Wavelet Thresholding for Image De-noising

Rohit Sihag
M.Tech Research Scholar
Electronics & Comm. Deptt.
NIT Hamirpur(H.P)-177005

Rakesh Sharma
Assistant Professor
Electronics & Comm. Deptt.
NIT Hamirpur (H.P)-177005

Varun Setia
M.Tech Research Scholar
Electronics & Comm. Deptt.
NIT Hamirpur (H.P)-177005

ABSTRACT
The de-noising is a challenging task in the field of signal and image processing. De-noising of the natural image corrupted by Gaussian noise using wavelet techniques are very effective because of its ability to capture the energy of a signal in few energy transform values. The wavelet denoising scheme thresholds the wavelet coefficients arising from the standard discrete wavelet transform. In this paper, we analyzed several methods of noise removal from degraded images with Gaussian noise by using adaptive wavelet threshold (Bayes Shrink, Normal Shrink and Neigh Shrink) and compare the results in term of PSNR.

Keywords: **Image De-noising, Wavelet Thresholding, Bayes Shrink, Normal Shrink, Neigh Shrink.**

1. INTRODUCTION
An image is often corrupted by noise during its acquisition or transmission. The de-noising process is to remove the noise while retaining and not distorting the quality of the processed image. The traditional way of image de-noising is filtering. Recently, a lot of research about non-linear methods of signal de-noising has been developed. These methods are mainly based on thresholding the Discrete Wavelet Transform (DWT) coefficients, which have been affected by additive white Gaussian noise [1]. Simple denoising algorithms that use DWT consist of three steps.

- Discrete wavelet transform is adopted to decompose the noisy image and get the wavelet coefficients.
- These wavelet coefficients are denoised with wavelet threshold.
- Inverse transform is applied to the modified coefficients and get denoised image.

The second step, known as thresholding, is a simple non-linear technique, which operates on one wavelet coefficient at a time. In its most basic form, each coefficient is thresholded by comparing threshold, if the coefficient is smaller than threshold, set to zero; otherwise it kept as it is or it is modified. Replacing the small noisy coefficient by zero and inverse wavelet transform on the resulted coefficient may lead to reconstruction with the essential signal characteristics and with less noise [2].
The paper is organized as follows. Section 2 introduces discrete wavelet transform. Section 3 explains wavelet thresholding. Section 4 explains the parameter estimation for Bayes Shrink, Normal Shrink and Neigh Shrink. Experimental results and discussion are presented in section 5. Finally, the conclusion and references are given in section 6 and 7.

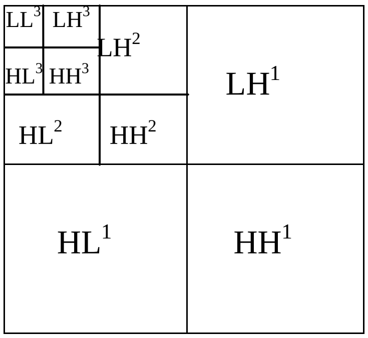
2. DISCRETE WAVELET TRANSFORM
The Discrete Wavelet Transform (DWT) of image signals produces a non-redundant image representation, which provides better spatial and spectral localization of image formation, compared with other multi scale representations such as Gaussian and Laplacian pyramid. Recently, Discrete Wavelet Transform has attracted more and more interest in image de-noising. The DWT can be interpreted as signal decomposition in a set of independent, spatially oriented frequency channels. The signal S is passed through two complementary filters and emerges a two signals, approximation and details. This is called decomposition or analysis. The components can be assembled back into the original signal without loss of information. This process is called reconstruction or synthesis. The mathematical manipulation, which implies analysis and synthesis, is called discrete wavelet transform and inverse discrete wavelet transform [3]. Another consideration of the wavelets is the sub-band coding theory or multi resolution analysis. The signal passes successively through pairs of low pass and high pass filters, the analysis filters, which produce the transform coefficients. These coefficients, if passes successively through the synthesis filters, may reproduce the initial signal at the decoder's side. In case of a 2D image, an N level decomposition can be performed resulting in 3N+1 different frequency bands namely, LL (low frequency or approximation coefficients), LH (vertical details), HL (horizontal details) and HH (diagonal details) as shown in figure 1. In figure 1, the number written next to sub-band name shows the level. The next level of wavelet transform is applied to the low frequency sub-band image LL only.

3. WAVELET TRESHOLDING
Let the signal be $\{f_{ij}\} i=1,\cdots,M,\ j=1,\cdots,N$, where M, N is some integer power of 2. It has been corrupted by additive noise and one observes

$$g_{ij} = f_{ij} + \sigma n_{ij}, \quad i = 1, \cdots, M,\ j = 1, \cdots, N \tag{1}$$

where $\{n_{ij}\}$ are independent and identically distributed (*iid*) zero mean, white Gaussian Noise with standard deviation σ i.e. as normal $n_{ij} \sim N(0, \sigma^2)$. The goal is to estimate $\{f'_{ij}\}$ from noisy observation $\{g_{ij}\}$ such that Mean Squared Error (MSE) is minimum, that is given by

$$MSE = \frac{1}{MN}\sum_{j=1}^{M}\sum_{i=1}^{N}[\{f_{ij}^{'}\} - \{f_{ij}\}]^{2} \quad (2)$$



1, 2, 3-Decomposition Levels
H-----High Frequency Bands
L------Low Frequency Bands

Fig 1. Sub-bands of the 2-D orthogonal wavelet transform [3]

It is convenient to label the sub-bands of the transform as in Fig. 1. The sub-bands HH^k, HL^k, LH^k are called the details, where k is the level ranging from 1 to J, where J is the largest level. The sub-band LL^J is the low resolution residual.The wavelet-thresholding de-noising method filters each coefficient from the detail sub-bands with a threshold function to obtain modified coefficients. The de-noised estimated by inverse wavelet transform of the modified coefficients. Here, the threshold plays an important role in the de-noising process. There are two thresholding methods frequently used. The *soft-threshold* function (also called the shrinkage function)

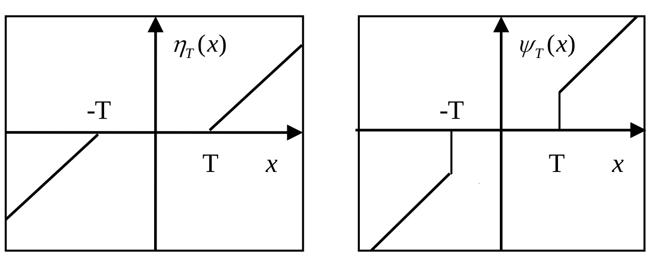
$$\eta_T(x) = \mathrm{sgn}(x)\cdot \max(|x| - T, 0) \quad (3)$$

takes the argument and shrinks it towards zero by the threshold T. The other popular alternative is the *hard thresholding* function.

$$\psi_T(x) = x \cdot 1\{|x| > T\} \quad (4)$$

which keeps the input if it is larger than the threshold; otherwise, it is set to zero. The wavelet thresholding procedure removes noise by thresholding only the wavelet coefficients of the detail sub-bands, while keeping the low resolution coefficients unaltered [3], [4].

Hard thresholding is a keep or kill rule whereas soft thresholding shrinks the coefficients above the threshold in absolute value. It is a shrink or kill rule as shown in Fig 2.



(a) (b)

Fig.2. Thresholding function: (a) Soft threshold, (b) Hard threshold

4. WAVELET BASED IMAGE DE-NOISING

All digital images contain some degree of noise. Image de-noising algorithm attempts to remove this noise from the image. Ideally, the resulting de-noised image will not contain any noise or added artifacts. De-noising of natural images corrupted by Gaussian noise using wavelet techniques is very effective because of its ability to capture the energy of a signal in few energy transform values. The basic frame work of the wavelet transform based image de-noising is showed in Fig. 3.

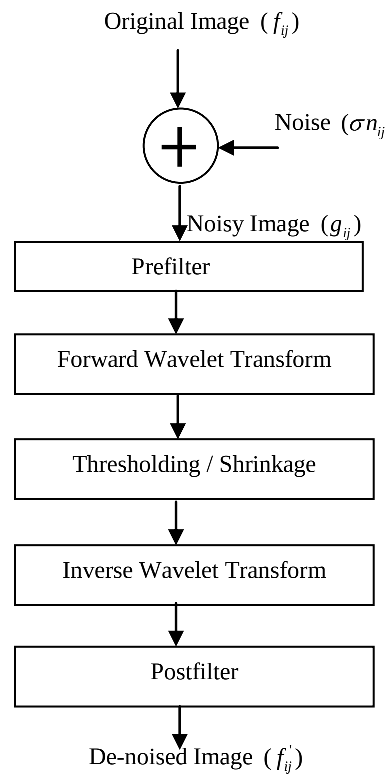


Fig. 3.The basic frame work of the wavelet transform based image de-noising

Finding an optimum threshold is a tedious process. A small threshold value will retain the noisy coefficients whereas a large threshold value leads to the loss of coefficients that carry image signal details [3].

The following are the methods of threshold selection for image de-noising based on wavelet transform

Method 1: Bayes Shrink (BS)

The Bayes Shrink method is effective for images including Gaussian noise. The observation model is expressed as follows:

$$Y = X + V \quad (5)$$

Here Y is the wavelet transform of the degraded image, X is the wavelet transform of the original image, and V denotes the wavelet transform of the noise components following the Gaussian distribution N (0, σ_v^2). Here, since X and V are mutually independent, the variances σ_y^2, σ_x^2 and σ_v^2 of y, x and v are given by:

$$\sigma_y^{\,2} = \sigma_x^{\,2} + \sigma_v^{\,2} \quad (6)$$

It has been shown that the noise variance σ_v^2 can be estimated from the first decomposition level diagonal sub-band HH_1 by the robust and accurate median estimator [4].

$$\hat{\sigma}_v^{\,2} = \left[\frac{median(|HH_1|)}{0.6745}\right]^2 \quad (7)$$

The variance of the sub-band of degraded image can be estimated as:

$$\hat{\sigma}_y^{\,2} = \frac{1}{M}\sum_{m=1}^{M} A_m^{\,2} \quad (8)$$

where A_m are the wavelet coefficients of sub-band under consideration, M is the total number of wavelet coefficient in that sub-band.
The bayes shrink thresholding technique performs soft thresholding, with adaptive data driven, sub-band and level dependent near optimal threshold given by [4]:

$$T_{BS} = \begin{cases} \dfrac{\hat{\sigma}_v^{\,2}}{\hat{\sigma}_x} & if\,\hat{\sigma}_v^{\,2} < \hat{\sigma}_y^{\,2} \\ \max\{|A_m|\} & otherwise \end{cases} \quad (9)$$

Where $\hat{\sigma}_x = \sqrt{\max(\hat{\sigma}_y^{\,2} - \hat{\sigma}_v^{\,2}, 0)}$

In the case, where $\hat{\sigma}_v^{\,2} > \hat{\sigma}_y^{\,2}$, $\hat{\sigma}_x$ is taken to be zero, i.e. $T_{BS} \rightarrow \infty$, or, in practice, $T_{BS} = \max(|A_m|)$, and all coefficients are set to zero.

Method 2: Normal Shrink (N)

The optimum threshold value for the Normal Shrink (T_N) is given by [2]:

$$T_N = \frac{\lambda \hat{\sigma}_v^{\,2}}{\hat{\sigma}_v} \quad (10)$$

Where, the parameter λ is given by the following equation:

$$\lambda = \sqrt{\log\left(\frac{L_K}{J}\right)} \quad (11)$$

L_k is the length of the sub-band at k_{th} scale. And, J is the total number of decomposition. $\hat{\sigma}_v$ is the estimated noise variance, calculated by equation (7) and $\hat{\sigma}_y$ is the standard deviation of the sub-band of noisy image, calculated by using equation (8).
Normal Shrink also performs soft thresholding with the data driven sub-band dependent threshold T_N, which is calculated by the equation (10).

Method 3: Neigh Shrink (NS)

Let **g**= {g_{ij}} will denote the matrix representation of the noisy signal. Then, $w = \mathrm{W}g$ denotes the matrix of wavelet coefficients of the signal under consideration. For every value of w_{ij}, let B_{ij} is a neighbouring window around w_{ij}, w_{ij} denotes the wavelet coefficient to be shrinked. The neighbouring window size can be represented as $L \times L$, where L is a positive odd number. A 3×3 neighbouring window centered at the wavelet coefficient to be shrinked is shown in Fig 4.

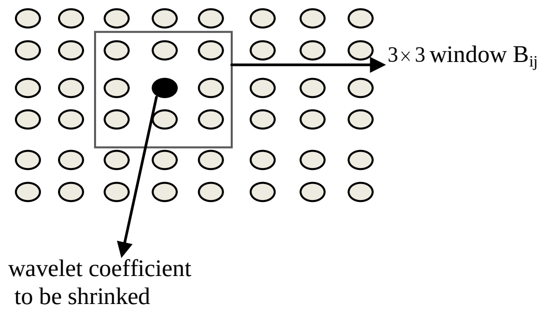


Fig. 4. An illustration of the neighbouring window of size 3×3 centered at the wavelet coefficient to be shrinked [5]

Let

$$s_{ij} = \sum_{(k,l)\in B_{ij}} w_{kl}^{\,2} \quad (12)$$

We omit the corresponding terms in the summation when the above summation has pixel indexes out of the wavelet sub-band range. The shrinked wavelet coefficient according to the neighshrink is given by this formula [5]:

$$w'_{ij} = w_{ij}\beta_{ij} \quad (13)$$

The shrinkage factor β_{ij} can be defined as:

$$\beta_{ij} = (1 - T_{UNI}^{\,2} / s^2_{ij})_+ \quad (14)$$

here, the + sign at the end of the formula means to keep the positive value while set it to zero when it is negative and T_{UNI} is the universal threshold, which is defined as [6]:

$$T_{UNI} = \sqrt{2\sigma^2 \ln(n)} \quad (15)$$

where n is the length of the signal.
Different wavelet coefficient sub-bands are shrinked independently, but the universal threshold T_{UNI} and neighbouring window size L kept unchanged in all sub-bands. The estimated de-noised signal $f' = f'_{ij}$ is calculated by taking the inverse wavelet transform of the shrinked wavelet coefficients w'_{ij} i.e. $f' = \mathrm{W}^{-1}w'$.

5. Experimental Results and Discussion

The experiments are conducted on natural gray scale test images like Lena and Boat of size 512×512. The kind of noise, added to original image, is Gaussian of different noise levels σ = 10, 20, 30, 40. First, the image is transformed into the orthogonal domain by taking wavelet transform. Then, the wavelet coefficients are modified according to the thresholding or shrinkage algorithm. Finally, inverse wavelet transform of the modified wavelet coefficients is taken to reconstruct the de-noised image. In this paper, different wavelet bases are used, at one scale of decomposition, in all methods. The window size for neigh shrink is taken in this experiment is 5×5. For taking the wavelet and inverse wavelet transform of the image, available MATLAB commands are used. In each sub-band, individual pixels of the image are de-noised according to the method used. For measuring the performance of the methods Peak Signal to Noise Ratio (PSNR) is used, which is calculated using the formula:

$$PSNR(db)= 10\log_{10}(255)^2 /MSE \qquad (16)$$

where MSE is the mean squared error between the original image and the reconstructed de-noised image, which is calculated by the equation (2).

The experiment results are shown in Table I and Fig.5. The PSNR from various methods are compared in Table I and the data are collected from an average of four runs on the image Lena of size 512×512. It is a comparison between Bayes Shrink, Normal Shrink and Neigh Shrink. So, the better one is highlighted in bold font for each test. Table I also shows the result of PSNR with respected to wavelet bases. Among all wavelet bases, the best one is highlighted in italic font for each test. In the result we found that the Neigh Shrink gives the better performance with respected to other methods. It is also found in the experiment that the coiflet performance better in image de-noising. But, when we taking the time into account, then, the average elapsed time by Neigh Shrink for coiflet wavelet bases in a single test, is near about 15 seconds, which is much more than the other two methods. The average time taking by the Bayes Shrink and Normal Shrink is 1.50 seconds and 1.48 seconds, respectively.

TABLE I. Comparison of PSNR Values For Different Wavelet Bases, Image De-Noising Techniques And Set Values of σ.

Wavelet	σ	Bayes Shrink	Normal Shrink	Neigh Shrink
Haar	10	31.4122	31.1543	**31.8963**
	20	26.7704	26.7234	**26.9708**
	30	23.8178	23.8245	**23.9174**
	40	21.5959	21.6112	**21.6618**
db5	10	32.2213	32.1160	**32.5413**
	20	27.3056	27.3453	**27.4533**
	30	24.1389	24.1899	**24.2421**
	40	21.7767	21.8334	**21.8815**
sym5	10	32.2278	32.1261	**32.5343**
	20	27.3659	27.3649	**27.4553**
	30	24.1965	24.2003	**24.2470**
	40	21.8429	21.8419	**21.8812**
coif5	10	*32.3218*	*32.2203*	***32.8963***
	20	*27.3982*	*27.4143*	***27.5007***
	30	*24.2091*	*24.2285*	***24.2702***
	40	*21.8519*	*21.8608*	***21.9012***

Noisy Lena σ = 20

Bayes Shrink

Normal Shrink

Neigh Shrink

Fig.5. Results of various Image de-noising methods.

6. Conclusion

In this paper, the image de-noising using discrete wavelet transform is analyzed. The experiments were conducted to study the suitability of different wavelet bases and also the different methods of threshold or shrinkage. The result shows that the Neigh Shrink gives the better result than the Bayes and Normal Shrink in term of PSNR. However, when we talk about the processing time then, the Normal Shrink is faster than the remaining both. And, it also shows that among all wavelet bases, the coiflet wavelet gives the better result for image de-noising because it has maximum PSNR.

7. References

[1] J. N. Ellinas, T. Mandadelis, A. Tzortzis, L. Aslanoglou, “Image de-noising using wavelets”, T.E.I. of Piraeus Applied Research Review, vol. IX, no. 1, pp. 97-109, 2004.

[2] Lakhwinder Kaur and Savita Gupta and R.C.Chauhan, “Image denoising using wavelet thresholding”, ICVGIP, Proceeding of the Third Indian Conference On Computer Vision, Graphics & Image Processing, Ahmdabad, India Dec. 16-18, 2002.

[3] S.Kother Mohideen, Dr. S. Arumuga Perumal, Dr. M.Mohamed Sathik. “Image De-noising using Discrete Wavelet transform”, *IJCSNS International Journal of Computer Science and Network Security*, vol .8, no.1, January 2008.

[4] S. Grace Chang, Bin Yu and M. Vattereli, “Adaptive Wavelet Thresholding for Image Denoising and Compression”, *IEEE Trans. Image Processing*, vol. 9, pp. 1532-1546, Sept. 2000.

[5] G. Y. Chen and T. D. Bui, “Multi-wavelet De-noising using Neighboring Coefficients,” *IEEE Signal Processing Letters*, vol.10, no.7, pp.211-214, 2003.

[6] D. L. Donoho and I. M. Johnstone, “Ideal spatial adaptation via wavelet shrinkage,” *Biomefrika,* vol. 81, pp. 425455, 1994.

[7] D. L. Donoho, “Denoising by soft-thresholding,” *IEEE Transactions on Information Theory,* vol. 41, pp.613-627, 1995.