

# Performance of OFDM SYSTEM under High Doppler Spread

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## ABSTRACT

It is necessary to consider fast varying channel for Orthogonal Frequency Division Multiplexing (OFDM) system when OFDM symbols are transmitted over a frequency selective time varying fading channel. The time variations of channel during one OFDM symbol destroy the orthogonality of different subcarriers and result Inter Carrier Interference (ICI), which results in degradation of system performance. To solve this problem, the OFDM symbols are designed in such a way so that the performance degradation caused by ICI can be avoided. A simple and efficient technique is proposed to design the OFDM symbols based on eigen values of time varying channel. This method is described as Channel Matched OFDM (CMOFDM). Performance comparison between conventional OFDM and proposed matched filter based CMOFDM receiver structure under the same channel conditions are provided. The system performance is measured in terms of BER. The results show that the new technique is better over the conventional OFDM receiver in suppressing ICI in OFDM system.

## General Terms

Performance, Design, Theory.

## Keywords

OFDM, CMOFDM, Eigen Values, Inter-Carrier-Interference (ICI), Time varying Channel, Frequency Selective Fading Channel.

## 1. INTRODUCTION

The dispersive characteristic of wireless channels leads to Inter Symbol Interference (ISI), where the energy of a given symbol spills over into the observation interval of adjacent symbols at the receiver. The traditional approach to combat ISI is to use equalization and coding and recently multicarrier modulation is being used, where signal design and optimization of transmitted power are two major areas. In related work, channel is assumed to be stationary over a bit duration or symbol duration. However there is need to consider fast varying channels for OFDM transmission, as when OFDM symbols are transmitted over time varying frequency selective channel with high Doppler spread, the Inter Carrier Interference (ICI) occurs. Due to ICI the symbols do not remain orthogonal. Various equalization and cancellation

techniques have been devised like self cancellation, ML detection, Kalman filtering etc. at the receiver as in [1], [2], [3] and [4]. In [5], block minimum mean-squared error equalizer is proposed over time-varying multipath channels, where the band structure of the frequency-domain channel matrix is exploited. In [6], ICI suppression is presented by a parallel canceling scheme via frequency-domain equalization techniques, with the assumption that the channel impulse response varies linearly during a block period. In [7], MMSE based block decision feedback equalizer based on basis expansion model is proposed. [8] Proposes a new pulse shape to reduce the average ICI power of OFDM systems. And by simulation results it is shown that the proposed pulse is superior to the conventional pulse shapes in terms of ICI power reduction, SIR and BER performance. In [9], two methods are investigated to combat the effects of ICI: the extended Kalman filter (EKF) method and a form of the sequential Monte Carlo (SMC) method called sequential importance sampling (SIS) and through simulations the performance improvement of the OFDM modulation scheme is shown. In [10], the weighting coefficients are designed so that the ICI caused by the channel frequency errors can be minimized. More recently in [11], a multistep windowing is proposed to reduce the ICI and complexity involved in ICI reconstruction and cancellation. [12] Presents an ICI cancellation scheme, which estimates and compensates the Carrier Frequency Offset (CFO) first and then adopts ICI self-cancellation to suppress the residual CFO. All these work do not consider the variation of channel within a symbol.

Since the wireless channel is changing continuously, it is necessary to change the symbols according to channel conditions. Thus channel information is needed every time and must be estimated at every instant. However, it will require large training information. Thus to limit it, the receiver has to make the assumption on maximum variation speed of the channel coefficients ie, Doppler spread. To combat with the problem, the objective is to design the symbol such that ICI can be avoided. Transformation of time varying parameters into time variant coefficient over an observation interval by expanding the parameter waveform into a set of basis function has been introduced in early 80s for nonstationary ARMA model and reintroduced in 1996 [13]. Guilloed and Slock proposed in [14] to represent temporal channel variations suited for CPOFDM system. This channel model is used to design the signature sequences for each of OFDM symbol. The rest of the paper is

organized as follows: In Section 2.1 the channel model is briefly reviewed. Section 2.2 describes OFDM system model. In section 2.3 a scheme based on eigen value of channel matrix is proposed. In Section 2.4, an example is presented to illustrate the concept of symbol designing. Simulation environments and results for the proposed scheme are presented in Section 3 and conclusions are drawn in section 4.

## 2. CHANNEL MODEL

Consider the transmission of a complex signal over a fast fading channel. The signal is defined in discrete time by the complex values  $m_k$ ,  $k = -L \dots L$  and is transmitted at sample rate  $T_s$ .

### 2.1 Continuous Time Channel Model

Consider a multipath, time varying complex channel  $h(t, \tau)$  where  $t$  is the time and  $\tau$  is the lag. Hence  $h(t, \tau)$  is the channel impulse response as seen by the signal received at time  $t$ . For the sake of simplicity, it is assumed that channel starts at time zero and finite delay spread  $\tau_{max}$ . Thus the received signal  $r(t)$  is represented by

$$r(t) = \sum_{k=-\infty}^{k=\infty} m_k h(t, t - kT_s) + n(t)$$

### 2.2 Discrete time channel model

The received signal  $r(t)$  is converted into discrete form at sampling rate  $T_s$ , Accordingly

$$r = r(t_0 + pT_s) \quad 0 \leq t_0 \leq T_s$$

Also consider the discretization of  $h$  along both temporal dimensions (time  $t$  and lag  $\tau$  with the same sampling rate  $T_s$  and epoch  $t_0$ )

$$h_{pl} = h(t_0 + pT_s, t_0 + lT_s)$$

for integer  $p$  and  $l$ . Let  $L = \tau_{max}/T_s$  be the number of non zero coefficients in the discretized impulse response. Hence the received signal  $r$  can be written as

$$r_p = \sum_{k=-\infty}^{\infty} m_k h(t_0 + pT_s, t_0 + pT_s - kT_s) + n(t_0 + pT_s)$$

And let  $l = p - k$

$$r_p = \sum_{k=-\infty}^{\infty} m_k h_{p, p-k} + np$$

$$r_p = \sum_{l=0}^{L-1} m_{p-l} h_{p, l} + np$$

Due to finite coefficients in channel impulse response, the infinite terms are reduced to  $L$  terms.

## 3. SYSTEM MODEL

In basic OFDM system, the information bits are coded and interleaved. The coded bits are then mapped to data symbols depending upon the modulation types. Another stage of interleaving and coding can be performed for the modulated symbols. Although the symbols are in the time domain, the data up to this point is considered in the frequency domain. The serial data symbols are converted to parallel blocks and an IFFT is applied to these parallel blocks to obtain the time domain OFDM symbols. For OFDM symbol  $n$ , consider the  $N$  frequency domain constellation symbols

$$C_n = [c_{n0} \ c_{n1} \ \dots \ c_{nN-1}]$$

Where,  $c_{n1}$  is the first frequency domain constellation in  $n^{\text{th}}$  symbol. The time domain equivalent ( $m_n$ ) is obtained via  $N$  point inverse DFT( $F^{-1}$ ) operation.

$$m_n = F^{-1} C_n$$

In order to avoid interference between consecutive OFDM symbols, we assume that a cyclic prefix of length  $P$  ( $P \geq L - 1$ ) is used ie the last  $P$  samples of  $m_n$  are prepended in the time domain to the OFDM symbol itself before transmission. The cycle prefix insertion is represented by  $C$ .

$$C = \begin{bmatrix} 0_{P \times N-P} & I_P \\ & I_N \end{bmatrix}$$

It yields the transmitted signal

$$m'_n = [m_{n(0+p)-p} \ \dots \ m_{n(N+p)+N-1}]$$

$$= C m_n$$

And the corresponding received signal is given as

$$r'_n = [r_{n(0+p)-p} \ r_{n(1+p)-p} \ \dots \ r_{n(N+p)+N-1}]$$

In order to simplify the notation, Let us consider the OFDM symbol  $n = 0$  and denote  $m' = m_n$  and  $c = c_0$ . The multipath channel is represented by

$$H = \begin{bmatrix} h_{p,0} & 0 & \dots & 0 \\ \vdots & h_{1-p,0} & \ddots & \vdots \\ h_{L-1-p,L-1} & \vdots & \ddots & 0 \\ 0 & h_{L-p,L-1} & \ddots & \\ \vdots & \ddots & \ddots & \vdots \\ \vdots & \ddots & & \\ 0 & \dots & 0 & h_{N+L-2,L-1} \end{bmatrix}$$

Which yields

$$r' = H' m' + n$$

Then, the cyclic prefix removal operation consists in discarding the first P values of the received signal, which in general contains interference from the previous OFDM symbol. The last L-1 samples in  $r'$  are themselves interfering with the following OFDM symbol, and should be ignored as well. Therefore, in the sequel, we will only consider

$$r = Dr'$$

Where

$$D = [0_{N+P} \ I_N \ 0_{N \times L-1}]$$

Finally, the frequency-domain equivalent of  $r$  is obtained by DFT

$$u = Fr = FDH'CF^{-1}c + w$$

In particular, the effect of pre- and post-multiplication by C and D respectively, is to create an equivalent square channel matrix with the following structure

$$H = \begin{bmatrix} h_{0,0} & & h_{L-1,L-1} & \dots & h_{0,1} \\ \vdots & h_{1,0} & \ddots & & \vdots \\ \vdots & & & & h_{L-1,L-1} \\ h_{L-1,L-1} & \vdots & & & \\ & h_{L,L-1} & \ddots & & \\ & & \ddots & & h_{N-1,0} \end{bmatrix}$$

In a quasi-static environment (defined by  $NB_D T_s \ll 1$ ), where channel is assumed to be constant over an OFDM symbol and CP is larger than channel impulse response length L then  $h_{p,l}$  is same for all p's, thus making H as a circulant matrix and the frequency domain expression of channel  $FHF^{-1}$  results in a diagonal matrix and the diagonal values represent the channel gain on each frequency subband, since no cross term between subcarriers is present, that is, no ICI occurs and in this case and equalization is a trivial operation. However when the channel varies over an OFDM symbol the ICI occurs, and for the equalization of the channel at each time sample of OFDM symbol is needed, that is, at each p samples. For the frequency domain estimation the requirement translates in to the knowledge of the channel coefficients at each carrier frequency as well as their cross terms. The number of unknowns in time domain estimations are NL, whereas the no. of unknowns in the frequency domain (The entries of  $FHF^{-1}$  are  $N^2$ ). In either case, the number of unknowns will be higher than the number of equations and hence a system of underdetermined equations will result in. In the work it is considered that these channel coefficients are perfectly known.

## 4. PROPOSED METHOD

Consider the case, where the channel co-efficients vary in time during one OFDM symbol. In this case H is not circulant matrix, therefore to avoid ICI, signal is processed at the transmitter, instead of receiver in the form of equalization. The existing

system is modified by assigning the signatures to each subcarrier and these signatures are the Eigen vectors of the channel matrix H, which is time varying channel. The modified system is shown in Fig.1. These signatures are assigned according to channel variations to each subcarrier. After the addition of CP, which is larger than the expected maximum excess delay of the channel and D/A conversion, the signal is sent through the time varying wireless channel. Due to rapidly time varying multipath fading channels some of the subcarriers are severely degraded.

### 4.1 Eigen Vectors as Signature sequences

On Eigen value decomposition of channel matrix H,

$$A = H^*H = V \Lambda V^*$$

Where, \* represents complex conjugate operation, V is a unitary matrix and

$$A = \text{diag} [\lambda_1 \ \lambda_2 \ \lambda_3 \ \dots \ \lambda_n]$$

with the eigen values arranged in descending order at the diagonal of  $\Lambda$ . Let  $s_n$  be the Eigen vector corresponding to the Eigen values  $\lambda_n$  and if

$$r_n = Hs_n c \quad n = 1, 2, \dots, N$$

Then

$$\begin{aligned} r_n^* r_m &= s_n^* H^* H s_m \\ &= s_n^* A s_m \\ &= s_n^* \lambda_m s_m \\ &= \lambda_n, \quad n = m \\ &= 0 \quad n \neq m \end{aligned}$$

Thus, orthogonality among the received subcarriers is maintained when unit energy eigen vectors of A are used as signature sequences. At the receiver the signal is received with noise and interference. After perfect synchronization, down sampling and the removal of CP, the signal is passed through matched filter structure for each subcarrier, where each subcarrier is multiplied by its signature, which was assigned at the transmitter. Since the signatures are the eigen vectors of channel matrix H, so at the output only data bits corresponding to subcarriers can be detected perfectly. These data bits are passed through FFT block to get desired data bits. Once the data bits are detected, they are demodulated, deinterleaved and decoded.

### 4.2 Illustration

The complete system can be explained with following example. let us consider,  $N = 3, L = 2, P = 2$  then for OFDM symbol  $n$ , consider the N frequency domain constellation symbols

$$c_n = [c_1 \ c_2 \ c_3]^T$$

The time domain equivalent is obtained via N point inverse DFT

$$m_n = F^{-1}c_n = [m_1 \ m_2 \ m_3]^T$$

Multiply the symbol by a signature matrix

$$S = [s_1 \ s_2 \ s_3]$$

each signature is a column vector ( $N \times 1$ ), which is eigen vector of channel matrix H. We get

$$s' = Sm_n = [s'_1 \ s'_2 \ s'_3]^T$$

Addition of cyclic prefix C gives

$$m'_n = Cs' = [s'_3 \ s'_1 \ s'_2 \ s'_3]^T$$

This transmitted signal is passed through time varying channel, H  
 And the received signal is given as

$$r'_n = H'm'_n = \begin{bmatrix} h_{-1,0} s'_3 \\ h_{0,1} s'_3 + h_{0,0} s'_1 \\ h_{1,1} s'_1 + h_{1,0} s'_2 \\ h_{2,1} s'_2 + h_{2,0} s'_3 \\ h_{3,1} s'_3 \end{bmatrix} + n$$

Now after discard we get

$$r_n = \begin{bmatrix} h_{0,1} s'_3 + h_{0,0} s'_1 \\ h_{1,1} s'_1 + h_{1,0} s'_2 \\ h_{2,1} s'_2 + h_{2,0} s'_3 \end{bmatrix} + n$$

The received signal is passed through matched filter receiver structure, where each subcarrier is multiplied with corresponding signature sequences

$$\begin{aligned} r_{1,n} &= \frac{s_1^T H s_1 m_1}{\lambda_1} + \frac{s_1^T H s_2 m_2}{\lambda_1} + \frac{s_1^T H s_3 m_3}{\lambda_1} + \frac{n s_1}{\lambda_1} \\ &= s_1^T s_1 m_1 + 0 + 0 + \frac{n s_1}{\lambda_1} \\ &= m_1 + n \end{aligned}$$

Thus  $m_1$ , can be detected, similarly others. After this take the FFT and we get the desired results ie  $c_1$ ,  $c_2$  and  $c_3$

## 5. PERFORMANCE VALIDATION AND SIMULATIONS

In this section, the verification on the performance improvement achieved by signature allocation scheme through computer simulation is presented. It is assumed that the channel variations are known at the receiver in advance.

To manifest the performance improvement of the proposed scheme the following schemes are used for comparison.

- OFDM signal is passed through time varying channel and decision feedback equalizer is used at the receiver.
- OFDM signal is assigned signatures before it passes through time varying channel and matched filter receiver structure is used(CMOFDM method) mentioned above.

It is observed that BER is higher in adaptive equalizer with decision feedback algorithm. (See, Figure 2). It is because this equalizer does not take in to account of fast varying channel. In this initially with the help of pilot carriers, the channel is estimated and assumed to be constant. Under the same channel conditions next symbols are demodulated and detected, thus performance is degraded due to ICI.

However the proposed scheme CMOFDM (with sequences) has performance better than the traditional equalizer based methods because the scheme copes with the time varying nature of channel over the symbol duration. In each symbol sequences are designed according to channel behavior. Bit Error rate plots are shown in Figure 2, 3 and 4 for L= 2 & 4 multi-paths, FFT size N=16 and 32 and Channel variations =2 & 4 within a symbol.

## 6. CONCLUSION

We have proposed a scheme to compensate ICI caused by time varying frequency selective fading channel with high Doppler spread. It is shown that allocation of signature sequences to each subcarrier improves the BER in comparison to equalizers. Thus we have shifted the complexity of the receiver to the Base station. Here we have assumed that the channel variations are known at the receiver, which is not possible in practical scenario. Thus we need to estimate these channel coefficients exactly. It is observed that in proposed scheme the Bit Error Rates are lower than equalizer based OFDM system.

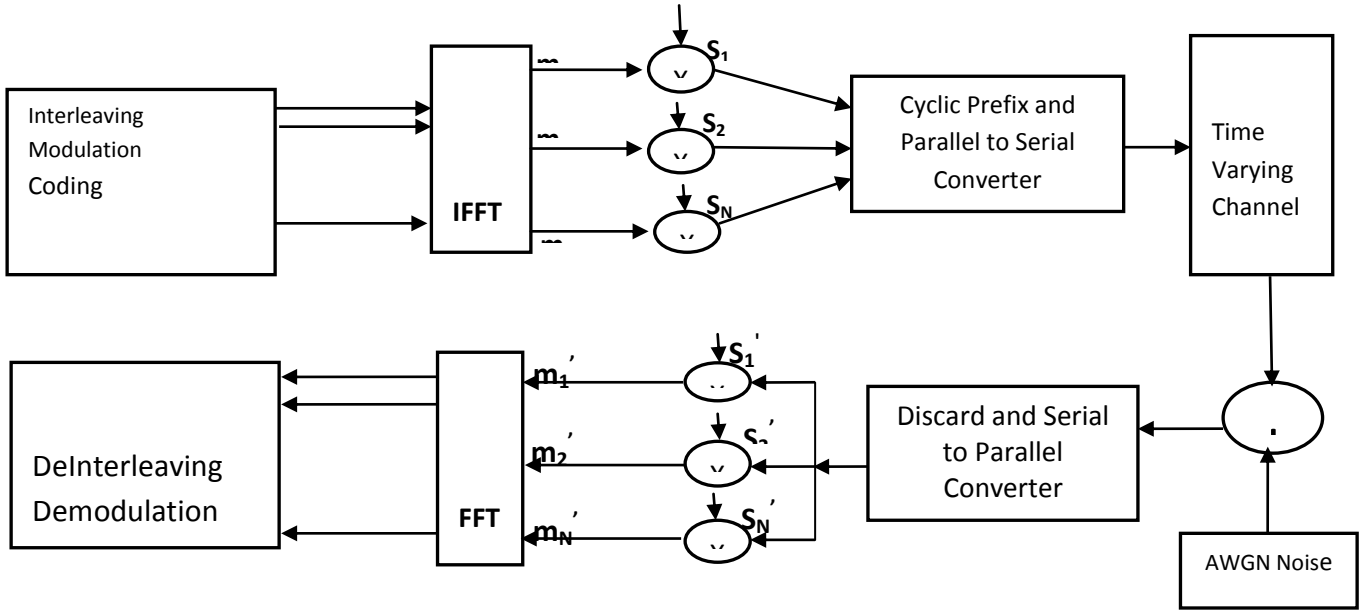


Figure.1: Modified OFDM System (Channel Matched OFDM System)

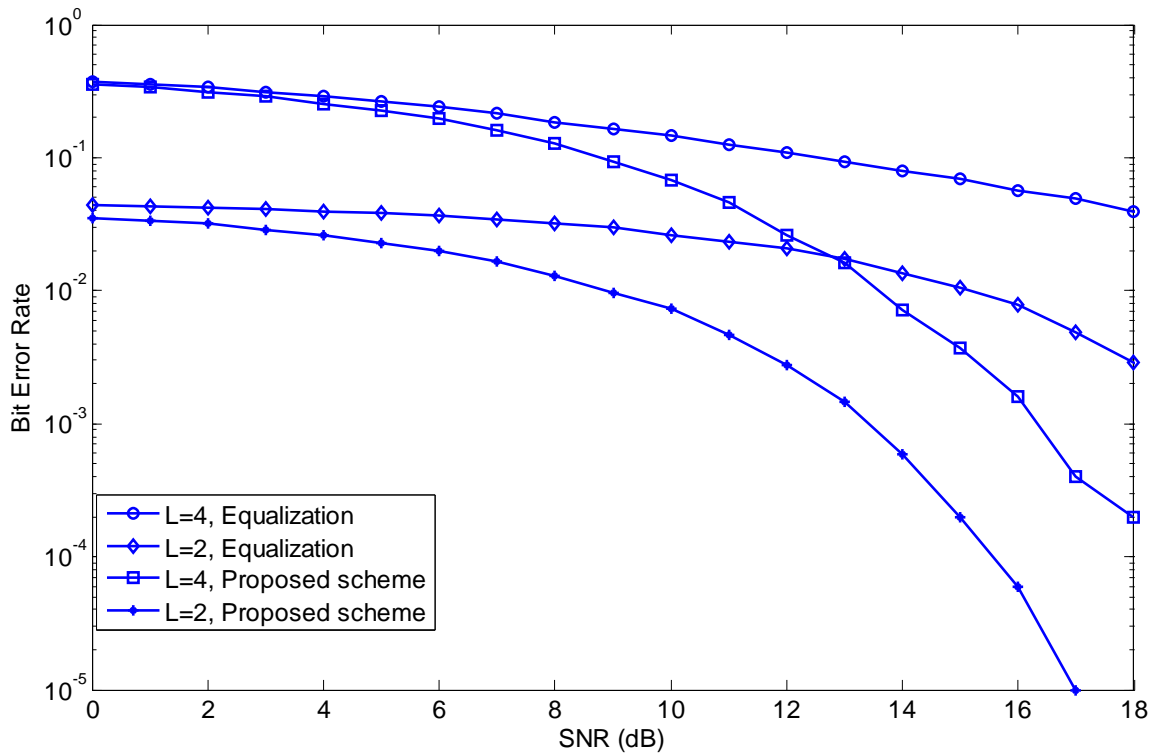


Figure 2: BER vs SNR for multi-paths L= 2 and 4, No. of channel variations=2 and FFT Size = 16

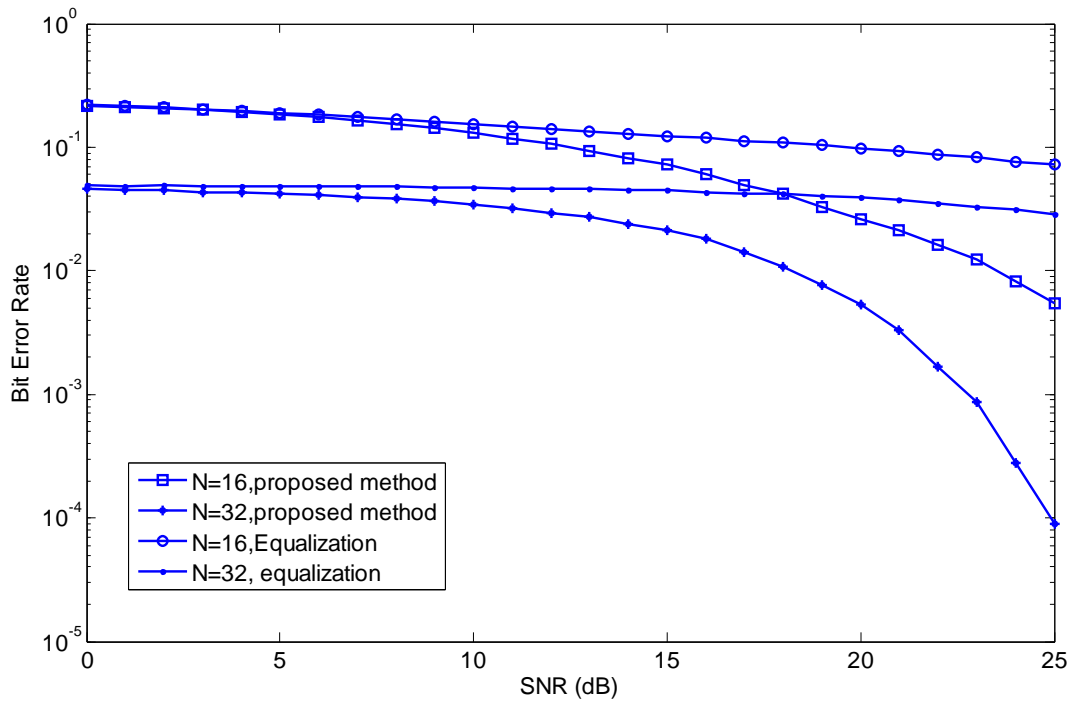


Figure 3: BER vs SNR for FFT sizes N= 16 and 32, No. of channel variations=4 and L=4

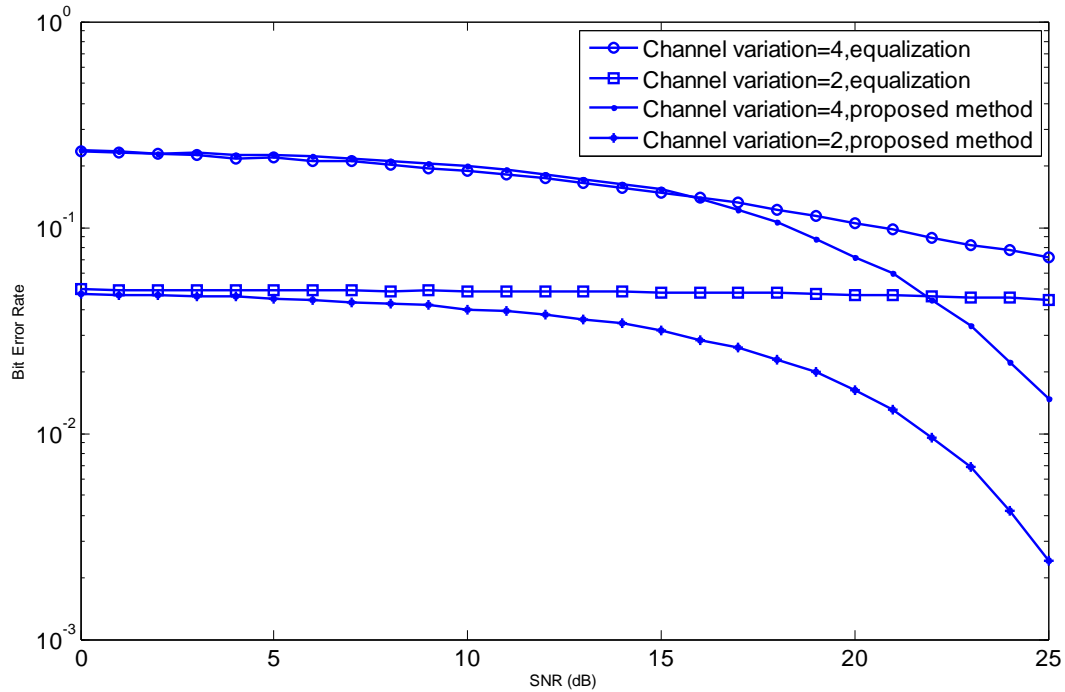


Figure 4: BER vs SNR for FFT No. of channel variations = 2 and 4, FFT Size N=32 and L=4

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