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Analysis of PAPR of Real and Complex OFDM systems

Saumendra Ku. Mohanty, ITER, Siksha 'O' Anusandhan University
Shubhendu Ku. Sarangi, ITER, Siksha 'O' Anusandhan University.
Mihir N. Mohanty, ITER, Siksha 'O' Anusandhan University
Laxmi.P.Mishra, ITER, Siksha 'O' Anusandhan University

ABSTRACT

For wireless applications, Multicarrier transmission, also known as orthogonal frequency division multiplexing (OFDM) or discrete multitone (DMT), based system can be of huge interest because it provides greater immunity to multipath fading and impulse noise, and eliminates the need for equalizers, while efficient hardware implementation can be realized using fast Fourier transform (FFT) techniques with high-speed wireless communications and recent advances in digital signal processing technology

In this paper, two aims will be studied. First, it introduces a practical technique for evaluating the continuous-time PAPR of OFDM signals using complex modulation is presented. Using the proposed scheme, it is observed that the TWO-times or more over sampled discrete-time PAPR is a good approximation of the continuous-time PAPR even for complex OFDM signals. Second, it introduces a conventional OFDM systems with the limitation of their behavior with peak-to-average-power ratio (PAPR). Computing the continuous-time PAPR of OFDM signals is computationally challenging. It is shown that the instantaneous envelope power function (EPF) can be transformed into a linear sum of Chebyshev polynomials. Consequently, the roots of the derivative of EPF can be obtained by solving a polynomial. The pioneering work of calculating PAPR of single carrier FDMA, multi-carrier BPSK-OFDM(Real-Valued Modulation) and multi-carrier QPSK-OFDM(Complex Modulation) is achieved.

Keywords

OFDM, DMT, peak-to-average-power ratio, EPF, multicarrier modulation, SC-FDMA, MC-BPSK, MC-QPSK

1. INTRODUCTION

One of the major challenges of Orthogonal Frequency Division Multiplexing (OFDM) is that the output signal may have a potentially very large peak-to-average power ratio (PAPR, also known as PAR). The resulting technical challenges, as well as PAPR-reduction techniques and related issues, have been widely studied and reported in the research literature [1], [2].

The most widely PAPR reduction techniques known are based on amplitude clipping or on some forms of coding [2]. However, comparative analysis of those methods could be a complex task, because the effects of those methods are usually analyzed using simulations or by simple case study, and no general analytical framework for such analysis exists. In this work, we try to characterize analytically the statistics of the PAPR problem in OFDM by considering the probability that the PAPR of an OFDM

Since the actual signal that enters the power amplifiers is a continuous-time signal, we ultimately want to reduce the PAPR of the continuous-time OFDM signal (we call this the "continuous-time PAPR" for convenience). However, the evaluation of the continuous-time PAPR is analytically non-trivial and computationally expensive. Therefore, most PAPR-reduction techniques focus on discrete-time approximations of the continuous-time PAPR. The discrete-time approximations result in what we call the "discrete-time PAPR".

The central limit theorem effectively decides the envelope of the OFDM signal and it is shown that, effectively, the PAPR grows as $2 \ln N$ and not linearly with N [7] ,where N is the total number of subcarriers .In [3], Tellambura investigated the differences between the continuous-time PAPR and discrete- time PAPR. To do this, Tellambura introduced a practical scheme to compute the continuous-time PAPR, using Chebyshev polynomials of the first kind. The scheme was then used to obtain numerical results. Based on these results, a common rule-of-thumb that has since emerged in the OFDM research community is that the discrete-time PAPR with four-time over sampling is a sufficiently accurate approximation of the continuous-timePAPR [2].

Unfortunately, Tellambura's method [3] applies only to real-valued modulation schemes like BPSK (and results were only presented for N=512 BPSK-OFDM, but not complex-valued schemes like QPSK. To circumvent this shortcoming, [4] extended Tellambura's method to complex modulation schemes, using Chebyshev polynomials of both the first and second kinds. However, neither [3] nor [4] present any analysis of the error from using the discrete-time PAPR instead of continuous- time PAPR. Thus, even though the empirical distribution of the continuous-time PAPR and the four-time oversampled discrete-time PAPR may look close, there is no guarantee that the error is bounded. Some analytical bounds have been provided in [5]–[6]. However, due to the lack of computationally feasible methods to obtain the continuous-time PAPR, [5]–[6] used the discrete-time PAPR to verify their continuous- time PAPR bounds.

In this paper, we introduce a computational method that is more general than Tellambura's [3], to find the peaks for OFDM signals with arbitrary complex-valued modulations. We express the instantaneous envelope power as a polynomial of powers of $\tan(\pi t)$. In contrast with [4], the proposed method only employs Chebyshev polynomials of the first kind. Also, because of the one-to-one relationship between $\tan(\pi t)$ and t in$0\leq t\leq 1$, the new method does not require breaking the problem into two domains ($0 \leq t\leq 0.5$and $0.5 \leq t\leq 1$) and carefully mapping the roots differently for each domain. Furthermore, comparisons are made between the distribution of the continuous-time PAPR obtained through the proposed method with the discrete-time PAPR obtained from over sampled signals and some of the analytical upper bounds derived in [5]–[6].

2. ANALYTICAL MODEL

The baseband continuous-time OFDM signal with N carriers can be expressed as

$$X(t) = \left(\frac{1}{\sqrt{N}}\right) \sum_{n=0}^{N-1} Sn \exp(j2\pi nt) \dots\dots\dots\dots (1)$$

where $\{s_n\}$ are data symbols and t is normalized with respect to the OFDM symbol duration.With unity average power, the continuous-time PAPR, γ_c, is defined as

$$\gamma_c = \max |x(t)|^2 \dots\dots\dots \quad (2)$$

γ_c measures the instantaneous envelope peak power of the baseband signal and represents the maximal PAPR. It is non-trivial to compute. Tellambura's method [3] works only for the special case of real-valued modulation.

As a computationally feasible alternative, the discrete-time PAPR, γ_d, is often used instead of γ_c and defined as

$$\gamma_d = \max_{0 \le k \le LN-1} |C_k|^2 \dots\dots (3)$$

where

$$C_k = \left(\frac{1}{\sqrt{N}}\right) \sum_{n=0}^{N-1} S_n\, e^{j2\pi nk/LN} \dots\dots\dots (4)$$

with L being the over sampling rate.

Let $P_a(t)=|x(t)|^2$.Without loss of generality, no assumptions are made on the modulation scheme used to generate$\{Sn\}$. It can be easily shown that,

$$P_a(t) = 1 + \frac{2}{N}\sum_{k=1}^{N-1}[\beta_k \cos(2\pi kt) + \alpha_k \sin(2\pi kt)] \dots\dots (5)$$

where β_k and α_k are defined as follows

$$\beta_k = R\left\{\sum_{m=0}^{N-1-k} S_m S_{m+k}^*\right\}, k = 1,2,\dots\dots, N-1, \dots\dots (6)$$

and

$$\alpha_k = I\left\{\sum_{m=0}^{N-1-k} S_m S_{m+k}^*\right\}, k = 1,2,\dots\dots, N-1, \dots\dots (7)$$

with $(\cdot)^*$denoting complex conjugation and $R\{\cdot\}$and $I\{\cdot\}$ being the real and imaginary part of the enclosed quantity, respectively.

Clearly, a necessary condition for $P_a(t)$ to achieve its maximum at t^*,i.e $\max_t P_a(t)=P_a(t^*)$, is

$$\frac{\partial P_a(t)}{\partial t} = 0 \text{ at } t = t^*$$

Thus, a practical approach to computing $P_a(t^*)$ is to first find the roots of $\partial P_a(t)/\partial t$ followed by comparing the values of $P_a(t)$ at only the real roots. Following the approach of [2], we denote by $T_k(t)=\cos(k \cos^{-1} t)$ the kth-order Chebyshev polynomial. For each k, $T_k(x)$ can be expressed as a kth-degree polynomial in terms of x, where $T_0(x)=1, T_1(x)=x$ and$T_{k+1}(x)=2xT_k(x)-T_{k-1}(x)$ for $k>1$. Exploiting the equalities $T_k(\cos\theta) = \cos k\theta$ and $\sin(\theta) = \cos\left(\frac{\pi}{2}-\theta\right)$, we can rewrite (5) in terms of Chebyshev polynomials as

$$P_a(t) = 1 + \frac{2}{N}\sum_{k=1}^{N-1}\beta_k T_k(\cos(2\pi t)) + \frac{2}{N}\sum_{k=1}^{N-1}\alpha_k T_k\left(\cos\left(2\pi t - \frac{\pi}{2k}\right)\right) \dots\dots\dots\dots\dots (8)$$

Being different from the BPSK-OFDM systems considered in [3], the complex OFDM signal introduces the second term on the right hand side (R.H.S.) of (8), which presents a major challenge in obtaining exact γ_c values.

3. PROPOSED METHOD

All trigonometric functions of an angle θ may be expressed as rational expressions in terms of $t=\tan(\theta/2)$[8]. Let $x=\tan(\pi t)$.Substituting$(1-x^2)/(1+x^2)$ for $\cos(2\pi t)$ and $2x/(1+x^2)$ for $\sin(2\pi t)$, and letting $\gamma_k = \cos(\pi/2k)$ and $\zeta_k = \sin(\pi/2k)$, we have

$$P_a(x) = 1 + \frac{2}{N}\sum_{k=1}^{N-1}\beta_k T_k\left[\frac{1-x^2}{1+x^2}\right] + \frac{2}{N}\sum_{k=1}^{N-1}\alpha_k T_k\left[\gamma_k \frac{1-x^2}{1+x^2} + \delta k2x1+x2 \dots\dots\dots\dots (9)\right.$$

We need only to find the roots of $\partial P_a(x)/\partial x$, since $\partial P_a(t)/\partial t = \partial P_a(x)/\partial x(\pi \sec^2(\pi t))$. Because $T_k(x)$ is an order-k polynomial, the highest power of $1/(1+x^2)$ in(9) is $N-1$. Hence we can remove the denominator and thus obtain a polynomial $Q(x)$ by writing

$$Q(x) = (1+x^2)^N \frac{\partial P_a(x)}{\partial x} \dots\dots\dots\dots (10)$$

$Q(x)$ is a polynomial of degree at most $2N$ in x and all roots of $\partial P_a(x)/\partial x$ are also roots of $Q(x)$. Thus, $\partial P_a(x)/\partial x$ has at most $2N$ roots. $P_a(x)$ can be routinely computed from (9) by expanding the Chebyshev polynomials, factoring out $1/(1+x^2)^N$,and collecting terms. We may then evaluate the values of $P_a(x)$ at the real roots, and the maximum is γ_c.

4. NUMERICAL PROCEDURE SUMMARY

The proposed method for computing the continuous-time PAPR for a given symbol set $\{s_n\}$ and number of sub carriers N is summarized as follows.

1) Compute β_k and α_k for $k = 1, 2, \cdots, N-1$according to (6) and (7);

2) Compute $P_a(x)$ according to (9), expanding and collecting the coefficients of the different powers of x;

3) Find the derivative of $P_a(x)$;

4) Find the roots of $Q(x)$, and hence of $\partial P_a(x)/\partial x$ using standard polynomial root finding algorithms;

5) Keep only the real roots of $Q(x)$;

6) Evaluate and compare the values of $P_a(x)$ at the real roots, and obtain γ_c.

Each step is straightforwardly handled by common mathematical software like Mathematica or Matlab. In our experiments, we have found that step 2 (expanding and simplifying $P_a(x)$), while conceptually easy, may dominate the computation time, especially for large N. In particular, expanding and simplifying $T_k[\gamma_k(1-x^2)/(1+x^2) + \zeta_k(2x)/(1+x^2)]$ is a time consuming operation for large k. For a given N, pre-computing these terms helps to significantly reduce the computation time

5. RESULTS

In this section, we evaluate the proposed scheme using QPSK-OFDM system for N=512 with different sampling rates

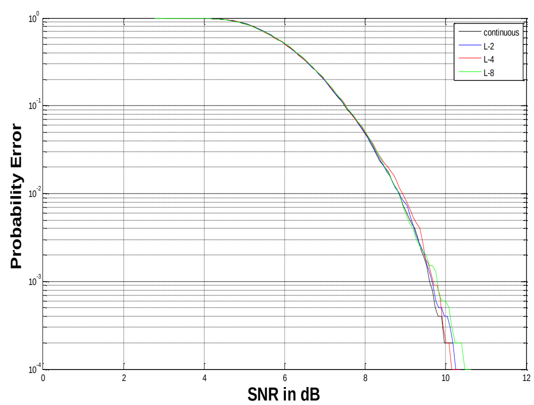

Fig1. Simulation of PAPR of QPSK-OFDM (for different Sampling rate)

Fig.1 shows the complementary cumulative distribution function (CCDF) of γ_d with different over sampling rates, $L = 2, 4, 8$. The CCDF of γ_c labeled as "continuous-time" is also plotted in Fig.1. As indicated in Fig.1, γ obtained from over sampled signals approaches γ_c as L increases, and γ_d obtained with a over sampling rate greater than or equal to $L = 2$ is an accurate approximation of γ_c. These result agrees to some extend with those reported in [3], where real-valued OFDM signals were considered.

In Fig.2, we evaluate the performance of PAPR of different real and complex modulation schemes used in OFDM systems

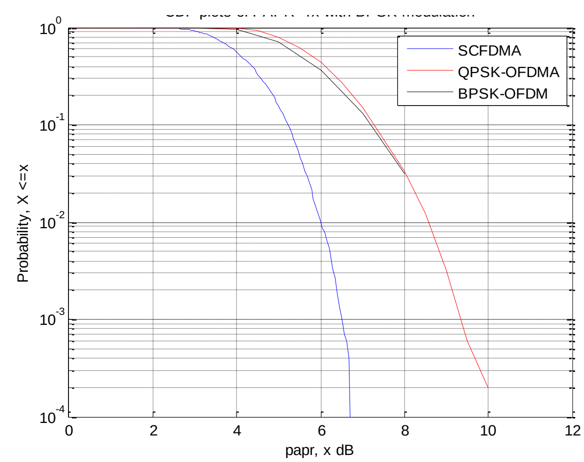

Fig2. Comparison Of PAPRs (with different modulation schemes)

Fig.2 shows that the transmitted SC-FDMA signal with a single carrier has the probability of errors is very less as it is a continuous-time real valued modulation scheme . In fact, for a PAPR of ~7dB, we get a probability of error ~0.0001, as shown in the plot.

For a transmitted BPSK-OFDM signal with multicarrier has the probability of errors is high for a slight increase in PAPR as it is a continuous-time real valued multicarrier modulation technique . In fact, for a PAPR of ~8dB, we get a probability of error ~0.01, as shown in the plot.

It is found from the transmitted QPSK-OFDM signal that for a multi carrier ,effect on the probability of errors is very low for a slight increase in PAPR as it is a discrete-time real valued multicarrier modulation technique . In fact, for a PAPR of ~10dB, we get a probability of error ~0.0001, as shown in the plot

6. CONCLUSION

Using the proposed scheme, we have shown(Fig.1) for complex-valued modulations (like QPSK-OFDM) that the discrete-time PAPR obtained from TWO-times or more oversampled signals may be considered a sufficiently accurate approximation of the continuous-time PAPR.

We have also used our scheme to examine the empirical plots (Fig.2), where we can conclude that , the discrete time PAPR of QPSK-OFDM has the less probability of errors even with the higher order nonlinearity in the system. This means that the signal is highly resistant to clipping distortions caused by the power amplifier used in transmitting the signal. It also means that the signal can be purposely clipped by up to ~2dB so that the probability of errors in both the cases (BPSK & QPSK) be reduced allowing an increased transmitted power

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