$$= \min\{\overline{\chi}_G(p), \overline{\chi}_G(q)\}.$$

This completes the proof.

Theorem 3.2 *Let* G *be a non-empty subset of* M. *If* $G = (\chi_G, \overline{\chi}_G)$ *is prime of* M, *then* G *is prime.* **Proof.** Suppose that $G = (\chi_G, \overline{\chi}_G)$ is prime of M and $pq \in G$. In this case, $g = pq$ for some $g \in G$. Therefore,

$$1 = \chi_G(g) = \chi_G(pq) = 1 = \max\{\chi_G(p), \chi_G(q)\}.$$

Hence $\chi_G(p) = 1$, or $\chi_G(q) = 1$ i.e., $p \in G$ or $q \in G$. Thus G is prime.

Now, assume that $G = (\chi_G, \overline{\chi}_G)$ is a prime of M and $x'y' \in G$.

Then $g' = x'y'$ for some $g' \in G$.

Now, from the property of prime, we get

$$\overline{\chi}_G(g') = 1 - \chi_G(g') = 0 = \overline{\chi}_G(p'q')$$

$$= \min\{\overline{\chi}_G(p'), \overline{\chi}_G(q')\}$$

$$= \min\{1 - \chi_G(p'), 1 - \chi_G(q')\}$$

and so $1 - \chi_G(p') = 0$ or $1 - \chi_G(q') = 0$.

Therefore $\chi_G(p') = 1$ or $\chi_G(q') = 1$, i.e., $p' \in G$ or $q' \in G$.

This completes the proof.

Definition 3.3 *Let us consider* μ *be a fuzzy subset of an ordered semigroup* M. *If* $\mu(p) \ge \mu(p^2)$, *for all*

$p \in M$, then μ is called semi-prime. A fuzzy ideal μ of M is called a fuzzy semi-prime ideal of M if μ is a fuzzy semi-prime subset of M.

Definition 3.4 *Let $P = (\alpha_P, \beta_P)$ be a Pythagorean fuzzy set in M. Then $P = (\alpha_P, \beta_P)$ is called Pythagorean fuzzy semi-prime of M if following criterias are satisfied:*

(i) $\alpha_P(p) \geq \alpha_P(p^2)$,

(ii) $\beta_P(p) \leq \beta_P(p^2),\ forall\ \ p \in M$.

Theorem 3.3 *If G is semi-prime, then $\square_G = (\chi_G, \square\chi_G)$ is a Pythagorean fuzzy semi-prime of M.*

Proof. Let g be any element of M. If $g^2 \in G$, then since G is semi-prime, we have $g \in G$. Thus

$$\chi_G(g) = 1 \geq \chi_G(g^2)$$
$$and\ \ \overline{\chi}_G(g) = 1 - \chi_G(g) = 0 \leq \overline{\chi}_G(g^2).$$

If $g^2 \notin G$, then we have $\chi_G(g^2) = 0$.

Therefore,

$$\chi_G(g) \geq 0 = \chi_G(g^2)$$
$$and\ \ \overline{\chi}_G(g^2) = 1 - \chi_G(g^2) = 1 \geq \overline{\chi}_G(g).$$

This completes the proof.

Theorem 3.4 *Let G be a non-empty subset of M. If $\square_G = (\chi_G, \square\chi_G)$ is Pythagorean fuzzy semi-prime of M, then G is semi-prime.*

Proof. Suppose that $\square_G = (\chi_G, \square\chi_G)$ is a Pythagorean fuzzy semi-prime of M and $p^2 \in G$.

In this case, $g = p^2$ for some $g \in G$. It follows that

$$1 = \chi_G(g) = \chi_G(p^2) \leq \chi_G(p).$$

Hence $\chi_G(p) = 1$, i.e., $p \in G$.

Thus G is semi-prime.

Now, assume that $\square_G = (\chi_G, \square\chi_G)$ is a Pythagorean fuzzy semi-prime of M and $p_0{}^2 \in G$. Then $g_0 = p_0{}^2$ for some $g_0 \in G$.

Therefore

$$\square\chi_G(p_0) \leq \square\chi_G(p_0{}^2) = 1 - \chi_U(p_0{}^2) = 1 - 1 = 0,$$

$$i.e.,\ \square\chi_G(p_0) = 1 - \chi_G(p_0) = 0.$$

Hence, $\chi_G(p_0) = 1$, and so $p_0 \in G$.

This completes the proof.

Theorem 3.5 *For any Pythagorean fuzzy subsemigroup $P = (\alpha_P, \beta_P)$ of M, if $P = (\alpha_P, \beta_P)$ is Pythagorean fuzzy semi-prime, then $P(p) = P(p^2)$ holds.*

Proof. Let p be an element of M. Since α_P is a fuzzy subsemi group of M, then

$$\alpha_P(p) \geq \alpha_P(p^2) = \min\{\alpha_P(p), \alpha_P(p)\} = \alpha_P(p)$$

and so we have $\alpha_P(p) = \alpha_P(p^2)$.

Also, we have

$$\beta_P(p) \leq \beta_P(p^2) \equiv \max\{\beta_P(p), \beta_P(p)\} = \beta_P(p).$$

Thus $\beta_P(p) = \beta_P(p^2)$.

This proves the theorem.

Definition 3.5 *An ordered semigroup M is called left (resp. right) regular if, for each element a of M, there exists an element x in M such that $a \leq xa^2 (resp. a \leq a^2 x)$.*

Theorem 3.6 *Let M be left regular. Then, for every Pythagorean fuzzy left ideal $P = (\alpha_P, \beta_P)$ of M, $P(p) = P(p^2)$ holds for all $p \in M$.*

Proof. Let p be any element of M. Since M is left regular, there exists an element x in M such that $p \leq xp^2$. Thus we have

$$\alpha_P(p) \geq \alpha_P(xp^2) \geq \alpha_P(p^2) \geq \alpha_P(p),$$

and so we have $\alpha_P(p) = \alpha_P(p^2)$.

Also, we have

$$\beta_P(p) \leq \beta_P(xp^2) \leq \beta_P(p^2) \leq \beta_P(p).$$

Thus $\beta_P(p) = \beta_P(xp^2)$. So, $P(p) = P(p^2)$.

This completes the proof.

Theorem 3.7 *Let M be left regular. Then, every Pythagorean fuzzy left ideal of M is Pythagorean fuzzy semi-prime.*

Proof. Let $P = (\alpha_P, \beta_P)$ be a Pythagorean fuzzy left ideal of M and let $p \in M$. Then, there exists an element x in M such that $p \leq xp^2$ since M is left regular. So, we have

$$\alpha_P(p) \geq \alpha_P(xp^2) \geq \alpha_P(p^2),$$
$$and\ \ \beta_P(p) \leq \beta_P(xp^2) \leq \beta_P(p^2).$$

This completes the proof.

Definition 3.6 *An ordered semigroup M is called intra-regular if, for each element p of M, there exist elements x and y in M such that $p \leq xp^2y$.*

Definition 3.7 *Let $P = (\alpha_P, \beta_P)$ be a Pythagorean fuzzy set in M. Then $P = (\alpha_P, \beta_P)$ is called a Pythagorean fuzzy interior ideal of M if it satisfies axioms:*
(i) $x \leq y$ implies $\alpha_P(x) \geq \alpha_P(y)$ and $\alpha_P(xsy) \geq \alpha_P(s)$,

(ii) $x \leq y$ implies $\beta_P(x) \leq \beta_P(y)$, and $\beta_P(xsy) \leq \beta_P(s)$, $\forall x, y \in M$.

Theorem 3.8 *Let $P = (\alpha_P, \beta_P)$ be a Pythagorean fuzzy set in an intra-regular ordered semigroup M. Then, $P = (\alpha_P, \beta_P)$ is a Pythagorean fuzzy interior ideal of M if and only if $P = (\alpha_P, \beta_P)$ is an Pythagorean fuzzy ideal of M.*

Proof. Let p, q be any elements of M, and let $P = (\alpha_P, \beta_P)$ be a Pythagorean fuzzy interior ideal of M. Then, since M is intra-regular, there exist elements x, y, u and v in M such that $p \leq xp^2y$ and $q \leq uq^2v$. Then, since α_P is a fuzzy interior ideal of M, we have

$$\alpha_P(pq) \geq \alpha_P((xp^2y)q) = \alpha_P((xp)p(yq)) \geq \alpha_P(p)$$

and

$$\alpha_P(pq) \geq \alpha_P(p(uq^2v)) = \alpha_P((pu)q(qv)) \geq \alpha_P(q).$$

Also, we have

$$\beta_P(pq) \leq \beta_P((xp^2y)q) = \beta_P((xp)p(yq)) \leq \beta_P(p)$$

and

$$\beta_P(pq) \leq \beta_P(p(uq^2v)) = \beta_P((pu)q(qv)) \leq \beta_P(q).$$

On the other hand, let $P = (\alpha_P, \beta_P)$ be a Pythagorean fuzzy ideal of M. Then, since α_P is a fuzzy ideal of M, we have

$$\alpha_P(xpy) = \alpha_P(x(py)) \geq \alpha_P(py) \geq \alpha_P(p)$$

$$\text{and } \beta_P(xpy) = \beta_P(x(py)) \leq \beta_P(py) \leq \beta_P(p)$$

for all x, a and $y \in M$. This completes the proof.

Theorem 3.9 *Let $P = (\alpha_P, \beta_P)$ be a Pythagorean fuzzy ideal of M. If M is intra-regular, then $P = (\alpha_P, \beta_P)$ is Pythagorean fuzzy semi-prime.*

Proof. Let p be any element of M. Then since M is intra-regular, there exist x and y in M such that $p \leq xp^2y$. So, we have

$$\alpha_P(p) \geq \alpha_P(xp^2y) \geq \alpha_P(p^2y) \geq \alpha_P(p^2)$$

and $\beta_P(p) \leq \beta_P(xp^2y) \leq \beta_P(p^2y) \leq \beta_P(p^2)$.

This proves the theorem.

Theorem 3.10 *Let $P = (\alpha_P, \beta_P)$ be a Pythagorean fuzzy interior ideal of M. If M is an intra-regular, then $P = (\alpha_P, \beta_P)$ is intuitionistic fuzzy semi-prime.*

Proof. Let p be any element of M. Then since M is intra-regular, there exist x and y in M such that $p \leq xp^2y$. So, we have

$$\alpha_P(p) \geq \alpha_P(xp^2y) \geq \alpha_P(p^2)$$

and $\beta_P(p) \leq \beta_P(xp^2y) \leq \beta_P(p^2)$.

This proves the theorem.

Theorem 3.11 *Let M be intra-regular. Then, for all Pythagorean fuzzy interior ideal $P = (\alpha_P, \beta_P)$ and for all $p \in M$, $P(p) = P(p^2)$ holds.*

Proof. Let p be any element of M. Then since M is intra-regular, there exist x and y in M such that $p \leq xp^2y$. So, we have

$$\alpha_P(p) \geq \alpha_P(xp^2y) \geq \alpha_P(p^2)$$

$$\geq \alpha_P((xp^2y)(xp^2y))$$

$$= \alpha_P((xp)p(yxp^2y)) \geq \alpha_P(p)$$

$$\text{and } \beta_P(p) \leq \beta_P(xp^2y) \leq \beta_P(p^2)$$

$$\leq \beta_P((xp^2y)(xp^2y))$$

$$= \beta_P((xp)p(yxp^2y)) \leq \beta_P(p)$$

So, we have $P(p) = P(p^2)$. This completes the proof.

Theorem 3.12 *Let M be intra-regular. Then, for all Pythagorean fuzzy interior ideal $P = (\alpha_P, \beta_P)$ and for all $p, q \in M$, $P(pq) = P(qp)$ holds.*

Proof. Let p be any element of M. Then since M is intra-regular, there exist x and y in M such that $p \leq xp^2y$. So, we have

$$\alpha_P(pq) = \alpha_P((pq)^2) = \alpha_P(p(qp)q) \geq \alpha_P(qp)$$

$$= \alpha_P((qp)^2) = \alpha_P(q(pq)p) \geq \alpha_P(pq)$$

and

$$\beta_P(pq) = \beta_P((pq)^2) = \beta_P(p(qp)q) \leq \beta_P(qp)$$

$$= \beta_P((qp)^2) = \beta_P(q(pq)p) \leq \beta_P(pq).$$

So, we have $P(pq) = P(qp)$. This proves the theorem.

Definition 3.8 *An ordered semigroup M is called archimedean if, for any elements p, q there exists a positive integer n such that $p^n \in MqM$.*

Theorem 3.13 *Suppose S be an ordered archimedean semigroup. Then, each intuitionistic fuzzy semi-prime fuzzy ideal of S is a constant function.*

Proof. Let $P = (\alpha_P, \beta_P)$ be any Pythagorean fuzzy semi-prime fuzzy ideal of M and $p, q \in M$. Then since M is archimedean, there exist x and y in M such that $p^n = xqy$ for some integer n . Then, we have

$$\alpha_P(p) = \alpha_P(p^n) = \alpha_P(xqy) \geq \alpha_P(q)$$
$$and\ \alpha_P(q) = \alpha_P(q^n) = \alpha_P(xpy) \geq \alpha_P(p).$$

Thus, we have $\alpha_P(p) = \alpha_P(q).$

Also, we have

$$\beta_P(p) = \beta_P(p^n) = \beta_P(xqy) \geq \beta_P(q)$$
$$and\ \beta_P(q) = \beta_P(q^n) = \beta_P(xpy) \geq \beta_P(p).$$

Therefore, we have $P(p) = P(q)$ for all $p, q \in M$.

This proves the theorem.

4. CONCLUSION

The Pythagorean fuzzy set is an effective expansion of the intuitionistic fuzzy set for dealing with knowledge uncertainty. In this context, the concepts of Pythagorean fuzzy prime ideals and semi-prime ideals of ordered semigroups in this study. Several of its appealing characteristics have also been studied. Also explore various findings on Pythagorean fuzzy regular ideals and intraregular ideals of ordered semigroups, along with promote the implementation of Pythagorean fuzzy regular ideals.

In future, interval-valued Pythagorean fuzzy sets are being used to solve difficulties with uncertain data. An investigation of the Interval-valued Pythagorean fuzzy will be conducted out oredered semigroups, near-rings and Interval-valued Pythagorean prime and semi-prime ideals, as well as their algebraic features

5. REFERENCES

[1] A. K. Adak, M. Bhowmik and M. Pal, Interval cut-set of generalized interval-valued intuitionistic fuzzy sets, *International Journal of Fuzzy System Applications*, 2 (3) (2012) 35-50.

[2] A. K. Adak, M. Bhowmik and M. Pal, Distributive Lattice over Intuitionistic Fuzzy Matrices, *The Journal of Fuzzy Mathematics*, 21(2) (2013) 401-416.

[3] D. Manna and A. K. Adak, Interval-valued Intuitionistic Fuzzy R-subgroup of Near-rings, *Journal of fuzzy mathematics*, 24(4) (2016) 985-994.

[4] A. K. Adak, D. D. Salokolaei, Some properties of Pythagorean fuzzy ideal of near-rings, *International Journal of Applied Operational Research* , 9(3) (2019) 1-9.

[5] A. K. Adak, Interval-Valued Intuitionistic Fuzzy Subnear Rings, *Handbook of Research on Emerging Applications of Fuzzy Algebraic Structures* IGI-Global, (2020) 213-224.

[6] A. K. Adak, D. D. Salokolaei, Some Properties Rough Pythagorean Fuzzy Sets, *Fuzzy Information and Engineering (IEEE)* , 13(4) (2021) 420-435.

[7] A. K. Adak, Characterization of Pythagorean Q-Fuzzy Ideal of Near-Ring, *Handbook of Research on Advances and Applications of Fuzzy Sets and Logic* IGI-Global, (2022) 229-242.

[8] K. Atanassov, Intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, 20 (1986) 87-96.

[9] R. Biswas, Fuzzy subgroups and anti-fuzzy subroups, Fuzzy Sets and Sys, vol 35(1) (1990) 121-124.

[10] H. Garg, A new generalized Pythagorean fuzzy information aggregation using Einstein operations and its application to decision making. *Int J Intell Syst.* 31(9) (2016) 886-920.

[11] H. Garg, Generalized Pythagorean fuzzy geometric aggregation operators using Einstein t-norm and t-conorm for multicriteria decision-making process. *Int J Intell Syst.* 32(6) (2017) 597-630.

[12] Y. B. Jun, K. H. Kim, Y. H. Yon, Intuitionistic fuzzy ideals of near-rings, *J. Inst. Math. Comp. Sci.* 12 (3) (1999) 221-228.

[13] K. H. Kim and Y. B. Jun, Intuitionistic fuzzy ideals of semigroups, *Indian J. Pure Appl. Math.* 33(4) (2002) 443-449.

[14] K. H. Kim and Y.B. Jun, Intuitionistic fuzzy interior ideals of semigroups, *Int. J. Math. Math. Sci.* 27 (5) (2001) 261-267.

[15] K. H. Kim and J. G. Lee, On fuzzy bi-ideals of semigroups, *Turk. J. Math.* 29 (2005) 201–210.

[16] S. P. Kuncham, S. Bhavanari, Fuzzy prime ideal of a gamma near ring. *Soochow Journal of Mathematics* 31 (1) (2005) 121-129.

[17] N. Kuroki, On fuzzy ideals and fuzzy bi-ideals in semigroups, *Fuzzy Sets and Systems* 5 (1981), 203-215.

[18] N. Kuroki, Fuzzy semiprime ideals in semigroups, *Fuzzy Sets and Systems* 8 (1982), 71-79.

[19] N. K. Saha, On Γ-semigroup II, *Bull. Calcutta Math. Soc.* 79(6) (1987) 331–335.

[20] N. K. Saha, On Γ-semigroup III, $\widehat{\mathrm{B}}$ ull. Calcutta Math. Soc. 80(1) (1988) 1-13.

[21] S. K. Sardar, S. K. Majumder and M. Mandal, Atanassov's intuitionistic fuzzy ideals of Γ-semigroups, *Int. J. Algebra* 5(7) (2011) 335-353.

[22] M. K. Sen and N. K. Saha, On Γ-semigroup I, *Bull. Calcutta Math. Soc.* 78(3) (1986) 180–186

[23] R. R. Yager, Abbasov AM. Pythagorean membeship grades, complex numbers and decision making. *Int J Intell Syst* 28 (2013) 436-452.

[24] R. R. Yager, Pythagorean fuzzy subsets. *In: Proc Joint IFSA World Congress and NAFIPS Annual Meeting*, Edmonton, Canada, (2013) 57-61.

[25] R. R. Yager, Pythagorean membership grades in multicriteria decision making. *IEEE Transaction on Fuzzy Systems* 22 (2014) 958-965.

[26] X. Yun Xie, F. Yan, Fuzzy ideal extension of ordered semigroups, *Lobach. J. Math.* 19 (2005), 29-40.

[27] L. A. Zadeh, Fuzzy sets, *Information and Control*, 8(1965) 338-353.