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Torus Embedded Hypercube Interconnection Network: A Comparative Study

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ABSTRACT
A design analysis and comparison of a product network generated from torus and hypercube networks known as torus embedded hypercube scalable interconnection network suitable for parallel computers is presented in this paper. It is shown here that with minor modifications in architecture of the existing mesh embedded hypercube interconnection network how good a torus embedded hypercube interconnection network could be. Also it has been proved with the computational results that the torus embedded hypercube interconnection network is highly scalable and more efficient in terms of communication.

Categories and Subject Descriptors
C.2.1 [**Computer Systems Organization**]: Computer-Communication Networks – Network Architecture and Design

General Terms
Performance, Design

Keywords
Hypercube network, Torus network, Mesh embedded hypercube network, Scalability, Network parameters.

1.INTRODUCTION
The effectiveness of parallel computers is often determined by its communication network. The interconnection network is an important component of a parallel processing system. A good interconnection network should have less topological network cost and meanwhile keep the network diameter as shorter as possible [2].

The mesh is a network with constant node degree in its internal nodes where as torus network has constant node degree with all its nodes [1]-[3]. The advantages of torus network can be imposed on to the mesh embedded hypercube network [4], [5] to give rise to an embedded architecture [6]-[8] called torus embedded hypercube scalable interconnection network and hence an architectural enhancement of mesh embedded hypercube network.

In this paper, we have described about the torus embedded hypercube network [9]. Also it has been proved how efficient a torus embedded hypercube interconnection network compared to the existing mesh embedded hypercube interconnection network.

2.ARCHITECTURAL PROPERTIES
Let $l \times m$ be the size of several concurrent torus networks and N be the number of nodes connected in hypercube configuration in it. Several hypercube connections can be derived from such group of N nodes within the torus configuration [9], [10] as shown in Figure 1. With this torus embedded hypercube network can be labeled as a network of size (l, m, N).

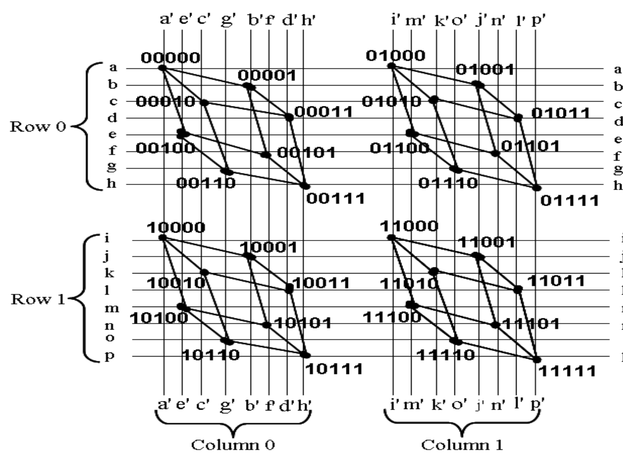

Figure 1. A (2, 2, 8) - Torus embedded hypercube network

Each node in the network can be addressed with three components; row number i and column number j of torus appended with the address of node k of hypercube. Hence, a (l, m, N)–torus embedded hypercube network will have $l \times m \times N$ number of nodes and a node will be addressed as (i, j, k) where $0 \leq i < l$, $0 \leq j < m$ and $0 \leq k < N$.

Combining the data routing functions of torus and hypercube networks to provide with the routing functions of the torus embedded hypercube [4] as

$$T_{h_1}(i,j,k) = (i,(j+1)\, mod\, m, k) \quad (1)$$

$$T_{h_2}(i,j,k) = (i,(m+j-1)\, mod\, m, k) \quad (2)$$

$$T_{h_3}(i,j,k) = ((i+1)\, mod\, l, j, k) \quad (3)$$

$$T_{h_4}(i,j,k) = ((l+i-1)\, mod\, l, j, k) \quad (4)$$

$$T_{C_d}(k_{n\text{-}1}..k_{d+1}\, k_d\, k_{d-1}\, ..k_0) = (k_{n\text{-}1}..k_{d+1}\, \overline{k_d}\, k_{d-1}..k_0) \quad (5)$$

We provide more explanation for the above discussion in the APPENDIX with a torus of size 2 × 2 and a 3-cube hypercube and derive a (2, 2, 8)-torus embedded hypercube network by combining them as in Figure 1 and in (1) - (5).

In Figure1, the ring connections of row/column of each torus are not shown for simplicity and without that the network will be a (2, 2, 8)-mesh embedded hypercube network. A wraparound connection is done along each row/column of the mesh if they have same label in Figure1 to deduce it to (2, 2, 8)-torus embedded hypercube network.

Scalability of a network [4], [5] is defined as the property by which the size of the system can be expanded with nominal changes in the existing configuration provided that system expansion results with improvement in performance. The torus embedded hypercube network is highly scalable network.

3.COMPARISON OF RESULTS AND DISCUSSION

For the analysis and comparison purpose we have considered network metrics such as Network Diameter and Topological Network cost. The diameter of a network can be defined as maximum number of hops an average message takes to reach to its destination [4], [9], [10].

The topological cost of a network depends on its node degree and diameter. A network with less node degree usually will have a large diameter, and a network with fewer diameters will possess larger node degree. Consequently, a network with large degree contains a large number of links while a network with low degree contains a small number of links [4].

3.1Network Diameter Analysis

If the diameter is too large, it implies that a large number of nodes

Table 1. Comparison of Network Diameter

Network type \ No. of processors	512	1024	2048	4096	8192	16384
(16,16,N)– Mesh embedded Hypercube	31 N=2	32 N=4	33 N=8	34 N=16	35 N=32	36 N=64
(l,m,16) - Mesh embedded Hypercube	14	18	26	34	48	66
(16,16,N)–Torus embedded Hypercube	17 N=2	18 N=4	19 N=8	20 N=16	21 N=32	22 N=64
(l,m,16) - Torus embedded Hypercube	10	12	16	20	26	36

will have to be busy to get connected to the destination node. In other words, the message from source to destination will have to pass through larger number of intermediate nodes. This in turn brings down the performance of the whole system and hence system may slow down.

In the results given in Table 1 and Figure 2, as far as the network diameter is concerned, the torus embedded hypercube network needs lesser network diameter to get connected between a source node and a destination node. Hence the torus embedded hypercube network is much superior than mesh embedded hypercube network as far as performance metrics is concerned and much faster than an equivalent mesh embedded hypercube network.

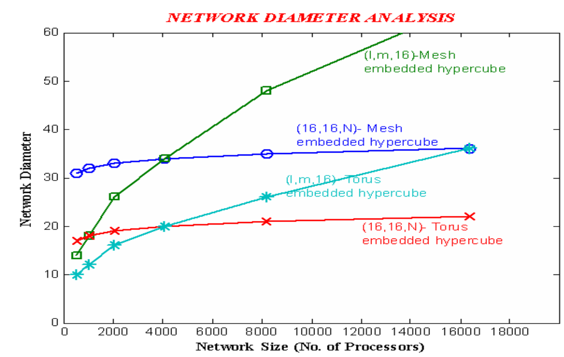

Figure 2. Network diameter analysis

3.2Network Cost Analysis

The topological network cost analysis result is given in Table 2 and Figure 3.

Table 2. Comparison of Topological Network Cost

Network type \ No. of processors	512	1024	2048	4096	8192	16384
(16,16,N)– Mesh embedded Hypercube	155 N=2	192 N=4	231 N=8	272 N=16	315 N=32	360 N=64
(l,m,16) - Mesh embedded Hypercube	112	144	208	272	384	528
(16,16,N)–Torus embedded Hypercube	85 N=2	108 N=4	133 N=8	160 N=16	189 N=32	220 N=64
(l,m,16) - Torus embedded Hypercube	80	96	128	160	208	288

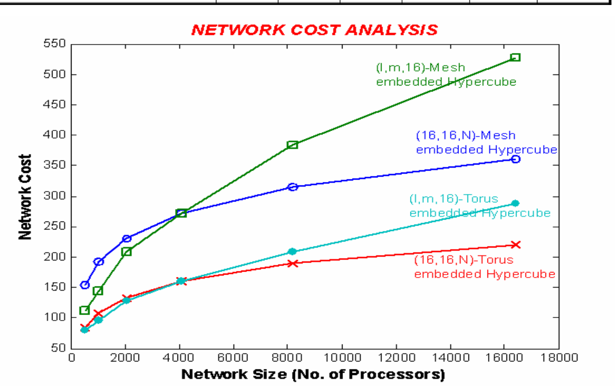

Figure 3. Network cost analysis

Topology is the study of connectivity and continuity. It is seen that the torus embedded hypercube network will have low network cost. Though the network diameter is found to be increasing in (l, m, 16)-torus embedded hypercube network, it has to be noted that the torus embedded hypercube has better values for network cost as the system is scaled up.

4.CONCLUSION

We have analyzed a torus embedded hypercube interconnection network and compared its network parameters with mesh embedded hypercube interconnection network for a parallel architecture. Network metrics such as network diameter and topological network cost are considered since they are the most important parameters to justify the efficiency of the network.

It is necessary to come up with a network that is scalable, minimum network diameter and a minimum topological cost. All afore mentioned requirements are met by the torus embedded hypercube network and hence it can supersede the mesh embedded hypercube network. The results show that torus embedded hypercube network is much faster than the mesh embedded hypercube in terms of communication. Hence this network could be chosen as interconnection network for parallel architecture.

5.APPENDIX

To provide the basic principles, we have considered simple torus and hypercube networks with their data routing functions to show the connectivity among the nodes.

The data routing functions as in (1.a) to (1.d) of torus network [1] are

$$T_1(i,\ j) = (i,\ (\ j+1)\ mod\ m) \qquad (1.a)$$

$$T_2(i,\ j) = (i,\ (m + j - 1)\ mod\ m) \qquad (1.b)$$

$$T_3(i,\ j) = ((i+1)\ mod\ l,\ j) \qquad (1.c)$$

$$T_4(i,\ j) = ((l + i - 1)\ mod\ l,\ j) \qquad (1.d)$$

where *i* and *j* are row and column numbers respectively.

According to these data routing functions, the various permutation cycles can be generated for a 2 × 2 torus network shown in Figure 4.

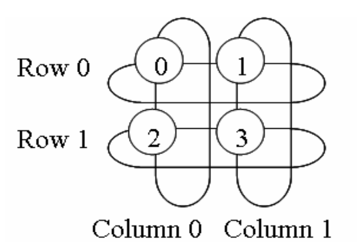

Figure 4. A 2 × 2 torus network

The data routing function as in (2.a) of hypercube network [1] is

$$C_d(k_{n-1}\dots k_{d+1}k_d k_{d-1}\dots k_0) = (k_{n-1}\dots k_{d+1}\overline{k_d}k_{d-1}\dots k_0)$$

for $d = 0, 1, \dots, n-1$ where k_j for (j = 0 to n-1) is the binary representation of node address k and n = log2(N) where N is the total number of nodes in the hypercube.

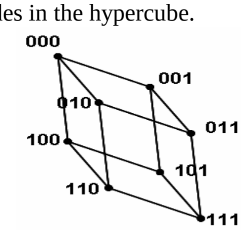

Figure 5. A 3-cube hypercube

According to the above hypercube data routing function the various permutation cycles can be generated for a 3-cube structured network shown in Figure 5.

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