Stability Criteria for Stochastic Recurrent Neural Networks with Two Time-Varying Delays and Impulses

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ABSTRACT

This paper is concerned with a stability problem for a class of stochastic recurrent impulsive neural networks with both discrete and distributed time-varying delays. Based on Lyapunov-Krasovskii functional and the linear matrix inequality (LMI) approach, we analyze the global asymptotic stability of impulsive neural networks. Two numerical examples are given to illustrate the effectiveness of the stability results.

Keywords

Global asymptotic stability; Linear matrix inequality; Lyapunov-Krasovskii functional; Time-varying delays; Stochastic recurrent neural networks; distributed delays; impulsive.

1. INTRODUCTION

Over the past decades, the recurrent neural networks with time varying delays have found their important applications in various areas such as image processing, pattern recognition, optimization solvers and fixed point computation [4, 5, 6]. Stability analysis is the basic knowledge for dynamical systems and is useful in the application to the real life systems. Many important results have been proposed to guarantee the global asymptotic or exponential stability for the recurrent neural networks with time delays, see example [3-6]. On the other hand time delays are often encountered in neural networks due to the finite switching speed of amplifiers and the inherent communication time of neurons.

Neural networks have a spacial nature due to the presence of parallel pathways with a variety of axon sizes and lengths, so it is desirable to model them by introducing unbounded delays. In recent years there has been a growing research interest in the study of neural networks with both discrete and distributed delays [17, 23, 24, 25, 29]. It should be mentioned that using linear matrix inequality (LMI) approach the sufficient global asymptotic stability conditions have been derived in [23] for a general class of neural networks with both discrete and distributed delays. Dynamical systems are often classified into two categories of either continuous-time or discrete-time systems. These two dynamic systems are widely studied in population models and neural networks, yet there is a somewhat new category of dynamical systems, which is neither continuous-time nor purely discrete-time; these are called dynamical systems with impulses. A fundamental theory of impulsive differential equations has been developed in [20]. For instance, in the implementation of neural networks, the state of the networks is subject to instantaneous perturbations and experiences abrupt changes at certain instants, which may be caused by the switching phenomenon, frequency change or other sudden noise that is it exhibits impulsive effects [27, 28, 30]. Neural networks are often subject to impulsive perturbations that in turn affect dynamical behaviors of the systems. Therefore, it is necessary to take both time delays and impulsive effects in to account on the dynamical behaviors of neural networks.

When performing the computation, there are many stochastic disturbances that affect the stability of neural networks. A neural network could be stabilized or destabilized by certain stochastic inputs [1]. It implies that the stability of stochastic neural networks also has primary significance in the research of neural networks. Hence the stability analysis problem for stochastic neural networks becomes increasingly significant and some results related to this problem have recently been published, see [1, 12-14, 16, 19, 21, 22, 24, 26]. We establish new stability conditions for the recurrent impulsive neural networks to be globally asymptotically stable by utilizing Lyapunov-Krasovskii functional method and using some well-known inequalities. Compared with the earlier results in the literature, those results are less restrictive and less conservative.

Motivated by the above discussions, this paper aims to develop the global asymptotic stability in the mean square for stochastic recurrent impulsive neural networks with both discrete and distributed delays. For the best of author's knowledge there were no global stability results for stochastic recurrent impulsive neural networks with both discrete and distributed delays. Based on Lyapunov stability theory and linear matrix inequality technique, the stability conditions are given in terms of LMIs which can be easily checked by LMI control toolbox in MATLAB. We provide two numerical examples to illustrate the effectiveness of the stability results.

2. PRELIMINARIES

Consider the delayed stochastic recurrent neural networks with impulses and time-varying delays as follows:

$$\dot{x}_i(t) = [-a_i x_i(t) + \sum_{j=1}^{n} b_{ij} f_j\left(x_j(t)\right) + \sum_{j=1}^{n} w_{ij} f_j(x_j(t-\tau(t))) + \sum_{j=1}^{n} c_{ij}\int_{t-h}^{t} f_j(x_j(s))ds + I_i]$$

1. $+\sum_{j=1}^{n} \sigma_{ij}\left(t, y_j(t), y_j(t-\tau(t))\right) dw_j$, **1.** $t \neq$ …. (1)

$\Delta x_i(t_k) = I_k(x_i(t_k)), t = t_k, i = 1,2,\dots,n, k = 1,2,\dots$

where $x_i(t)$ is the state of the i[th] neuron at time t, $a_i > 0$ denotes the passive decay rate, b_{ij}, w_{ij}, c_{ij}, are the synaptic connection strengths, f_j denotes the neuron activations, I_i is the constant input from outside the system, $\tau(t)$ represents the discrete transmission delay with $\dot{\tau}(t) \le \mu < 1$ and $1 \le h < \infty$. The stochastic disturbance $w(t) = (w_1(t), w_2(t), \dots, w_m(t))^T$ is an m-dimensional Brownian motion defined on a complete probability space $(\Omega, \mathcal{F}, P)$ with a natural filtration $\{\mathcal{F}(t)\}_{t\ge 0}$. Moreover,

$$trace[\sigma^T(t, y(t), y(t-\tau(t)))\sigma(t, y(t), y(t-\tau(t)))]$$
$$\le y^T(t)F_0 y(t) + y^T(t-\tau(t))F_1 y(t-\tau(t)) \quad \dots (2)$$

Let $\sigma(t,x,y): \mathbb{R}_+ \times \mathbb{R}^n \to \mathbb{R}^{n\times m}$ is locally Lipschitz continuous and satisfies the linear growth condition as well. $\Delta x_i(t_k) = I_k(x_i(t_k))$ = $x_i(t_k^+) - x_i(t_k^-)$ and $t_1 < t_2 < \cdots$ is a strictly increasing sequence such that $\lim_{k\to\infty} t_k = +\infty$.

We assume that the neuron activation function $f_j, j = 1,2,\dots,n$ is bounded and satisfy the following Assumption:

Assumption 1.

$0 \le \left|f_j(\delta_1) - f_j(\delta_2)\right| \le \Sigma_j|\delta_1 - \delta_2|$, for all $\delta_1, \delta_2 \in \mathbb{R}$,

Assume that $x^* = (x_1^*, x_2^*, \dots, x_n^*)^T$ is an equilibrium point of Eqn. (1). It can be easily to verify that the transformation $y_i = x_i - x_i^*$ transforms system (1) into the following system:

$$\dot{y}(t) = [-Ay(t) + Bg(y(t)) + Wg\left(y(t-\tau(t))\right) + C\int_{t-h}^{t} g(y(s))ds]$$
$$+\sigma\left(t, y(t), y(t-\tau(t))\right) dw(t),\ t \neq t_k \quad \dots (3)$$
$$\Delta(y_k) = I_k y(t_k), t = t_k, k = 1,2,\dots$$

where $y = (y_1, y_2, \dots, y_n)^T$, $A = diag(a_1, a_2, \dots, a_n)$, $B = [b_{ij}]$, $W = [w_{ij}]$, $C = [c_{ij}]$, $g(y) = [g_1(y_1), g_2(y_2), \dots, g_n(y_n)]^T$, with $g_j(y_j(t)) = f_j(y_j(t) + x_j^*) - f_j(x_j^*)$. Let $C^{2,1}(\mathbb{R}^n \times \mathbb{R}_+ : \mathbb{R}_+)$ denote the family of all nonnegative functions $V(y,t)$ on $\mathbb{R}^n \times \mathbb{R}_+$ which are continuously twice differentiable in y and once differentiable in t. For each values of V, we define an operator $\mathcal{L}V$ associated with delayed stochastic recurrent neural networks as

$$\mathcal{L}V(y(t),t) = V_t(y,t) + V_y(y,t)[-Ay(t) + Bg(y(t)) + Wg\left(y(t-\tau(t))\right)$$
$$+ C\int_{t-h}^{t} g(y(s))ds] + \frac{1}{2}trace[\sigma^T V_{yy}\sigma]$$

where $V_t(y,t) = \frac{\partial V(y,t)}{\partial t}$, $V_y(y,t) = \left(\frac{\partial V(y,t)}{\partial_{y_1}}, \frac{\partial V(y,t)}{\partial_{y_2}}, \dots, \frac{\partial V(y,t)}{\partial_{y_n}}\right)$

and $V_{yy}(y,t) = \left(\frac{\partial^2 V(y,t)}{\partial_{y_i}\partial_{y_j}}\right)_{n\times n}$, i,j=1,2,…,n.

Lemma 2.1 (Schur Complement[2]). The following LMI

$$\begin{bmatrix} Q(x) & S(x) \\ S^T(x) & R(x) \end{bmatrix} > 0$$

where $(x) = Q^T(x)$, $R(x) = R^T(x)$ and $S(x)$ depend on x, is equivalent to $R(x) > 0$, $Q(x) - S(x)R^{-1}(x)S^T(x) > 0$.

Lemma 2.2 For any constant symmetric matrix 1. , 1. $M = M^T$;, scalar 1. γ ;,

$$\gamma \int_0^{\gamma} w^T(s)Mw(s)ds \geq [\int_0^{\gamma} w(s)ds]^T M [\int_0^{\gamma} w(s)ds]$$

3. GLOBAL STABILITY RESULTS

In this section, we establish the global asymptotic stability of the addressed network by using linear matrix inequality method and Lyapunov functional technique.

Theorem 3.1 If there exists a positive scalar $\delta > 0$ and positive definite matrices $P > 0,\ R > 0$, positive diagonal matrices $Q_1 > 0, Q_2 > 0$, such that the following linear matrix inequalities hold:

$$P < \delta I \qquad \text{.... (4)}$$

$$I_k P I_k - P I_k - I_k P < 0 \qquad \text{.... (5)}$$

$$\Omega = \begin{bmatrix} \Phi_1 & PW & PC & 0 \\ * & -(1-\mu)Q_1 & 0 & 0 \\ * & * & -R & 0 \\ * & * & * & -(1-\mu)Q_2 + \delta F_1 \end{bmatrix} < 0 \qquad \text{.... (6)}$$

where $\Phi_1 = -PA - A^T P + \delta F_0 + 2PB\Sigma + \Sigma Q_1 \Sigma + Q_2 + h\Sigma R\Sigma, \Sigma = diag\{\sigma_1, \sigma_2, \dots, \sigma_n\} > 0$.

Then system (3) is globally asymptotically stable.

Proof. We use the following Lyapunov function to derive the stability result

$$V_1(y(t),t) = y^T(t)Py(t),$$

$$V_2(y(t),t) = \int_{t-\tau(t)}^{t} g^T(y(s))Q_1 g(y(s))ds,$$

$$V_3(y(t),t) = \int_{t-\tau(t)}^{t} y^T(s)Q_2 y(s)ds,$$

$$V_4(y(t),t) = \int_{-h}^{0}\int_{t+s}^{t} g^T(y(s))Rg(y(s))d\eta\, ds,$$

By using Ito's differential formula, the time derivative of V_1, V_2, V_3, V_4 can be calculated as

$$\mathcal{L}V_1 = 2y^T(t)P[-Ay(t) + Bg(y(t)) + Wg\left(y(t-\tau(t))\right) + C\int_{t-h}^{t} g(y(s))ds]$$

$$+\text{trace } [\sigma^T\left(t, y(t), y(t-\tau(t))\right)P\sigma\left(t, y(t), y\left(t-\tau(t)\right)\right)]$$

$$\mathcal{L}V_2 = g^T(y(t))Q_1 g(y(t)) - (1-\mu)g^T\left(y(t-\tau(t))\right)Q_1\, g(y(t-\tau(t)))$$

$$\mathcal{L}V_3 = y^T(t)Q_2 y(t) - (1-\mu)y^T(t-\tau(t))Q_2 y(t-\tau(t))$$

$$\mathcal{L}V_4 = hg^T(y(t))Rg(y(t)) - \int_{t-h}^{t} g^T(y(s))Rg(y(s))ds$$

$$\leq hg^T(y(t))Rg(y(t)) - \left(\int_{t-h}^{t} g(y(s))\,ds\right)^T R\left(\int_{t-h}^{t} g(y(s))\,ds\right)$$

When $t \neq t_k$, we have

$$\mathcal{L}V = \mathcal{L}V_1 + \mathcal{L}V_2 + \mathcal{L}V_3 + \mathcal{L}V_4$$

$$\begin{aligned}\mathcal{L}V \leq\ & y^T(t)[-PA - A^TP]y(t) + y^T(t)2PBg(y(t)) + y^T(t)2PWg\left(y(t-\tau(t))\right) \\ & + y^T(t)2PC\int_{t-h}^{t} g(y(s))\,ds + y^T(t)(\delta F_0)y(t) + y^T(t-\tau(t))(\delta F_1)y(t-\tau(t)) \\ & + g^T(y(t))Q_1g(y(t)) - (1-\mu)g^T\left(y(t-\tau(t))\right)Q_1\,g\left(y(t-\tau(t))\right) \\ & + y^T(t)Q_2y(t) - (1-\mu)y^T(t-\tau(t))Q_2y(t-\tau(t)) + hg^T(y(t))Rg(y(t)) \\ & - \left(\int_{t-h}^{t} g(y(s))\,ds\right)^T R\left(\int_{t-h}^{t} g(y(s))\,ds\right) \\ \leq\ & y^T(t)[-PA - A^TP + \delta F_0 + 2PB\Sigma + \Sigma Q_1\Sigma + Q_2 + h\Sigma R\Sigma]y(t) + y^T(t)2PW \\ & \times g\left(y(t-\tau(t))\right) + y^T(t)2PC\left(\int_{t-h}^{t} g(y(s))\,ds\right) + y^T(t-\tau(t)) \\ & \times[-(1-\mu)Q_2 + \delta F_1]\,y(t-\tau(t)) + g^T\left(y(t-\tau(t))\right)[-(1-\mu)Q_1] \\ & \times g\left(y(t-\tau(t))\right) - \left(\int_{t-h}^{t} g(y(s))\,ds\right)^T R\left(\int_{t-h}^{t} g(y(s))ds\right) \\ =\ & \Xi^T(t)\,\Omega\,\Xi(t) \qquad \text{…. (7)}\end{aligned}$$

where $\Xi^T(t) = [y^T(t)\ \ g^T\left(y(t-\tau(t))\right)\left(\int_{t-h}^{t} g(y(s))\,ds\right)^T\ \ y^T(t-\tau(t))]$

Thus, for ensuring negativity of $\mathcal{L}V$ for any possible state, it suffices to require Ω be a negative definite matrix. From (7), $\mathcal{L}V \leq 0$. $\mathcal{L}V = 0$, if and only if $(y(t), t) = 0$.

When $t = t_k$, we have

$$\begin{aligned}V(y(t_k^+), t_k^+) =\ & y^T(t_k)(E_0 - I_k)^TP(E_0 - I_k)y(t_k) + \int_{t_{k^+} - \tau(t_{k^+})}^{t_{k^+}} g^T(y(s))Q_1g(y(s))ds \\ & + \int_{t_{k^+}-\tau(t_{k^+})}^{t_{k^+}} y^T(s)Q_2y(s)ds + \int_{-h}^{0}\int_{t_{k^+}-\tau(t_{k^+})}^{t_{k^+}} g^T(y(s))Rg(y(s))d\eta\ ds \\ =\ & y^T(t_k)Py(t_k) - y^T(t_k)PI_ky(t_k) - y^T(t_k)I_k^TPy(t_k) + y^T(t_k)I_k^TPI_ky(t_k) \\ & + \int_{t_{k-\tau(t_k)}}^{t_k} g^T(y(s))Q_1g(y(s))ds + \int_{t_{k-\tau(t_k)}}^{t_k} y^T(s)Q_2y(s))ds \\ & + \int_{-h}^{0}\int_{t_{k-\tau(t_k)}}^{t_k} g^T(y(s))Rg(y(s))ds \\ =\ & V(y(t_k), t_k) + y^T(t_k)[I_kPI_k - PI_k - I_kP]y(t_k)\end{aligned}$$

Based on the Lyapunov-krasovskii stability theorem, the impulsive neural network (3) is globally asymptotically stable.

Theorem 3.2 If there exists a positive scalar $\delta > 0$, positive definite matrices $P > 0,\ R > 0$ and positive diagonal matrices $D > 0,\ \ Q_1 > 0, Q_2 > 0$ such that the following linear matrix inequalities hold:

$$P + d_0\Sigma < \delta I \qquad \text{…. (8)}$$

$$I_kPI_k - PI_k - I_kP < 0 \qquad \text{…. (9)}$$

$$\Omega_1 = \begin{bmatrix} \Phi_2 & PB & PW & PC & 0 \\ * & \Phi_3 & DW & DC & 0 \\ * & * & -(1-\mu)Q_1 & 0 & 0 \\ * & * & * & -R & 0 \\ * & * & * & * & -(1-\mu)Q_2+\delta F_1 \end{bmatrix} < 0 \quad \text{.... (10)}$$

where $\Phi_2 = -PA - A^T P + \delta F_0 + Q_2,$ $\Phi_3 = DB + B^T D + Q_1 - 2DA\Sigma^{-1} + hR,$
$\Sigma = diag\{\sigma_1, \sigma_2, \ldots, \sigma_n\} > 0, D = diag(d_i), d_0 = \sum_{i=1}^{n} d_i.$

Then the dynamics of the delayed stochastic impulsive neural network (3) is globally asymptotically stable.

Proof. Let us consider the Lyapunov function

$$V_1(y(t),t) = y^T(t)Py(t), \quad V_2(y(t),t) = 2\sum_{i=1}^{n} d_i \int_0^{y_i(t)} g_i(\alpha)\,d\alpha$$

$$V_3(y(t),t) = \int_{t-\tau(t)}^{t} g^T\big(y(s)\big)Q_1 g(y(s))ds, \quad V_4(y(t),t) = \int_{t-\tau(t)}^{t} y^T(s)Q_2 y(s)ds,$$

$$V_5(y(t),t) = \int_{-h}^{0}\int_{t+s}^{t} g^T\big(y(s)\big)Rg(y(s))d\eta\, ds,$$

By Ito's formula, the derivatives of $V(y(t),t)$ is worked out as

$$\mathcal{L}V_1 = 2y^T(t)P[-Ay(t) + Bg\big(y(t)\big) + Wg\left(y\big(t-\tau(t)\big)\right) + C\int_{t-h}^{t} g(y(s))ds]$$

$$+\text{trace}\,[\sigma^T\big(t, y(t), y(t-\tau(t))\big)P\sigma\left(t, y(t), y\big(t-\tau(t)\big)\right)]$$

$$\mathcal{L}V_2 = 2g^T\big(y(t)\big)D[-Ay(t) + Bg\big(y(t)\big) + Wg\left(y\big(t-\tau(t)\big)\right) + C\int_{t-h}^{t} g(y(s))ds]$$

$$+\text{trace}\,[\sigma^T\big(t, y(t), y(t-\tau(t))\big)DL\sigma\left(t, y(t), y\big(t-\tau(t)\big)\right)]$$

$$\mathcal{L}V_3 = g^T\big(y(t)\big)Q_1 g\big(y(t)\big) - (1-\mu)g^T\left(y\big(t-\tau(t)\big)\right)Q_1\, g(y(t-\tau(t)))$$

$$\mathcal{L}V_4 = y^T(t)Q_2 y(t) - (1-\mu)y^T\big(t-\tau(t)\big)Q_2 y(t-\tau(t))$$

$$\mathcal{L}V_5 \le hg^T\big(y(t)\big)Rg\big(y(t)\big) - \left(\int_{t-h}^{t} g(y(s))\,ds\right)^T R\left(\int_{t-h}^{t} g(y(s))\,ds\right)$$

Therefore,

$$\begin{aligned}\mathcal{L}V \le\; & y^T(t)[-PA - A^T P + \delta F_0 + Q_2]y(t) + y^T(t)2PBg\big(y(t)\big) + y^T(t) \\ & \times 2PWg\left(y\big(t-\tau(t)\big)\right) + y^T(t)2PC\left(\int_{t-h}^{t} g(y(s))\,ds\right) - g^T\big(y(t)\big) \\ & \times 2DA(y(t) - \Sigma^{-1}g\big(y(t)\big)) - g^T\big(y(t)\big)2DA\Sigma^{-1}g\big(y(t)\big) + g^T\big(y(t)\big) \\ & \times [DB + B^T D + Q_1 + hR]\, g\big(y(t)\big) + g^T\big(y(t)\big)2DWg\left(y\big(t-\tau(t)\big)\right) \\ & + g^T\big(y(t)\big)2DC\left(\int_{t-h}^{t} g(y(s))\,ds\right) + g^T\left(y\big(t-\tau(t)\big)\right)[-(1-\mu)Q_1] \\ & \times g\left(y\big(t-\tau(t)\big)\right) - \left(\int_{t-h}^{t} g(y(s))\,ds\right)^T R\left(\int_{t-h}^{t} g(y(s))\,ds\right)\end{aligned}$$

$$+y^T(t-\tau(t))[-(1-\mu)Q_2+\delta F_1]\,y(t-\tau(t))$$

$$=\Psi^T(t)\,\Omega_1\Psi(t)-g^T(y(t))2DA(y(t)-\Sigma^{-1})\,g(y(t)) \qquad \text{…. (11)}$$

where $\Psi^T(t)=[y^T(t)\;g^T(y(t))\;g^T\left(y(t-\tau(t))\right)\;\left(\int_{t-h}^{t}g(y(s))\,ds\right)^T\;y^T(t-\tau(t))]$

From (11), $\mathcal{L}V\le 0$. $\mathcal{L}V=0$, if and only if $(y(t),t)=0$. When $t=t_k$, arguing similar to the proof of Theorem 1, we can show that the system (3) is globally asymptotically stable.

4. NUMERICAL EXAMPLES

Now, we give two numerical examples to illustrate the effectiveness of our main results.

Example 4.1 Consider the following two-neuron stochastic recurrent neural networks with two time-varying delays and impulses:

$$\dot{y}(t)=[-Ay(t)+Bg(y(t))+Wg\left(y(t-\tau(t))\right)+C\int_{t-h}^{t}g(y(s))ds]$$
$$+\sigma\left(t,y(t),y(t-\tau(t))\right)dw(t),\ t\neq t_k \qquad \text{…. (12)}$$
$$\Delta y(t_k)=I_k y(t_k^-),t=t_k\,,k=1,2,\ldots$$

where the activation function is described by $f_i(x)=tanhx$, $\tau=0.5$, $h=0.5$, and the delayed feedback matrices A,B,W and C are

$$A=\begin{bmatrix}6&0\\0&6\end{bmatrix},\ B=\begin{bmatrix}0.3&-0.2\\-0.2&0.1\end{bmatrix},\ W=\begin{bmatrix}0.3&0\\0.7&-0.5\end{bmatrix},\ C=\begin{bmatrix}0.5&0.4\\0.3&-0.5\end{bmatrix},$$
$$F_0=F_1=\begin{bmatrix}0.5&0\\0&0.62\end{bmatrix},$$

Clearly the activation function satisfies the Assumption 1, with

$$I_k=I=\begin{bmatrix}0.2&0\\0&0.2\end{bmatrix}$$

Solving the LMI in Theorem 3.1, a following feasible solution is obtained by using LMI toolbox

$$P=\begin{bmatrix}1.6516&-0.0552\\-0.0552&1.7333\end{bmatrix},\ Q_1=\begin{bmatrix}5.5397&0\\0&5.3657\end{bmatrix},\ Q_2=\begin{bmatrix}9.6057&0\\0&10.4748\end{bmatrix},$$
$$R=\begin{bmatrix}4.1762&0.0107\\0.0107&4.1975\end{bmatrix},\ \delta=28.7508$$

The obtained result shows that the delayed stochastic recurrent neural network (12) with impulsive effect is globally asymptotically stable.

Example 4.2 For the system (12) the delayed feedback matrices are described as:

$$A=\begin{bmatrix}5&0\\0&5\end{bmatrix},\ B=\begin{bmatrix}0.3&-0.2\\-0.2&0.1\end{bmatrix},\ W=\begin{bmatrix}0.5&0.2\\0&-0.5\end{bmatrix},\ C=\begin{bmatrix}0.6&0.5\\0.4&-0.6\end{bmatrix},$$
$$F_0=F_1=\begin{bmatrix}0.5&0\\0&0.32\end{bmatrix}$$

Clearly the activation function satisfies the Assumption 1, with τ, h are defined as in Example 4.1 and

$$I_k=I=\begin{bmatrix}0.5&0\\0&0.5\end{bmatrix}$$

Solving the LMI in Theorem 3.2, a following feasible solution is obtained by using LMI toolbox

$$P = \begin{bmatrix} 2.1064 & -0.0040 \\ -0.0040 & 1.8435 \end{bmatrix},\quad Q_1 = \begin{bmatrix} 6.6938 & 0 \\ 0 & 6.6750 \end{bmatrix},\quad Q_2 = \begin{bmatrix} 11.7509 & 0 \\ 0 & 10.2465 \end{bmatrix},$$

$$R = \begin{bmatrix} 5.5279 & 0.1260 \\ 0.1260 & 5.5395 \end{bmatrix},\quad D = \begin{bmatrix} 0.7687 & 0 \\ 0 & 0.7491 \end{bmatrix},\quad \delta = 14.6791$$

The obtained result shows that the delayed stochastic recurrent neural network (12) with impulsive effect is globally asymptotically stable.

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