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Bearing only Tracking of Maneuvering Targets using a Single Coordinated Turn Model

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ABSTRACT
The passive tracking of manoeuvring objects using line of sight (LOS) angle measurements only is an important field of research in the application areas of submarine tracking, aircraft surveillance, autonomous robotics and mobile systems. In this paper, the tracking of target dynamics is treated as a system identification problem. We propose to use the coordinated turn (CT) model along with extended Kalman filter to track all possible dynamics such as velocity, acceleration and coordinated turn of manoeuvring targets. Simulations are used to demonstrate the effectiveness of this approach and the results obtained are promising.

Categories and Subject Descriptors
C.4 [Performance of Systems]: C.4.3 - Measurement Techniques, C.4.4 - Modeling Techniques.

C.3.4 [Special Purpose and Application Based Systems (J7)]: - Signal Processing Systems.

1.6.4 – [Simulation and Madeling (G.3)] – Model Validation and Analysis

General Terms
Algorithms,Measurement,Performance.

Keywords
bearings-only tracking, manoeuvring target tracking, extended Kalman filter.

1. INTRODUCTION
The problem of bearing only tracking arises in a variety of important practical applications in diverse fields. There is a problem in tracking of manoeuvring target that it might have abrupt change of its state by sudden operation of acceleration pedal , break or steering. The most commonly used scheme under these conditions is the interactive multiple model (IMM) described in [1,2,3,13]. They make assumption that at any time in the observation period, the target motion obeys one of the three dynamic models: (a) CV model (b) clockwise CT model and (c) anticlockwise CT model. One model corresponds the typical motion of the target, while the other models possible deviations from that standard model.

In any multiple model algorithm, it is important to know when the tartget is manoeuvring so that the algorithm can switch from one model to another. In this paper , the problem of using manoeuvre detection and switching to corresponding model is alleviated using EKF and treating the tracking as a system identification problem, thus leading to an adaptive target tracking.

The target is modelled by general state space representation that consists of system model for the dynamics and observation model [5,6,7] for the measurement process with non-linear formula. The system model used is a single CT model (no separate models fpr clockwise and anticlockwise CTs as in [1]). The state vector is augmented with the angular turn-rate (Ω) and the states are estimated recursively using EKF, with Ω as a parameter. Thus the target tracking ultimately boils down to a sort of parameter estimation problem. The error performance of the developed filter is analysed by Monte Carlo (MC) simulations and compared with the therotical Cramer-Rao lower bounds (CRLBs).

The paper is organised as follows. Section 2 deals with the formulations of problem The need and scope of Kalman filter is dealt with in section 3. The concept of Cramer Rao lower bound in evaluating the performance in any estimation problem, is described in section 4. Simulation and results are discussed in section 5. Finally conclusions are drawn in section 6.

2. PROBLEM STATEMENT
The basic problem in bearing only tracking is to estimate the trajectory of a target from

noise – corrupted data. In a single sensor problem , the bearing data is obtained from a moving observer. Consider a typical target – observer geometry depicted in figure 1. The target located at Coordinates (x^t,y^t) moves with a constant velocity vector (x^t,y^t).The target state vector is defined as

$$X^t = \begin{bmatrix} x^t & \dot{x}^t & y^t & \dot{y}^t \end{bmatrix}^T \quad (1)$$

where (x,y) and x,y) are position and velocity components. The own-ship state is similarly defined as

$$X^o = \begin{bmatrix} x^o & \dot{x}^o & y^o & \dot{y}^o \end{bmatrix}^T \quad (2)$$

The relative state vector is defined by

$$X_1 = X^t - X^o = \begin{bmatrix} x & \dot{x} & y & \dot{y} \end{bmatrix}^T \quad (3)$$

In practice , two most common form of target motion in Cartesian plane are 1) a target that moves with constant course and speed and 2) a constant speed target that undergoes a constant radius turn.

Model 1- Constant velocity model

In this case the target-observer motion can be modelled by general discrete time state equation

$$X_{1k+1} = F^1 X_{1k} + \Gamma^1 v_k \tag{4}$$

where

$$F_1 = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ and} \tag{5}$$

$$\Gamma^1 = \begin{bmatrix} \frac{T^2}{2} & 0 \\ T & 0 \\ 0 & \frac{T^2}{2} \\ 0 & T \end{bmatrix} \tag{6}$$

T is the sampling period and $v_k \approx N(0,Q)$ is the process noise vector. $Q = \sigma_a I_2$ where σ_a is a scalar and I_2 is a 2x2 identity matrix. The available measurement at time k is the angle from the observer's platform to the target and is given by

$$Z_k = h[X_k, k] + w_k \tag{7}$$

where w_k is a zero mean independent Gaussian noise with variance σ_θ^2 and

$$h[X_k, k] = \theta_k = \tan^{-1}\left(\frac{x_k}{y_k}\right) \tag{8}$$

is the bearing angle.

Given a sequence of measurements Z_k , k=1,2,… the bearing only tracking problem is to obtain estimate of the state vector X_k. The problem is non linear as the measurements are non-linearly related to the state vector. To track the target with angle only measurements, it has been found[1,6] that the observer must outmanoeuvre the target.

In state estimation, tracking a constant course and speed target can be done accurately. Tracking a moanoeuvring target is more difficult since manoeuvre must be detected and a filter reasonably matched to the motion must be used to gain accuracy.

Model 2 – Coordinated turn model

The dynamics of manoeuvring target is modelled by multiple switching regimes. In this case the assumption is that at any time in the observation period, the target motion obeys one of the 3dynamic model..1) CV model 2) Clockwise Coordinated turn model and 3) anti clockwise Coordinated turn model. Then the target dynamic model can be mathematically written as

$$X_{k+1} = f(X_k, X_k^0, X_{k+1}^0, r_{k+1}) + \Gamma_k v_k \tag{9}$$

$$= F_k X_k + \Gamma_k v_k \tag{10}$$

Γ_k and v_k are as defined in

for the CV motion F_k is as defined in

The transition matrix corresponding to coordinated turn model is given by

$$F^C = \begin{bmatrix} 1 & \frac{Sin\Omega T}{\Omega} & 0 & -\frac{1-Cos\Omega T}{\Omega} \\ 0 & Cos\Omega T & 0 & -Sin\Omega T \\ 0 & \frac{1-Cos\Omega T}{\Omega} & 1 & \frac{Sin\Omega T}{\Omega} \\ 0 & Sin\Omega T & 0 & Cos\Omega T \end{bmatrix} \tag{11}$$

where $\Omega = \dfrac{a_m}{\sqrt{(\dot{x}^t + x^o)^2 + (\dot{y}^t + y^o)^2}}$

a_m >0 is typical manoeuvre acceleration. Positive values of a_m correspond to clockwise CT and negative values correspond to anticlockwise CT.

The noise gain is

$$\Gamma^C = \begin{bmatrix} \frac{T^2}{2} & 0 \\ T & 0 \\ 0 & \frac{T^2}{2} \\ 0 & T \end{bmatrix} \tag{12}$$

The turning rate is expressed as a function of target speed (a non-linear function of state vector) and the model is non-linear one

3. THE EXTENDED KALMAN FILTER

The Kalman filter [10,11,12] was first proposed in 1960s and it is the most commonly used technique in target tracking ever since. The basic Kalman filter has shown to be a form of Bayesian filter that is an optimal estimator with respect to the variance of the estimation errors. For linear Gaussian systems the KF gives the minimum variance estimate of the state vector. Given a series of noisy measurements , the Kalman Filter is capable of estimating the state of the system.

For non-linear systems(in this case target tracking), the extended Kalman [12,13] filter has to be used. The EKF is a sophisticated algorithm that is suitable for estimation of the parameters in many complex situations [14,15]. To apply the EKF for simultaneous state and parameter estimation, an extension to the Kalman Filter is the Extended Kalman filter. This enables data such as bearing only passive sonar data to be used in the KF. Due to the linearization step, the EKF is sub-optimal.

The system state and output equations are of the following form

$$X(k+1) = f[X(k),k] + v(k) \quad (13)$$

$$z(k) = h[X(k),k] + w(k) \quad (14)$$

Where f and h are non-linear functions depending on the system state.
X(k) and z(k) are the state vector and output vector. v(k) and w(k) are the corresponding system and measurement noises respectively. These noises are white with zero mean and characterized by

$$E\{W(k)\} = 0, E\{W(k)W^T(k)\} = Q(k) \quad (15)$$

$$E\{V(k)\} = 0, E\{V(k)V^T(k)\} = R(k) \quad (16)$$

Initialisations

$$X(0,0) = E[X(0)] \quad (17)$$

$$P(0,0) = P(0) \quad (18)$$

Prediction equations

$$\hat{X}(k+1) = f[\hat{X}(k,k),\hat{\Omega}(k,k)] + W(k) \quad (19)$$

$$P(k+1,k) = F(k)P(k,k)F(k)^T + Q(k) \quad (20)$$

Where $\hat{X}$ is the state estimate and P is the estimation error covariance and

$$F(k) = \frac{\partial}{\partial X} f(X(k),\Omega(k))\Big|_{X(k)=\hat{X}(k,k)}$$

.Derivation of the Jacobian is given in annexure I.

Update equations

$$\hat{X}(k+1,k+1) = \hat{X}(k+1,k) + K(k+1)S \quad (21)$$

$$S = [Y(k+1) - H(k+1)X(k+1,k)] \quad (22)$$

Kalman filter gain

$$K(k+1) = P(k+1)H^T(k+1)\Delta \quad (23)$$

where

$$\Delta = [H(k+1)P(k+1,k)H^T(k+1) + R(k+1)]^{-1}$$

Updated covariance

$$P(k+1,k+1) = P(k+1,k) - K(k+1)H(k+1)P(k+1,k) \quad (24)$$

Where

$$H(k+1) = \frac{\partial}{\partial X} h[X(k+1,k+1)]\Big|_{X(k)=\hat{X}(k+1,k)} \quad (25)$$

The Jacobian of the measurement model is given by

$$H_{k+1} = \begin{bmatrix} \frac{\partial h}{\partial x_{k+1}} & \frac{\partial h}{\partial \dot{x}_{k+1}} & \frac{\partial h}{\partial y_{k+1}} & \frac{\partial h}{\partial \dot{y}_{k+1}} & \frac{\partial h}{\partial \Omega_{k+1}} \end{bmatrix} \quad (26)$$

where

$$\frac{\partial h}{\partial x_{k+1}} = \frac{y_{k+1}}{x_{k+1}^2 + y_{k+1}^2}, \quad \frac{\partial h}{\partial y_{k+1}} = \frac{-x_{k+1}}{x_{k+1}^2 + y_{k+1}^2},$$

$$\frac{\partial h}{\partial \dot{x}_{k+1}} = 0, \quad \frac{\partial h}{\partial \dot{y}_{k+1}} = 0, \quad \frac{\partial h}{\partial \Omega_{k+1}} = 0$$

$$H_{k+1} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \quad (27)$$

.

4. CRAMER – RAO LOWER BOUNDS

Cramer- Rao lower bounds (CRLBs) [16,17,18] has been used as a bench mark for the comparison of implemented sub-optimal filtering and the assessment of the effects of introduced approximations. Such a performance bound is important in practice. It enables one to predict the best achievable performance based on the fundamental properties of dynamic and measurement models and it can be used to optimise sensor placement or scheduling.

According to the CRLB the mean square error corresponding to the estimator cannot be smaller than a certain quantity related to likelihood function. If an estimator variance is equal to CRLB, then such an estimator is called efficient.

$$E[(\hat{X}_k - X_k)(\hat{X}_k - X_k)^T] \geq [J_k]^{-1} \quad (28)$$

where

$$J_k = E[(\nabla_{xk} \log p(X_k, Z_k))(\nabla_{xk} \log(X_k, Z_k))^T] \quad (29)$$

Following [17] a recursion for J_k can be written as

$$J_{k+1} = D_k^{22} - D_k^{21}(J_k + D_k^{11})D_k^{12} \quad (30)$$

where in the case of additive Gaussian model the elements D_k^{ij} are given by

$$D_k^{11} = E\{F_k^T Q_k^{-1} F_k\} \quad (31)$$

$$D_k^{12} = -E\{F_k^T Q_k^{-1}\} = (D_k^{21})^T \quad (32)$$

$$D_k^{22} = Q_k^{-1} + E\{H_{k+1}^T R_{k+1}^{-1} H_{k+1}\} \quad (33)$$

where

$$F_k = [\nabla_{x_k}(f(X_k)^T]^T \quad (34)$$

$$H_{k+1} = [\nabla_{X_{k+1}} h_{k+1}^T(X_{k+1})\ ]^T \quad (35)$$

$R_{k+1} = \sigma_\theta^2$ is the variance of the bearing measurements and Q_k is the process noise covariance matrix.

5. SIMULATION AND RESULTS

To analyse the effectiveness of the CT model in tracking manoeuvring targets, a simulation experiment has been done as follows. At first a synthetic data have been generated to simulate the manoeuvring target. The bearing data are assumed to be actually observed and used for estimation. Variances of the observation are assumed to be known. For implementation of this algorithm, the initial estimates of the target state vector are chosen as follows. $X(0|0) = [6940 \quad 7.5 \quad -3980 \quad -13 \quad 0.1]^T$ .The elements of the initial covariance matrix are given as $P(0|0) = [3.2*10\text{^}8 \quad 0 \quad 3.2*10\text{^}8 \quad 0 \quad 0.8017*10\text{^}8]$. In the own ship-target geometry, the target is initially at 20000 Km moving at a speed of 12 km/sec on a course of 150 deg. The observer is making a manoeuvre at a speed of -15 Km/sec. The measurements are assumed to be corrupted with 1 deg. The target takes two co-ordinated turn for 25 minutes with a turn rate of the target is 0.1 rad/sec. The simulations for three different target – observer geometry are shown in figures 1-3. For all the scenarios the parameters mentioned remain the same. Only the coordinated turn rate and the duration for which it exists change. The target and own ship trajectories of the CT scenarios and the estimated target trajectories are shown in fig 1a 2a and 3a, for the process noise covariance matrix Q= 1*diag[1 1 1 1 1]. Figures 1b,2b and 3b indicate the velocity profile of the different target scenarios. The rmses of bearing, position, velocity and their CRLBs are shown in figures 1c-1e,2c-2e and 3e-3e. The simulation results clearly demonstrate the excellent tracking capability of the CT model for manoeuvring targets. Figures 2a-2e represent the performance measures of the observer-target scenario with two coordinated turn each of 0.1 rad/sec for durations of 1.6 minutes. Figure 3a-3e represents the performance for a target making a coordinated turn of 1 rad/sec for 25 minutes. All the results illustrate the ability of the CT model alone to track targets performing varied manoeuvres.

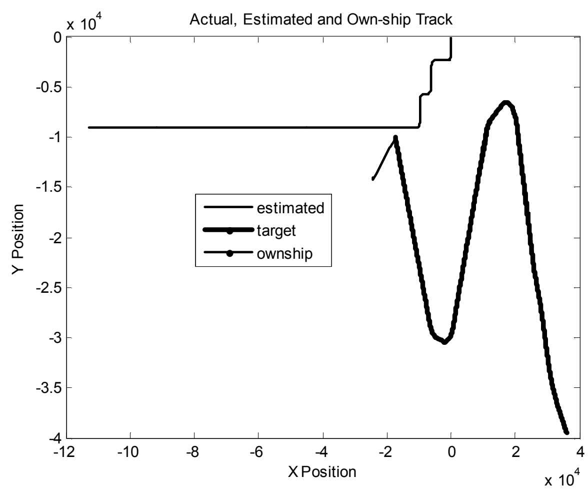

fig 1a

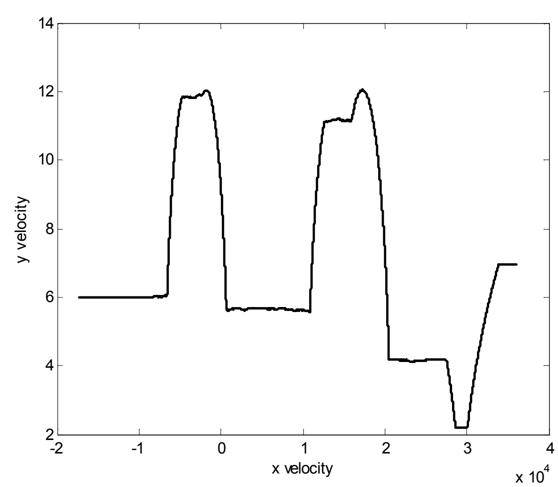

fig 1b

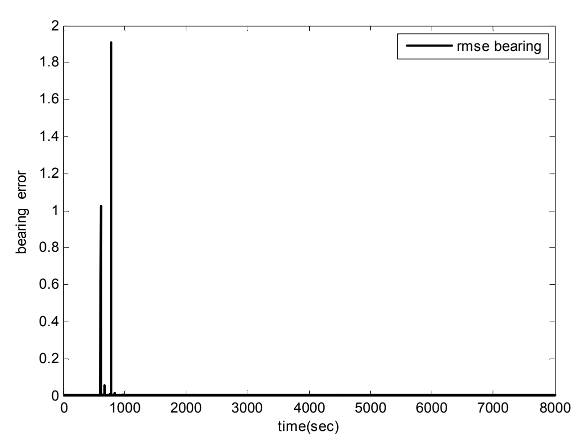

fig 1c

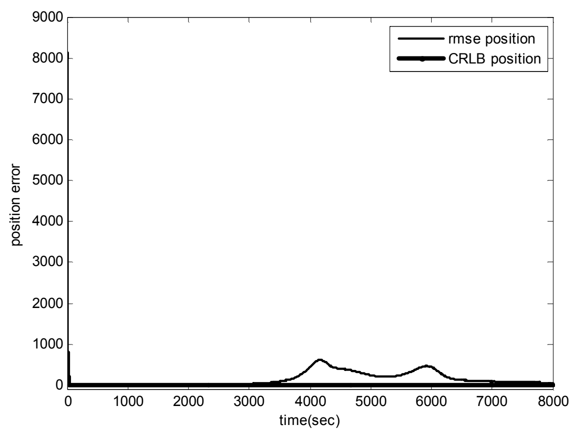

fig 1d

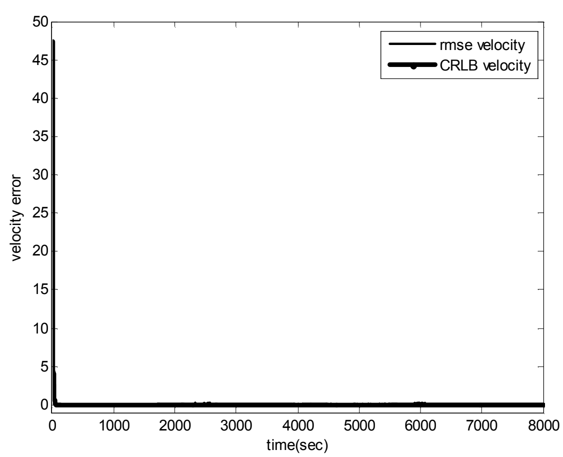

fig 1e

Fig 1a-1e Tracking performance for scenario I

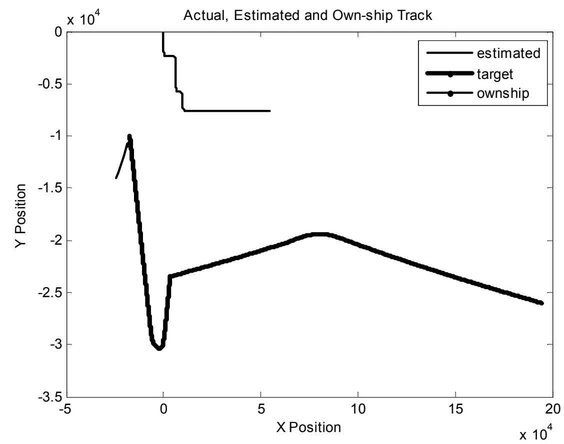

fig 2a

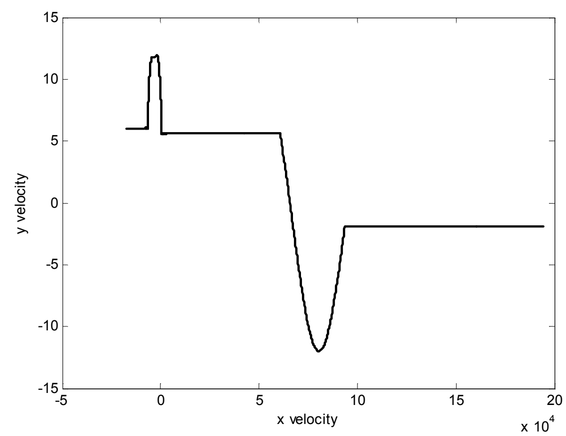

fig 2b

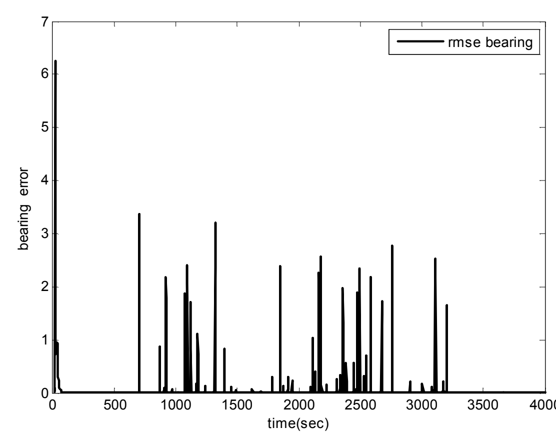

fig 2c

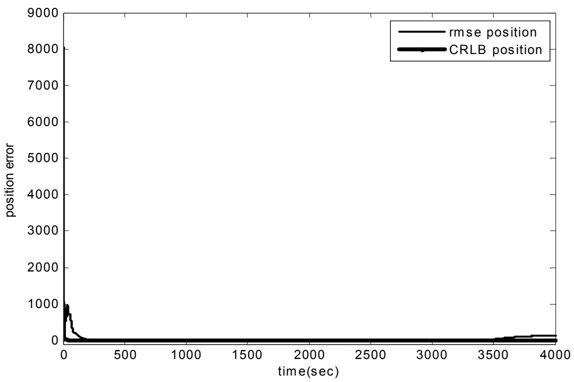

fig 2d

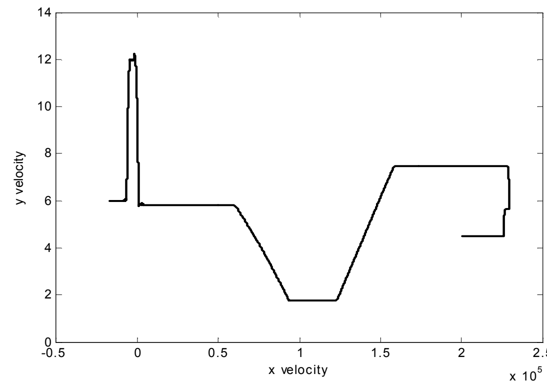

fig 3b

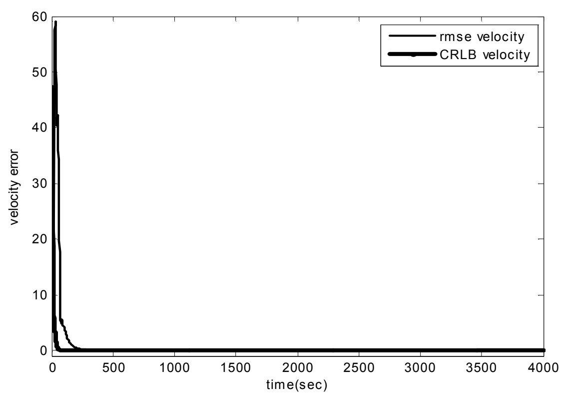

fig 2e

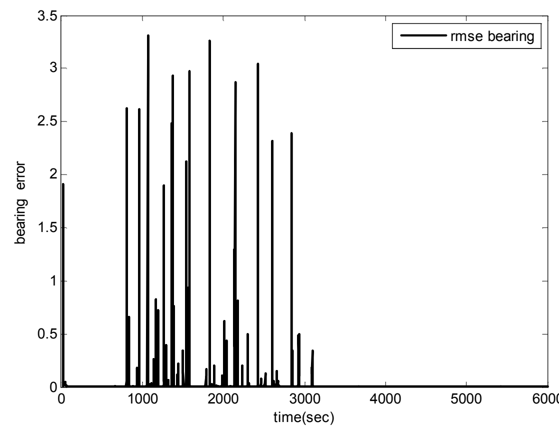

fig 3c

Fig 2a-2e Tracking performance for scenario II

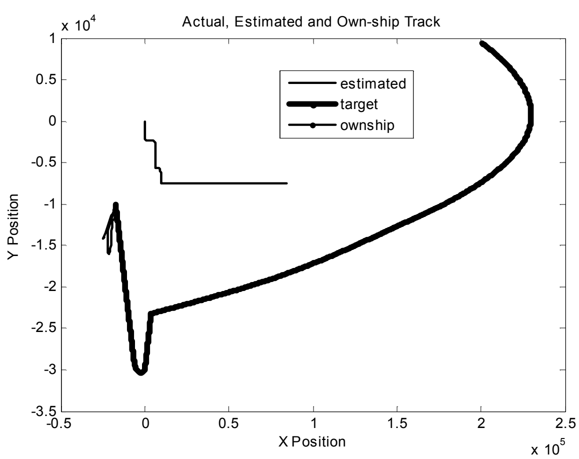

fig 3a

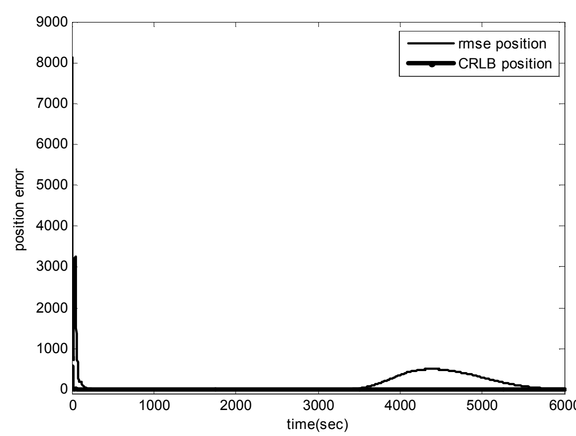

fig 3d

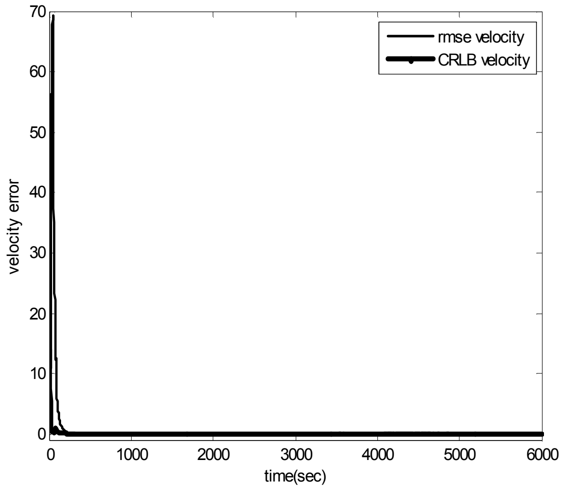

fig 3e

Fig 3a-3e Tracking performance for scenario III

7.CONCLUSION

In this paper we have employed a single CT model along with EKF to capture all (velocity, acceleration and coordinated turn) the dynamics of some manoeuvring target scenarios. The performance parameters such as position RMSE, velocity RMSE and bearing RMSE are computed and compared with the respective CRLBs that indicate the best possible performance one can expect for a given scenario and a set of parameters. It has been proved to provide excellent target tracking capabilities.

Annexure

The Jacobian of the CT model can be computed as

$$F_k = \begin{bmatrix} 1 & \frac{\partial f_1}{\partial \dot{x}_k} & 0 & \frac{\partial f_1}{\partial \dot{y}_k} & 0 \\ 0 & \frac{\partial f_2}{\partial \dot{x}_k} & 0 & \frac{\partial f_2}{\partial \dot{y}_k} & 0 \\ 0 & \frac{\partial f_3}{\partial \dot{x}_k} & 1 & \frac{\partial f_3}{\partial \dot{y}_k} & 0 \\ 0 & \frac{\partial f_4}{\partial \dot{x}_k} & 0 & \frac{\partial f_4}{\partial \dot{y}_k} & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\frac{\partial f_1}{\partial \dot{x}_k} = \frac{\sin(\Omega_k T)}{\Omega_k} + g_1(k)\frac{\partial \Omega_k}{\partial \dot{x}_k},$$

$$\frac{\partial f_1}{\partial \dot{y}_k} = \frac{-(1-\cos(\Omega_k T))}{\Omega_k} + g_1(k)\frac{\partial \Omega_k}{\partial \dot{y}_k},$$

$$\frac{\partial f_2}{\partial \dot{x}_k} = \cos(\Omega_k T) + g_2(k)\frac{\partial \Omega_k}{\partial \dot{x}_k},$$

$$\frac{\partial f_2}{\partial \dot{y}_k} = -\sin(\Omega_k T) + g_2(k)\frac{\partial \Omega_k}{\partial \dot{y}_k},$$

$$\frac{\partial f_3}{\partial \dot{x}_k} = \frac{(1-\cos(\Omega_k T))}{\Omega_k} + g_3(k)\frac{\partial \Omega_k}{\partial \dot{x}_k},$$

$$\frac{\partial f_3}{\partial \dot{y}_k} = \frac{\sin(\Omega_k T)}{\Omega_k} + g_3(k)\frac{\partial \Omega_k}{\partial \dot{y}_k},$$

$$\frac{\partial f_4}{\partial \dot{x}_k} = \sin(\Omega_k T) + g_4(k)\frac{\partial \Omega_k}{\partial \dot{x}_k},$$

$$\frac{\partial f_4}{\partial \dot{y}_k} = \cos(\Omega_k T) + g_4(k)\frac{\partial \Omega_k}{\partial \dot{y}_k},$$

$$\begin{bmatrix} f_{\Omega,1}(k) \\ f_{\Omega,2}(k) \\ f_{\Omega,3}(k) \\ f_{\Omega,4}(k) \end{bmatrix} = \begin{bmatrix} \frac{(\cos\Omega(k)T)\hat{x}(k)}{\Omega(k)} - \frac{(\sin\Omega(k)T)\dot{x}(k)}{\Omega(k)^2} - \frac{(\sin\Omega(k)T)\dot{y}(k)}{\Omega(k)} - \frac{(-1+\cos\Omega(k)T)\dot{y}(k)}{\Omega(k)^2} \\ -(\sin\Omega(k)T)T\dot{x}(k) - ((\cos\Omega(k)T)T\dot{y}(k) \\ \frac{(\sin\Omega(k)T)\dot{x}(k)}{\Omega(k)} - \frac{(1-\cos\Omega(k)T)\dot{x}(k)}{\Omega(k)^2} + \frac{(\cos\Omega(k)T)T\dot{y}(k)}{\Omega(k)} - \frac{(\sin\Omega(k)T)\dot{y}(k)}{\Omega(k)^2} \\ (\cos\Omega(k)T\dot{x}(k) - (\sin\Omega(k)T)\dot{y}(k) \end{bmatrix}$$

The Jacobian of the measurement model is given by

$$H_{k+1} = \begin{bmatrix} \frac{\partial h}{\partial x_{k+1}} & \frac{\partial h}{\partial \dot{x}_{k+1}} & \frac{\partial h}{\partial y_{k+1}} & \frac{\partial h}{\partial \dot{y}_{k+1}} & \frac{\partial h}{\partial \Omega_{k+1}} \end{bmatrix}$$

where

$$\frac{\partial h}{\partial x_{k+1}} = \frac{y_{k+1}}{x_{k+1}^2 + y_{k+1}^2} \quad , \quad \frac{\partial h}{\partial y_{k+1}} = \frac{-x_{k+1}}{x_{k+1}^2 + y_{k+1}^2} \quad ,$$

$$\frac{\partial h}{\partial \dot{x}_{k+1}} = 0 \ , \ \frac{\partial h}{\partial \dot{y}_{k+1}} = 0 \ , \ \frac{\partial h}{\partial \Omega_{k+1}} = 0$$

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