Order Reduction of Linear Dynamic System using Matlab Programming Method

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ABSTRACT
This paper presents an algorithm for model order reduction of linear dynamic systems using the in MATLAB programming method. The denominator and the numerator coefficients of the reduced order model is obtained by the using pole-zero relationship between given higher order model and the mentioned lower order model. This proposed method is implemented in MATLAB m-file; it retains the original characteristics of the higher order model. It is shown that the proposed method has several advantages like: The reduction procedure is simple compared to other conventional techniques and the error is also minimized. The proposed algorithm has also been used for the order reduction of linear multivariable systems.

Keywords: Higher order model-Model order reduction-MATLAB-steady state value.

1. INTRODUCTION
The approximation of high order systems by low order models is one of the important problems in system theory. The use of a reduced order model makes it easier to implement analysis, simulations and control system designs. Numerous methods are available in the literature for order-reduction of linear continuous systems in time domain as well as in frequency domain [1]-[6]. Each of these methods has both advantages and disadvantages when tried on a particular system. In spite of several methods available, no approach always gives the best results for all systems. The numerical technique uses algebraic method is one of the most basic techniques among the various model order reduction methods available in the literature.

This method preserves stability in the reduced model, if the original high-order system is stable, and retains the first two time-moments of the system, thus it ensures the matching for impulse, step and ramp inputs between the original high-order and reduced order systems. The proposed environment for order reduction is carried out in simple and efficient manner. Further, numerous methods of order reduction are also available in the literature [12]-[15], which are based on the minimization of the integral square error (ISE) criterion.

2. STATEMENT OF THE PROBLEM
Given an original system of order ‘ n ’ that is described by the transfer function $G(s)$ and its reduced model $R(s)$ of order ‘ r ’ be represented as,

$$G(s)=\frac{N(S)}{D(S)} = \frac{\sum_{i=0}^{n-1} A_i s^i}{\sum_{i=0}^{n} a_i s^i} \qquad (1)$$

Where, *N(s)* is the numerator polynomial and *D(s)* is the common denominator polynomial of the higher order system. Also, A_i and a_i are the constant matrices of numerator and denominator polynomial respectively.

Irrespective of the form represented in equation (1) of the original system G*(s)*, the problem is to find a m^{th} lower order model $R^m(s)$ where *m<n* in the following form represented by equation (2), such that the reduced model retains the important characteristics of the original system and approximates its response as closely as possible for the same type of inputs with minimum integral square error.

$$R^m(S) = \frac{N^m(S)}{D^m(S)} = \frac{\sum_{i=0}^{n-1} B_i s^i}{\sum_{i=0}^{n} b_i s^i} \qquad (2)$$

Where, $N^m(S)$and $D^m(S)$ are the numerator matrix polynomial and common denominator of the reduced order model respectively. Also, *Bi* and *bi* are the constant matrices of numerator and denominator polynomial of the same order respectively.

Mathematically, the integral square error [7] can be expressed as,

$$E=\sum_{t=0}^{\tau}(Y_t - y_t)^2 \qquad (3)$$

Where,
Yt is the unit step time response of the given higher order system at the t^{th} instant in the time interval $0\leq t \leq\tau$, where τ is to be chosen and y_t is the unit step time response of the lower order system at the t^{th} time instant. The objective is to model a system $R^m(s)$, which closely approximates *G(s)* for a specified set of inputs.

3. DESCRIPTION OF PROPOSED METHOD
Let the transfer function of the original high-order system (HOS) of order 'n' be

$$G_n(s) = \frac{N(S)}{D(S)} = \frac{a_n+a_1s+a_2s^2+\cdots+a_{n-1}s^{n-1}}{b_n+b_1s+b_2s^2+\cdots b_{n-1}s^{n-1}+s^n} \qquad (4)$$

and let the same of low-order system (LOS) of order 'r' to be synthesized is :

$$G_r(s) = \frac{N(S)}{D(S)} = \frac{d_n+d_1s+d_2s^2+\cdots+d_{n-1}s^{r-1}}{c_n+c_1s+c_2s^2+\cdots c_{n-1}s^{r-1}+s^r} \qquad (5)$$

Further, the method consists of following steps:

3.1. Steps for order reduction using MATLAB programming:

Step 1: From the given higher order system the transient gain and steady state gain are to be determined.

Step 2: By using pole-zero relationship in equation (6) the unknown parameters a_0 and b_0 are determined.

$$\alpha_0 = \frac{sum\ of\ poles \pm sum\ of\ zeros}{number\ of\ poles\ \pm number\ of\ zeros} \quad (6)$$

Step 3: The other unknown values in the reduced order model are obtained using the proposed method as shown in the Figure 1.

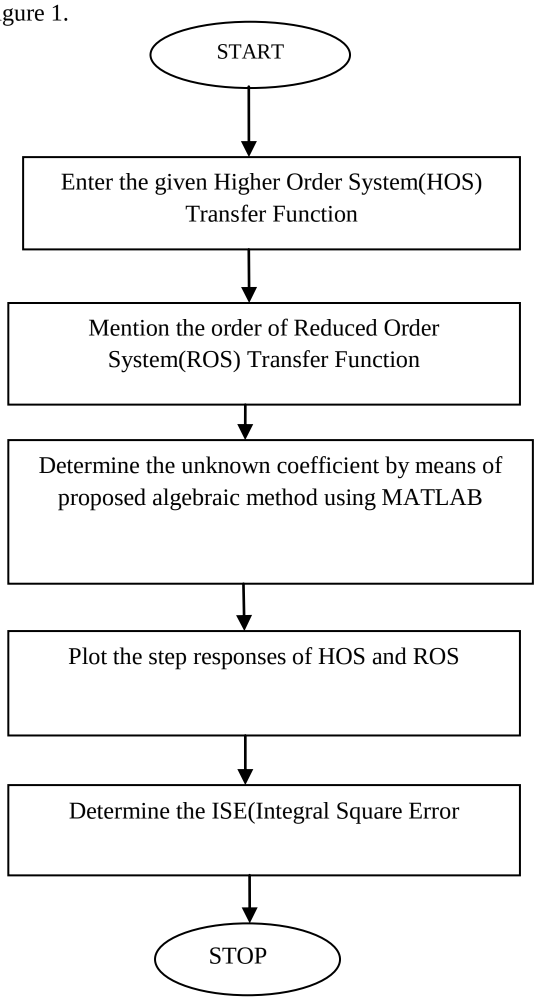

Figure 1. Flowchart for order reduction using MATLAB programming:

Step 4:The obtained reduced order models using proposed method are compared with that of other technique, and the error analysis is made.

4. NUMERICAL EXAMPLE

Consider a Fourth-order system described by the transfer function as ,[19]

$$G_4(s) = \frac{s^3 + 7s^2 + 24s + 24}{s^4 + 10s^3 + 35s^2 + 50s + 24} \quad (7)$$

If a second-order reduced model considered l $G_2(S)$ as,

$$G_2(S) = \frac{a_1 s + a_0}{b_2 s^2 + b_1 s + b_0} \quad (8)$$

Using the proposed technique, the unknown parameters of reduced order system are determined. The above steps are obtained by using MATLAB programming, (i.e) programmed using m-file in MATLAB, and the corresponding five lower order models are obtained for the given higher order model, and it is given in the Table.1,and the graphical results are shown in the Figure 2.

Table 1 Reduced Order Model and their Error analysis

S.no	Reduced Order Model Transfer Function	Error
1	$\frac{s + 28.25}{s^2 + 31.6s + 28.5}$	0.0009
2	$\frac{s - 17}{s^2 - 17.42s - 17}$	3.0810
3	$\frac{s + 2.429}{s^2 + 3.631s + 2.429}$	0.0014
4	$\frac{s + 3}{s^2 + 4.25s + 3}$	3.4549
5	$\frac{s + 28.25}{s^2 + 31.6s + 28.5}$	0.0012

The step responses of the higher order model and its reduced order model is shown in the below graph

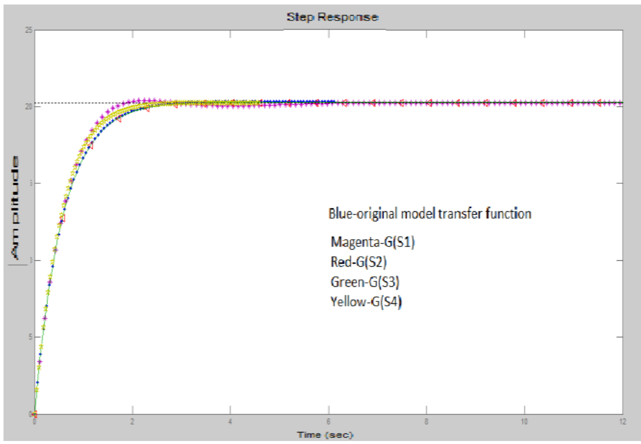

Figure2.step responses for fourth order system and reduced order model

the proposed order reduction method and various other methods are compared and error analysis is done in the below Table.2

Table 2 gives the comparison of different methods with that of proposed method.

Methods of model order reduction	**ISE**
Proposed method	0.0009
Parma and Prasad [5]	0.0016
Pal[6]	1.534272
Parthasarathy and jayasimha[7]	34.014055×10^{-3}
Davison [9]	220.237945×10^{-3}
Shiehand wei[10]	142.56072×10^{-3}
Chen[11]	2.665530×10^{-3}

5. CONCLUSIONS

The proposed model reduction method is used to drive stable reduced order models for linear dynamic systems. The algorithm has also been extended to the order reduction for and discrete systems. This MATLAB programming method is simple, rugged and computer oriented. The matching of step response is assured reasonably well in this method. The proposed method preserves more stability and minimizes the error between the initial or final values of the responses of original and reduced order models.

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