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SVD based Reconfigurable Tracking Controller for Three-tank System via output/State Feedback

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ABSTRACT
This paper deals with the control reconfiguration problem of partial actuator failure in a Three-Tank Multi-Input Multi-Output (MIMO) system. In order to tolerate known actuator failure, a reconfiguration in control parameter/structure is made through output/state feedback. A new Proportional-Integral (PI) Reconfigurable Controller (RC) design is proposed for servo tracking problem using Singular Value Decomposition (SVD) based Parametric Eigenstructure Assignment (PEA) technique. The main contribution of the paper is to design an optimal controller to achieve response shaping and guaranteed stability of the system. The effectiveness of the proposed controller is simulated by analyzing servo-regulatory responses of the three-tank system for various fault scenarios. Finally, performance of the proposed state and output feedback controllers are compared by using various performance indices. The comparison reveals the feasibility of controller used for practical applications

Keywords
SVD, MIMO, PI, PEA, RC

1.INTRODUCTION
Over the past few decades, Eigenstructure Assignment (EA) technique was used for reconfiguration of Multi-Input Multi-Output (MIMO) system. The existing method which deals with MIMO linear system is the Linear Quadratic Regulator (LQR). Out of these two, the eigenstructure assignment technique is most popular one, since, it guarantees the stability and provides acceptable performance. The LQR method based reconfigurable controller has been investigated by Theilliol et al(2002). Even though this method gives performance recovery, as far as stability is concerned, it is not suitable for critical systems. Therefore, EA is used for reconfigurable control system design while LQR is used for the nominal controller design. K.Konstantaopoulos (1996) has proposed a reconfigurable controller design through state and output feedback. This method is used to preserve the most dominant eigenvalues of the closed-loop system and guarantees the stability of remaining closed-loop eigenvalues. Proportional-Integral reconfigurable controller was proposed by Jin.Jiang (2000), where the state feedback gain matrix was calculated through a Singular Value Decomposition (SVD) based EA system. However, this design does not provide parametric solution and closed-loop design degrees of freedom. Wang

(2005) proposed a proportional type state feedback reconfigurable controller based on PEA for linear continuous system. Using this technique, all the eigenvalues of the original system are recovered and overall stability is guaranteed, provided

that full state feedback is permitted,. A few algorithms for eigenstructure assignment via output feedback were reported in the literature: Srinathkumar(1975),S.L.Shaw(1975), Lewis (1993), Alexandridis et al (1996), Sreenatha (1999), Esna Ashari (2005). All these methods are used to assign max(m,p), eigenvalues, where m, p are the number of inputs and outputs. But the disadvantage of the above methods is that, it is difficult to control over all the eigenvalues. Umamaheswari et al (1993) proposed eigenstructure assignment based output feedback controller for a decentralized system. In this paper the problem of eigenstructure assignment has been converted into a constrained nonlinear minimization problem. Duan (2003) applied a robust output dynamic compensator to magnetic bearing system. In practical servo system the importance is not only given to recover the closed loop eigenvalues of the system, but also for response shaping. The rest of this paper is organized as follows: The Three Tank System (TTS) is described in section-2. The section-3 presents Reconfigurable Controller (RC) design. Simulation results are presented in sections 4. Conclusion and references are presented at the end.

2.BENCHMARK THREE-TANK SYSTEM
The schematic diagram of benchmark Three-Tank System as defined by Patton (2005) is shown in Figure.1. The interacting TTS comprises of three identical tanks with cross section of A. The tanks are inter-connected by two cylindrical pipes with a cross-section S and outflow coefficients a_{z1} & a_{z2}. The nominal inflows (q_1 & q_2) are located at tank1 and tank3 respectively. The nominal outflow pipe has a cross section S with an outflow coefficient a_{z3} and located at tank 3.

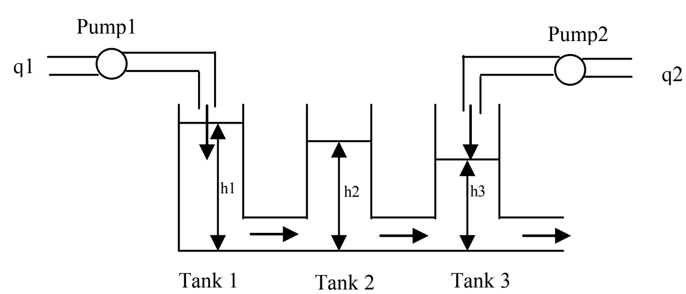

Figure.1 Three-tank system

The three-tank system is modeled and it is defined by the mass balance Equation (1):

$$\left.\begin{aligned}\frac{dh_1}{dt}&=\frac{q_1 - S_1 a_{z_1}\,\mathrm{sgn}(h_1-h_2)\sqrt{2g(h_1-h_2)}}{A}\\ \frac{dh_2}{dt}&=\frac{S_1 a_{z_1}\,\mathrm{sgn}(h_1-h_2)\sqrt{2g(h_1-h_2)}-S_2\,a_{z_2}\,\mathrm{sgn}(h_2-h_3)\sqrt{2g(h_2-h_3)}}{A}\\ \frac{dh_3}{dt}&=\frac{q_2+S_2\,a_{z_2}\,\mathrm{sgn}(h_2-h_3)\sqrt{2g(h_2-h_3)}-S_3\,a_{z_3}\sqrt{2gh_3}}{A}\end{aligned}\right\} \quad (1)$$

Table I Physical Parameters of the Three- Tank System

Parameter	Value
Tank cross section area	$A = 0.0171\,m^2$
Pipe cross section area	$S_1 = S_2 = S_3 = 0.00005\,m^2$
Pipeoutflow coefficients	$a_{z_1} = 0.511, a_{z_2} = 0.5279\ \&\ a_{z_3} = 0.7313$
Maximum level	$H_{max} = 0.68\,m$
Maximum in-flow level	$q_{max} = 1.0 * 10^{-4}\,m^3\,/\,s$

The nonlinear system is linearized around the steady state operating points (h_1 = 0.6 m, h_2= 0.5m and h_3= 0.4m). The linearized continuous model is described by a discrete linear state model with a sampling period T_s=1s as given in Equation (2)

$$\left.\begin{aligned}x(k+1)&=Ax(k)+Bu(k)\\ y(k)&=Cx(k)\end{aligned}\right\} \quad (2)$$

$$A=\begin{bmatrix}0.9896 & 0.0103 & 0.0001\\ 0.0103 & 0.9791 & 0.0106\\ 0.0001 & 0.0106 & 0.9819\end{bmatrix},\ B=\begin{bmatrix}58.18 & 0.0011\\ 0.3028 & 0.312\\ 0.0011 & 57.95\end{bmatrix}\ \text{and}\ C=\begin{bmatrix}1 & 0 & 0\\ 0 & 0 & 1\end{bmatrix};$$

$x(k) = [h_1\ h_2\ h_3]^T$, $u(k) = [q_1\ q_2]^T$ *and* $y(k) = [h_1\ h_3]^T$ are the state, input and output vectors respectively.

3. DESIGN OF RECONFIGURABLE CONTROLLER

3.1 PI Controller Design

Consider a servo system with state feedback and integral control is assumed to be completely controllable and observable. The process state and output equations are given in Equations (3a)-(3b)

$$x(k+1) = Ax(k) + Bu(k) \quad (3a)$$

$$y(k) = C\,x(k) \quad (3b)$$

where $x(k)\in R^n$ is the state vector, $u(k)\in R^m$ is the input vector and $y(k)\in R^p$ is the output vector. A , B and C are the system, input and output matrices respectively. The integrator state is represented in Equation (4)

$$e(k+1) = e(k) + T_s e(k) \quad (4)$$

where tracking error $e(k)=r(k)-y_r(k)$, r(k) is the reference input vector, $y_r(k)$ is the output vector that is required to follow the reference input vector and T_s is the sampling period. Many techniques are available for incorporating integral action into the controller design. One of the simplest techniques is state augmentation.

The feedback control is required to generate control signal such that the output vector $y_r(k)$ tracks the reference input vector r(k) and reaches steady state. In order to maintain controllability, the number of tracking outputs cannot exceed the number of control inputs (Theilliol et al., 2002). At steady state the value of y(k) is given by Equation(5)

$$\lim_{k\to\infty} y_r(k) = r(k) \quad (5)$$

Augmented system is derived from Equation(3)-(4) and is represented by Equation (6)

$$\left.\begin{aligned}\tilde{x}(k+1)&=\tilde{A}\tilde{x}(k)+\tilde{B}u(k)+Er(k)\\ yk)&=\tilde{C}\tilde{x}(k)\end{aligned}\right\} \quad (6)$$

where

$$\tilde{x}(k+1)=\begin{bmatrix}x(k+1)\\ e(k+1)\end{bmatrix},\ \tilde{x}(k)=\begin{bmatrix}x(k)\\ e(k)\end{bmatrix},\ \tilde{A}=\begin{bmatrix}A & 0\\ -T_sC_r & I\end{bmatrix},$$

$$\tilde{B}=\begin{bmatrix}B\\ 0\end{bmatrix},\ \tilde{C}=\begin{bmatrix}C & 0\\ 0 & I\end{bmatrix},\ E=\begin{bmatrix}0\\ T_sI\end{bmatrix}$$

and C_r is the tracking output matrix. If the augmented system matrix $(\tilde{A},\tilde{B})$ is controllable and the control law is $u = Ky = \begin{bmatrix}K_P & K_I\end{bmatrix}\tilde{C}\tilde{x}(k)$, then the closed loop Equation (6) can be rewritten and represented as given in Equation (7)

$$\tilde{x}(k+1)=(\tilde{A}+\tilde{B}K\tilde{C})\tilde{x}(k)+\tilde{B}u(k)+Er(k) \quad (7)$$

The design of a output feedback controller based on the augmented system can allow the computation of Proportional and Integral controller gains for the system.

3.1.1 Output feedback controller gain calculation using SVD based PEA

Any system can be represented in terms of its eigenstructure i.e. in terms of its eigenvalues and eigenvectors. The augmented system in terms of eigenvalues and eigenvectors is presented in Equation (8)

$$(\tilde{A}+\tilde{B}K\tilde{C})v_i = z_i v_i \quad (8)$$

where $i = 1, 2, 3....,n+p$

Define $\Lambda = diag(z_1, z_2, ...z_{n+p}), V = [v_1, v_2, ...v_{n+p}]$

The Equation (8) can be written in the matrix form as given in Equation (9):

$$(\tilde{A} + \tilde{B}K\tilde{C})V = V\Lambda \tag{9}$$

Let $W = K\tilde{C}V$ then, Equation (9) can be rewritten as

$$\tilde{A}V + \tilde{B}W = V\Lambda \tag{10}$$

Right Eigenstructure Assignment (RESA) is considered, when p>m. Hence, the solutions of above Equation (10) can be obtained from Theorem 3.1(Duan, 2003). If $(\tilde{A}, \tilde{B})$ is controllable, then while taking SVD, there exists orthogonal matrices ϕ_i and ψ_i satisfying the following Equation (11)

$$\psi_i \begin{bmatrix} \tilde{B} & \tilde{A}-zI \end{bmatrix} \phi_i = \begin{bmatrix} \Sigma_i & 0 \end{bmatrix}, \forall z \in C \tag{11}$$

Partitioning of ϕ_i is given in Equation (12), from which we can obtain the parametric expressions of eigenstructure assignment as given in Equation (12)

$$\phi_i = \begin{bmatrix} * & D_i \\ * & N_i \end{bmatrix}, N_i \in R^{(n+p)\times m}, D_i \in R^{m\times m} \tag{12}$$

In order to improve the transient response and the stability of system, the parametric vectors are chosen such a way that the norm of feedback gain matrix, condition number of eigenvector are small and satisfy the following constraints:

C_{R1}: $z_i = \overline{z_j} \leftrightarrow f_i = \overline{f_j}, \quad i, j = 1, 2, ...n + p$

C_{R2}: $\det(V) \neq 0$

C_{R3} : $W(I - (C * V)^T [(CV)(CV)^T]^{-1} CV) = 0$

Now, the parametric expression of feedback gain matrix $K \in R^{m\times p}$ can be written as given in Equation (13)

$$K = W(CV)^T [(CV)(CV)^T]^{-1} \tag{13}$$

where

$$V = \begin{bmatrix} v_1 & v_2 .... v_{n+p} \end{bmatrix}, \quad v_i = N_i f_i, \; i = 1, 2, ..., n + p, \text{and} \; f_i \in C^m$$

$$W = \begin{bmatrix} w_1 & w_2 .... w_{n+p} \end{bmatrix}, \quad w_i = D_i f_i$$

3.1.2 State feedback controller gain calculation using SVD based PEA

The procedure used for the output feedback can easily be extended to the case of state feedback. When the number of outputs is identical to the number of states $p = n$ i.e., $C = I_n$, the above expression in Equation (13) reduces to that represented in Equation(14)

$$K = W(CV)^{-1} \tag{14}$$

It is apparent that state-feedback compared to output feedback, offers a greater flexibility with regards to eigenstructure assignment. However, from the practical point of view, state feedback is quite undesirable, since, for large systems it requires measuring and feeding all the states of the system.

3.1.3 Reconfigurable controller

The above technique has been extended to recover the performance of the closed-loop system, when the system is affected by actuator fault. In this paper, known multiplicative type actuator failure (reduction in control effectiveness factor) is considered. The system equation can be represented by Equation (15)

$$\begin{aligned} x_f(k+1) &= A_f x_f(k) + B_f u(k) \\ y_f(k) &= C x_f(k) \end{aligned} \tag{15}$$

where x_f, y_f are the state and output vectors of the faulty system and A_f, B_f are the state and input matrices respectively. Faulty system can be recovered by reconfiguring the controller gain of nominal system. State feedback controller gain for recovered system (K_f) is determined by replacing $(\tilde{A}, \tilde{B})$ with augmented faulty system $(\tilde{A}_f, \tilde{B}_f)$, where $\tilde{A}_f = \begin{bmatrix} A_f & 0 \\ -T_s C_r & I \end{bmatrix}$, and $\tilde{B}_f = \begin{bmatrix} B_f \\ 0 \end{bmatrix}$.

3.1.4 Parametric optimization

The aim of optimal controller design is to choose the parametric vectors and eigenvalues to satisfy all the robust measures such as performance measure, closed-loop stability measure, tracking performance and optimal controller gain.

i) Robust stability

Guaranteed closed-loop robust stability can be achieved by minimizing eigenvalues sensitivities with respect to open-loop system parameter perturbations. One of the measures of sensitivity is the condition number of the closed loop eigenvector. For practical applications, Frobenius condition number is more conservative than spectral condition number (Ashari, 2005). The Frobenius condition number is given in Equation(16):

$$\kappa_F(V) = \|V\|_F \, \|V^{-1}\| \tag{16}$$

ii) Minimum control effort:

In some practical applications, constraints such as settling time and actuator usage limits the selection of eigenvalues. In order to minimize the control effort, closed-loop eigenvalues are assigned closer to open loop eigenvalues. Therefore, the performance index J_c is used to minimize the difference between open loop eigenvalues and closed loop eigenvalues with respect to the elements of eigenvalues constrained by $u(k) < u_{max}(k)$:

$$J = \sum_{i=1}^{n+p} \|\lambda_{oi} - \lambda_{ci}\|, \quad i = 1, 2, ...n + p \tag{17}$$

where λ_{oi} and λ_{ci} are open loop and closed loop eigenvalues respectively.

iii) Optimal dynamic performance

Optimal dynamic performance of the system is obtained by choosing optimal controller parameters and the control parameters are parameterized in terms of parametric vector. The following cost function reported by Radke et al (1987) is used to achieve minimum integral square error.

$$J = \sum_{k=1}^{\infty} e^2(k) \tag{18}$$

3.1.5 Reconfigurable Controller design via output feedback

The desired eigenvalues of closed loop augmented system are chosen as 0.9857+0.01342i, 0.9857-0.01342i, 0.9355, 0.9231 and 0.9109 and the parametric vectors are selected as

$$f_1 = \begin{bmatrix} 5+5i \\ 6+5i \end{bmatrix};\ f_2 = \overline{f_1};\ f_3 = \begin{bmatrix} 9 \\ -10 \end{bmatrix};$$

$$f_4 = \begin{bmatrix} 10 \\ 10 \end{bmatrix} \text{ and } f_5 = \begin{bmatrix} 16 \\ -14 \end{bmatrix}$$

The achievable eigenvalues which are very close to desired ones are: 0.9836 + 0.0080i, 0.9836 - 0.0080i, 0.9368, 0.9245 and 0.9111.

In this paper, 80% reduction in the flow rate of actuator 2 has been considered as actuator fault and it is introduced at the sampling instant k=1500. Due to the presence of integral controller, actuator fault acts on the system as a perturbation. Under the assumption that actuator fault information provided by the Fault Detection and Isolation (FDI) system is correct, the reconfigurable controller reconfigures its parameters provided in the form of state and input matrices. After the fault has occurred, the controller resynthesizes the state feedback law such that eigenstructure of the closed loop system can completely recover that of the normal (Fault-free) system. Eigenvectors and feedback gain matrix of normal (Fault-free) and recovered system are presented in Table II.

Table II Output feedback controller parameters

System state	Achievable eigenvectors and Controller gain
Fault-free	$V = \begin{pmatrix} 0.1386 \pm 0.0044i & 0.5790 & 0.7671 & 1.4188 \\ 0.0891 \mp 0.1825i & 0.0199 & -0.2776 & -0.0194 \\ 0.1528 \mp 0.0090i & -0.6439 & 0.7660 & -1.2438 \\ 4.9968 \pm 4.9998i & 8.9765 & 9.9757 & 15.9235 \\ 5.9969 \pm 4.9988i & -9.9836 & 9.9615 & -13.9599 \end{pmatrix}$ $K = \begin{pmatrix} -0.0019 & 0.0005 & 0.0001 & -0.00003 \\ 0.0006 & -0.0018 & -0.00004 & 0.0001 \end{pmatrix}$
Recovered	$V_f = \begin{pmatrix} 0.1386 \pm 0.0044i & 0.5790 & 0.7671 & 1.4188 \\ 0.0891 \mp 0.1825i & 0.0199 & -0.2776 & -0.0194 \\ 0.1528 \mp 0.0090i & -0.6439 & 0.7660 & -1.2438 \\ 4.9968 \pm 4.9998i & 8.9765 & 9.9757 & 15.9235 \\ 5.9969 \pm 4.9988i & -9.9836 & 9.9615 & -13.9599 \end{pmatrix}$ $K_f = \begin{pmatrix} -0.0019 & 0.0005 & 0.0001 & -0.00003 \\ 0.0031 & -0.0088 & -0.0002 & 0.0003 \end{pmatrix}$

3.1.6 Reconfigurable Controller design via statet feedback

The achievable eigenvalues of fault-free and recovered system via state feedback for the same faulty conditions are 0.9857+0.01342i, 0.9857-0.01342i, 0.9355, 0.9231 and 0.9109. The parametric vectors are chosen as

$$f_1 = \begin{bmatrix} 5+5i \\ 6+5i \end{bmatrix};\ f_2 = \overline{f_1};\ f_3 = \begin{bmatrix} 9 \\ -10 \end{bmatrix};$$

$$f_4 = \begin{bmatrix} 10 \\ 10 \end{bmatrix} \text{ and } f_5 = \begin{bmatrix} 16 \\ -14 \end{bmatrix}$$

Eigenvectors and feedback gain matrix of normal (Fault-free) and recovered system are presented in Table III.

Table III State feedback controller parameters

System state	Achievable eigenvectors and Controller gain
Fault-free	$V = \begin{pmatrix} 0.1386 \pm 0.0044i & 0.5790 & 0.7671 & 1.4188 \\ 0.0891 \mp 0.1825i & 0.0199 & -0.2776 & -0.0194 \\ 0.1528 \mp 0.0090i & -0.6439 & 0.7660 & -1.2438 \\ 4.9968 \pm 4.9998i & 8.9765 & 9.9757 & 15.9235 \\ 5.9969 \pm 4.9988i & -9.9836 & 9.9615 & -13.9599 \end{pmatrix}$ $K = \begin{pmatrix} -0.0019 & 0.0004 & 0.0006 & 0.0001 & -0.00003 \\ 0.0006 & 0.0002 & -0.0017 & -0.00004 & 0.0001 \end{pmatrix}$
Recovered	$V_f = \begin{pmatrix} 0.1386 \pm 0.0044i & 0.5790 & 0.7671 & 1.4188 \\ 0.0891 \mp 0.1825i & 0.0199 & -0.2776 & -0.0194 \\ 0.1528 \mp 0.0090i & -0.6439 & 0.7660 & -1.2438 \\ 4.9968 \pm 4.9998i & 8.9765 & 9.9757 & 15.9235 \\ 5.9969 \pm 4.9988i & -9.9836 & 9.9615 & -13.9599 \end{pmatrix}$ $K_f = \begin{pmatrix} -0.0019 & 0.0004 & 0.0006 & 0.0001 & -0.00003 \\ \text{[illegible]} & \text{[illegible]} & \text{[illegible]} & \text{[illegible]} & \text{[illegible]} \end{pmatrix}$

4.SIMULATION RESULTS

4.1Output feedback controller

The output response of system under normal conditions when excited with step input and the corresponding variations in manipulated variables are shown in Figure.2a-2b (solid line) and 3a-3c. For the purpose of validation, a step change in the tank level of h_1 0.06m and h_3 0.04m(10% changes around the operating point) is applied at 250th sampling instant and is maintained up to 3000th sampling instant. It can be observed in Figure 3a that the flow rate of actuator 1 increases from 3.5787×10^{-5} m^3/s to 4.7287×10^{-5} m^3/s and flow rate of actuator 2 increases from 6.5363×10^{-5} m^3/s to 8.625×10^{-5} m^3/s. The occurrence of 80% failure in actuator 2 (q_2) at 1500th sampling instant decreases its flow rate of it from 8.625×10^{-5} m^3/s to 7×10^{-5} m^3/s. It also decreases flow rate of actuator 1(q_1) from 4.7287 $\times10^{-5}$ m^3/s to 4.4167 $\times10^{-5}$ m^3/s (refer Figure 3b). The variation in manipulated variable drives the process output away from the desired trajectory as shown in Figure 2a and Fig 2b (refer dashed line). It can also be seen in Figure 2 that the process outputs are not able to track the set points at k=1500. Due to the actuator failure, level h1 decreases from 0.66 m to 0.6524 m (refer Figure 2a) and consequently, the level h_3 decreases from 0.44 m to 0.4269 m (refer Figure 2b).

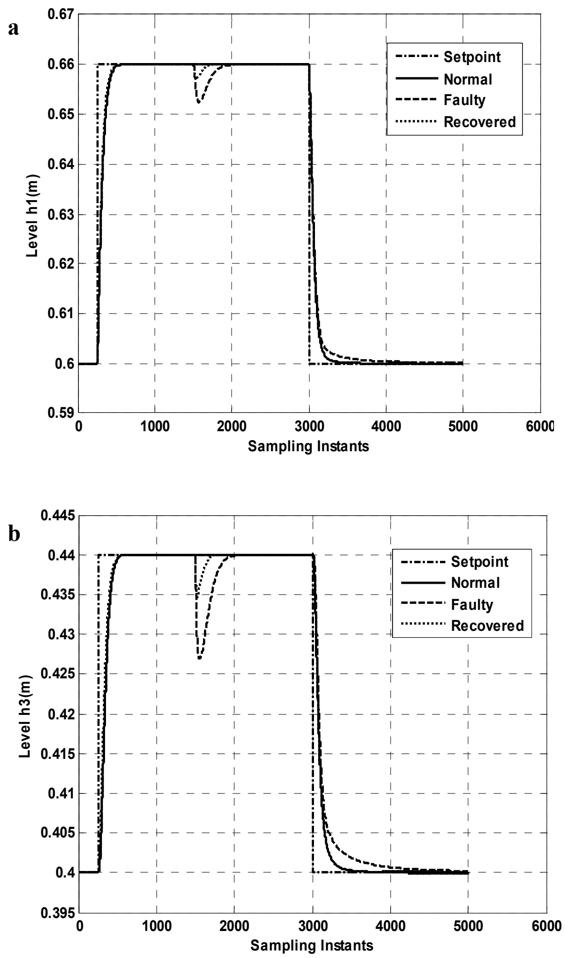

Figure 2. **Outputs responses for 80% failure in actuator 2: (a) Level h_1 (b) Level h_3**

The reconfigurable controller drives the three tank system and brings the output back to the desired trajectory as much as possible when the fault occurs. The tracking performance of the recovered system outputs and the associated manipulated variables are shown in Figure 2a-2b (refer dotted line) and Figure 3c respectively. It can be found from the Figure 2 that the reconfigurable controller recovers its nominal characteristics with reasonably small transients in the presence of actuator fault. After the reconfiguration, decrease in h1 is brought from 0.6524 m to 0.6572 m and h3 from 0.4269m to 0.4345 m (refer Figure 2a-2b dotted line).

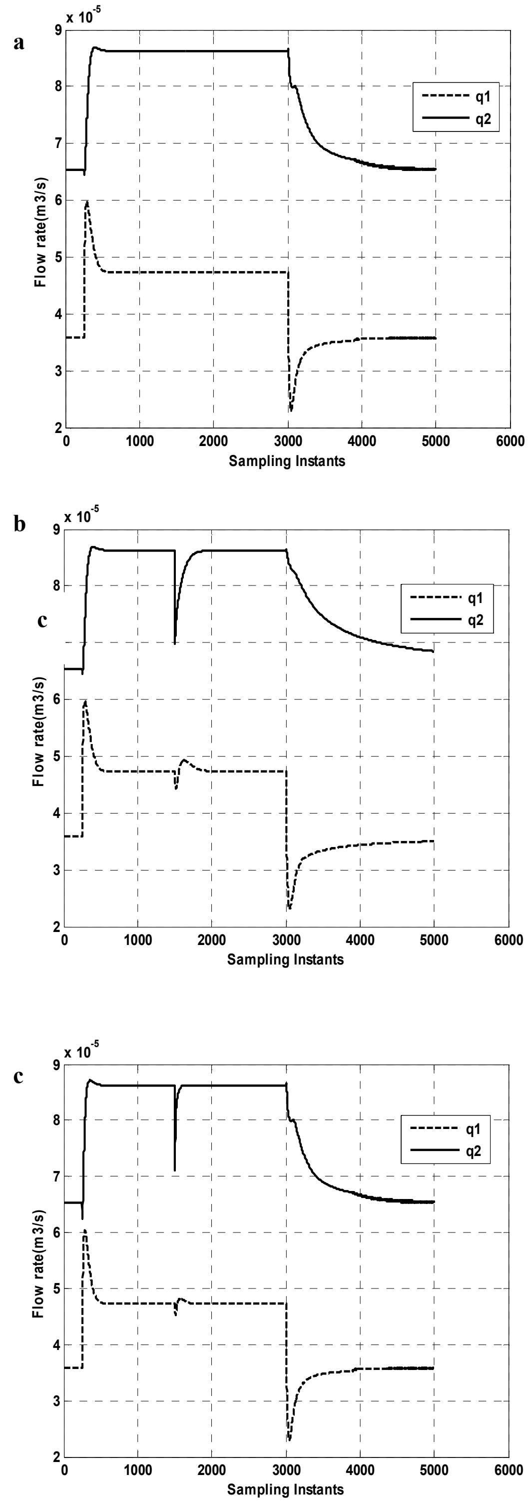

Figure 3. **Variations in manipulated variables under different conditions; (a) Normal (Fault free) (b) Faulty condition (c) Recovered condition**

Table IV Comparison of ISE performance indices of output feedback RCS

Error/State	Normal(Fault -free)	Faulty	Recovered
ISE_{eh1}	0.2772	0.2923	0.2707
ISE_{eh2}	0.2007	0.2409	0.1858

It can be seen from the Table IV that the ISE of system increases from its nominal values due to actuator failure. After the reconfiguration, its value becomes less than the normal one. Robust performance indices of fault which were mentioned earlier are presented in Table V. The $K_2(V)$ values for normal and recovered systems are same but control energy required for the recovered system is more than that of normal system. It is reflected in $\|K\|$.

Table V Robust stability and performance measures of output feedback RCS

Performance measure	Normal(Fault-free)	Recovered
$K_2(V)$	94.1058	94.1058
$\|K\|$	0.0024	0.0094

4.2State feedback controller

The reference input used for output feedback controller is used for this state feedback analysis also. Under normal operating conditions, the outputs perfectly track the corresponding reference inputs. It can be observed in Figure 5a that the flow rate of actuator1 increases from 3.5787×10^{-5} m^3/s to 4.7287×10^{-5} m^3/s and flow rate of actuator 2 increases from 6.5363×10^{-5} m^3/s to 8.625×10^{-5} m^3/s. The actuator faults are introduced on the actuator 2 at t =1500s as shown in Figure 5. The occurrence of 80% failure in actuator2 at 1500th sampling instant decreases the flow rate from 8.625×10^{-5} m^3/s to 7×10^{-5} m^3/s. It also decreases flow rate of actuator 1 from 4.7287×10^{-5} m^3/s to 4.1854×10^{-5} m^3/s(refer Figure 5b). Due to actuator failure level h1 decreases from 0.66 m to 0.6491 m (refer Figure 4a dashed line) and consequently, the level h_3 decreases from 0.44 m to 0.4244 m (refer Figure 4b dashed line).

a

b

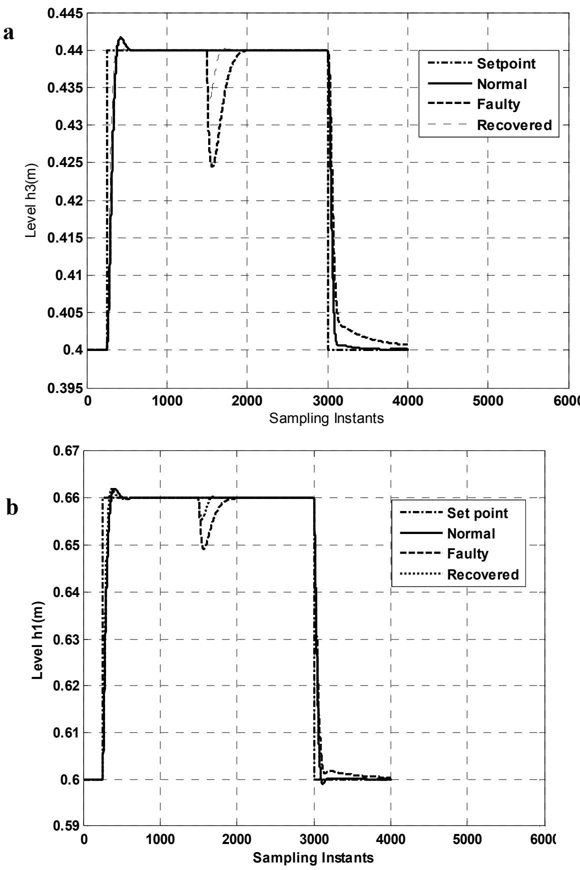

Figure 4. **Outputs responses for 80% failure in actuator 2: (a) Level h_1 (b) Level h_3**

The tracking performance of the recovered system outputs and the associated manipulated variables are shown in Figure 4a-4b and Figure 5c respectively. It can be found from the Figure 2 that the reconfigurable controller recovers its nominal characteristics with reasonably small transients in the presence of actuator fault. After the reconfiguration, decrease in h1 is brought from 0.6491 m to 0.6552 m and h3 from 0.4244 to 0.4332 m(refer Figure 4a-4b dotted line).It can also found that the small overshoot at the step changing instant.

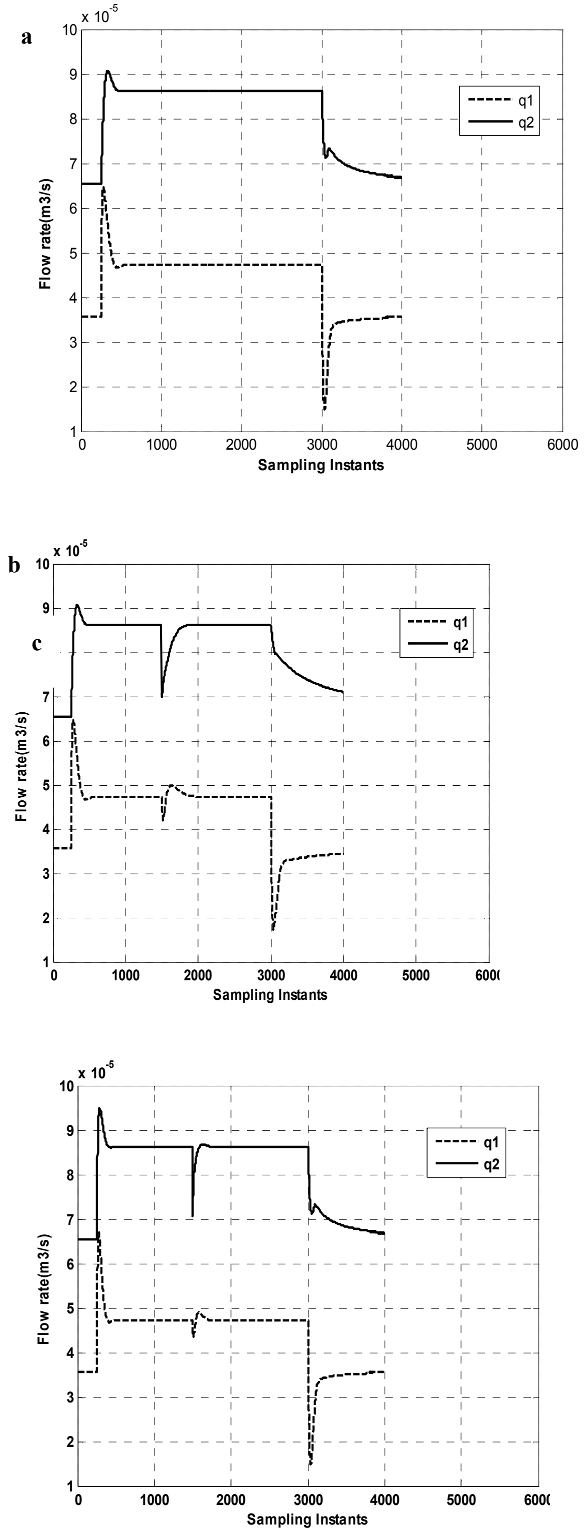

Figure 5. **Variations in manipulated variables under different conditions; (a) Normal (Fault free) (b) Faulty condition (c) Recovered condition**

4.2.1 Performance Evaluation of state feedback controller

The ISE of normal (fault-free) case, faulty case and recovered case are reported in Table VI. From Table IV and Table VI, it is confirmed that the ISE value of state feedback controller is lesser than that of the output feedback controller.

Table VI ISE performance indices of output feedback RCS

Error/State	Normal(Fault-free)	Faulty	Recovered
ISE_{eh1}	0.2216	0.2366	0.2146
ISE_{eh2}	0.1491	0.1919	0.1334

The robust stability and robust performance indices are listed in Table VII. The $K_2(V)$ value of recovered system through state feedback is smaller than one which is computed through output feedback.

Table VII Robust stability and performance measures of output feedback RCS

Error/State	Normal(Fault-free)	Recovered
$K_2(V)$	94.1058	94.1058
$\|K\|$	0.0024	0.0092

5. CONCLUSION

In this paper, parametric eigenstructure based output and state feedback PI reconfigurable controllers were proposed and were applied to the three tank system against actuator failure. With reference to Table IV-VII, it can be concluded that the state feedback controller provides better result as compared to output feedback controller. This is due to the fact that all closed-loop poles are preserved and located at the desired location. In order to achieve the guaranteed stability by output feedback controller design, optimal parametric vectors are chosen so that the achievable eigenstructure are very close to the desired one. An important issue in reconfigurable controller design is the trade-off between performance recovery and stability. The parametric vectors provide the degrees of freedom to compromise this issue. The proposed controllers work effectively around an operating point against partial actuator failure.

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