Bending of Skewed Cylindrical Shell Panels

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ABSTRACT
In this paper a nine node isoparametric plate bending element has been used for bending analysis of isotropic skewed cylindrical shell panels. Both thick and thin shell panels have been solved. In the present analysis first order shear deformation theory has been incorporated. The analysis has been performed considering shallow shell method. Both shallow and moderately deep shells have been solved. Skewed cylindrical shell panels having different types of transverse loads, shell thickness (h/a = 0.01 and 0.1), length to curvature ratios (a/R), skewed angle, and boundary conditions have been analyzed following shallow shell method.

Keywords
Skewed shell panel, Shallow shell method, FSDT, FEM.

1.INTRODUCTION
The finite element method [1] is regarded as one of the most versatile analysis tools specifically in structural mechanics problems. The analysis of plates and shells are the first problems where the finite element method was first applied. The initial attempts were made with Kirchoff's hypothesis where a number of problems were faced. The major problem concerned the satisfaction of normal slope continuity at the element edges which could not be solved satisfactorily by this time. In the subsequent study, the above problem has been avoided by adopting Mindlin's hypothesis where the effect of shear deformation has been considered.

Literature on skewed isotropic shells is very limited. The spline finite element method [2] has been used to analyze the bending of skew plates subjected to transverse uniform load and concentrated load with arbitrary boundary conditions. Natural vibration of parallelogram cylindrical shell panels fixed along all the edges has been investigated [3] and presented. Linear static analysis of first order shear deformable plates of various shapes has been analyzed [4] using energy method and presented. Free vibration analysis of laminated composite skewed cylindrical shell panels has been studied [5] thoroughly considering different boundary conditions, thickness ratios, skewed angles and shallow ness ratios. Dynamic and static analysis of open cylindrical shell freely supported along curved edges and having different boundary conditions along straight edges has been analyzed [6]. An improved finite element method [7] has been presented for the linear analysis of anisotropic and laminated composite doubly curved, moderately thick shell panels. Both shallow and deep shells have been investigated. A numerical investigation [8] of free vibration of skewed open cylindrical shell panels have been studied and presented. Thin and moderately thick shells have been studied [8].

In the present work, static analysis of isotropic skewed cylindrical shell panels has been studied using the concept of shallow shell method.

2.FINITE ELEMENT FORMULATION
The formulation is based on shallow shell theory. The effect of shear deformation has been taken into account following the Mindlin's hypothesis where it has been assumed that the normal to the middle plane of the shell before bending remains straight but not necessarily normal to the middle plane of the shell after bending. Taking middle surface of the shell as the reference surface, the formulation has been carried out following the usual assumptions of linear elastic analysis. Element is used in the present work is the nine node isoperimetric element. One of the major advantages of the element is that any plate/shell shape can be nicely handled through a simple mapping technique which may be defined as $x = \sum_{r=1}^{9} N_r x_r$ and $y = \sum_{r=1}^{9} N_r y_r$ (1)

Where (x, y) are the co-ordinates of any point within the element, (x_r, y_r) are the co-ordinates of the r[th] nodal point and N_r is the corresponding interpolation function of the element. In this element Lagrangian interpolation function has been used for N_r [9]. Taking the bending rotations as independent field variables, the effect of shear deformation may be expressed as

$$\begin{Bmatrix} \phi_x \\ \phi_y \end{Bmatrix} = \begin{Bmatrix} \theta_x - \dfrac{\partial w}{\partial x} \\ \theta_y - \dfrac{\partial w}{\partial y} \end{Bmatrix}$$

Where, φ_x, and φ_y are the average shear rotation over the entire shell thickness and θ_x and θ_y are the total rotations in bending. The other independent field variables are u, v and w, where w is the transverse displacement while u and v are the corresponding in-plane displacements.

The interpolation functions used for the representation of element geometry, Eqns. (1) are used to express the displacement field at a point within the element in terms of nodal variables as

$$u = \sum_{r=1}^{9} N_r u_r, \;\; v = \sum_{r=1}^{9} N_r v_r, \;\; w = \sum_{r=1}^{9} N_r w_r,$$

$$\theta_x = \sum_{r=1}^{9} N_r \theta_r \; and \;\; \theta_y = \sum_{r=1}^{9} N_r \theta_y \qquad (2)$$

The generalized stress-strain relationship with respect to its reference plane may be expressed as

$$\{\sigma\} = [D]\{\varepsilon\} \qquad (3).$$

Where

$$\{\sigma\}^T = \begin{bmatrix} N_x & N_y & N_{xy} & M_x & M_y & M_{xy} & Q_x & Q_y \end{bmatrix} \qquad (4),$$

$$\{\varepsilon\} = \begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} + \frac{w}{R_y} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \\ -\frac{\partial \theta_x}{\partial x} \\ -\frac{\partial \theta_y}{\partial y} \\ -\frac{\partial \theta_x}{\partial y} - \frac{\partial \theta_y}{\partial x} \\ \frac{\partial w}{\partial x} - \theta_x \\ \frac{\partial w}{\partial y} - \theta_y \end{Bmatrix} \qquad (5)$$

and

$$[D] = \begin{bmatrix} D_{11} & D_{12} & 0 & 0 & 0 & 0 & 0 & \\ D_{21} & D_{22} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & D_{33} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & D_{44} & D_{45} & 0 & 0 & 0 \\ 0 & 0 & 0 & D_{54} & D_{55} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & D_{66} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & D_{77} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & D_{88} \end{bmatrix} \qquad (6).$$

Where

$$D_{11} = D_{22} = \frac{E}{1-\nu^2}, \;\; D_{12} = D_{21} = \nu D_{11}, \;\; D_{33} = \frac{E}{2(1+\nu)},$$

$$D_{44} = D_{55} = \frac{Eh^3}{12(1-\nu^2)}, \;\; D_{45} = D_{54} = \nu D_{44},$$

$$D_{66} = \frac{1-\nu}{2} D_{44}, \;\; D_{77} = D_{88} = \frac{Gh}{2K(1+\nu)}.$$

With the help of Eqns. (2) and (5) the strain vector may be expressed in terms of the nodal displacement vector {δ} as

$$\{\varepsilon\} = \sum_{r=1}^{9} \begin{bmatrix} \frac{\partial N_r}{\partial x} & 0 & 0 & 0 & 0 \\ 0 & \frac{\partial N_r}{\partial y} & \frac{1}{R_Y} & 0 & 0 \\ \frac{\partial N_r}{\partial y} & \frac{\partial N_r}{\partial x} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{\partial N_r}{\partial x} & 0 \\ 0 & 0 & 0 & 0 & -\frac{\partial N_r}{\partial y} \\ 0 & 0 & 0 & -\frac{\partial N_r}{\partial y} & -\frac{\partial N_r}{\partial x} \\ 0 & 0 & \frac{\partial N_r}{\partial x} & N_r & 0 \\ 0 & 0 & \frac{\partial N_r}{\partial y} & 0 & -N_r \end{bmatrix}$$

or $\{\varepsilon\} = \sum_{r=1}^{9} [B]_r \{\delta_r\}_e$ or, $\{\varepsilon\} = [B]\{\delta\}$ (7).

Where $[B]$ is the strain displacement matrix containing interpolation functions and their derivatives and $\{\delta\}$ is the nodal displacement vector.

Once the matrices $[B]$ and $[D]$ are obtained, the stiffness matrix of an element $[K]_e$ can be easily derived with the help of virtual work method which may be expressed as

$$[K]_e = \int_{-1}^{+1} \int_{-1}^{+1} [B]^T [D][B] |J| \, d\xi \, d\eta \qquad (8)$$

The degrees of freedom of the inclined edges have been transformed from global to local axes. This transformation has been done in element level. In the similar manner the load vector $\{P_e\}$ may be expressed as

$$\{P_e\} = \iint q[N]^T |J| \partial\xi \partial\eta \qquad (9)$$

The integration of the above Eqns. (8) and (9) has been carried out numerically following Guass quadrature rule.

The stiffness matrix and load vector having an order of forty five have been evaluated for all the elements and they have been assembled together to form the overall stiffness matrix $[K]$ and load vector $\{P\}$ respectively.

After incorporating the boundary conditions in the overall system of equations it has been solved to get the nodal displacements of the structure. Once nodal displacements have been obtained, the stresses at any point within an element can be evaluated with the above equations.

3.Results and Discussions

A simply supported isotropic skew plate as shown in Fig. 1 under uniform transverse distributed load with skew angle $\alpha = 30^0$ and 45^0, aspect ratio b/a= 1 and thickness ratio $h/2a$ =0.01 has been analyzed. The present non-dimensional deflection and principal bending moments with different mesh divisions have been given in Table 1 with those of Butalia et al. [10] and Sengupta [11]. Butalia et al. [10] have used an eight node isoparametric element whereas Sengupta [11] has used a 16-node triangular element and fourth order polynomial for transverse displacement. Present results are close to the published results. This example has been presented to validate the present formulation for skewed plate only.

In the next example a simply supported skewed cylindrical shell panel as shown in Fig. 2 subjected to uniform distributed load has been investigated. The study has been made considering the plan form of the skewed shell panel with thickness ratio h/a = 0.01 and b/a = 1.0. The deflection and principal bending moments have been presented in Table 2 with different values of shallowness ratios (a/R) and skewed angle (α) taking mesh size 20x20. Analytical and finite element solution [12] of a cylindrical shell panel has also been presented with the presented solution to validate the present formulation for shell analysis. It is seen that the present results are very close to the analytical solutions.

In the third example a skewed cylindrical shell panel having left edge free and other three edges simply supported subjected to transverse hydrostatic load (zero at left edge and maximum at the right edge) has been investigated. The analysis has been performed considering b/a = 1.0 and different values of thickness ratio h/a, shallowness ratios a/R and skewed angles. The deflection and principal bending moments of the skewed shell panel have been presented in Table 3 with mesh size 20x20. The present formulation for hydrostatic load has been validated for a plate problem.

A simply supported skewed cylindrical shell panel subjected to transverse doubly sinusoidal load has been investigated. To validate the present finite element formulation a rectangular plate has also been studied. The analysis has been performed considering different aspect and shallowness ratios with constant thickness ratio h/a = 0.01. The deflections and principal bending moments have been presented in Table 4 with the thin plate solution [13]. From the table it is seen that present plate solutions is very close to the analytical results.

In the last example a skewed cylindrical shell panel subjected to transverse concentrated load at the centre of the panel has been analyzed. The study has been performed considering different boundary conditions and skewed angle with constant thickness, aspect and shallowness ratios. The deflection at the centre of the panel has been presented in Table 5.

Table 1. Central deflection ($w^* = w100D/qa^4$) and bending moments ($M^*_{max} = 10M_{man}/qa^2$, $M^*_{min} = 10M_{min}/qa^2$) of a simply supported skew plate under uniform distributed load. b/a=1.0, h/2a=0.01, ν = 0.3.

Skew angle (α)	Sources	w^*	M^*_{max}	M_{min}
30^0	Present (12x12)	4.0990	1.8006	1.3922
	Present (16x16)	4.0998	1.7775	1.3785
	Present (20x20)	4.1000	1.7627	1.3693
	Butalia et al. [10]	3.9832	1.6790	1.2980
	Sengupta [11]	4.2824	1.7455	1.3670
45^0	Present (12x12)	2.0965	1.5043	0.9924
	Present (16x16)	2.1014	1.4757	0.9843
	Present (20x20)	2.1029	1.4526	0.9747
	Butalia et al. [10]	1.91125	1.2266	0.7803
	Sengupta [11]	2.2028	1.3258	0.9008

Table 2. Central deflection and bending moments of a simply supported skew plate/shell panels under uniform distributed load. h/a = 0.01, b/a = 1.0, ν = 0.3.

For cylindrical shell panel. a/R = 0.5					
a/R	**Sources**	**α**	**Deflection $10^5wEh^3/qa^4$**	**10^4 x M_x/qa**	**10^4 x M_x/qa**
0.5	Present	0	179.2	-13.172	5.689
	Exact [12]		179.0	-13.0	6.0
	FEM [12]		177	-12	5.0
For skewed cylindrical shell panel.					
			Deflection $w10^4D/qa^4$	**1000x M_{man}/qa^2**	**1000 x M_{min}/qa^2**
0.1	Present	15^0	22.168	284.46	240.46
		30^0	17.938	304.18	221.81
		45^0	11.008	30.217	194.12
0.25	Present	15^0	6.9777	7.7555	4.8852
		30^0	6.8677	11.108	5.7806
		45^0	5.9367	15.964	8.2589
0.5	Present	15^0	1.7745	1.2235	-0.9236
		30^0	2.0696	3.1624	0.0170
		45^0	2.2725	6.156	1.5196

Table 3. Central deflection ($w^* = w10^4D/qa^4$) and bending moments ($M^*_{max} = 1000M_{man}/qa^2$, $M^*_{min} = 1000M_{min}/qa^2$) of a

simply supported skew plate/shell panels under hydrostatic load. b/a=1.0, ν = 0.3.

a/R	h/a	Skew angle	Sources	w^*	M^*_{max}	M_{min}
0	0.01	0	Present	31.290	33.13	21.39
			T.S.[13]*	31.3	33.1	21.4
0.5	0.01	15	Present	10.946	13.404	6.032
		30	Present	5.8302	14.547	-3.516
		45	Present	4.2829	22.937	-8.985
0.5	0.1	15	Present	36.954	40.75	21.5
		30	Present	31.423	56.175	10.00
		45	Present	19.164	67.3	-7.137

*** T. S: Thin plate solution**

Table 4. Central deflection ($w^* = w10^4qa^4/D$) and bending moments ($M^*_{max} = 1000M_{man}qa^2$, $M^*_{min} = 1000M_{min}qa^2$) of a simply supported skew plate/shell panels under double sinusoidal load. h/a = 0.01, ν = 0.3.

b/a	α	a/R	Sources	w^*	M^*_{max}	M_{man}
1	0	0	Present	25.680	32.966	32.966
			T.S [13]	25.665	32.93	32.93
	30	0.1	Present	13.350	25.080	18.587
		0.3	Present	4.0453	8.5726	4.5509
		0.5	Present	1.7632	4.4555	1.7024
2	0	0	Present	65.730	69.857	35.711
			T. S [13]	65.702	69.709	35.665
	30	0.1	Present	40.238	57.470	26.774
		0.3	Present	19.845	28.337	13.256
		0.5	Present	9.9995	14.302	6.5374
3	30	0.1	Present	54.304	74.415	27.852
		0.3	Present	40.952	56.073	21.152
		0.5	Present	27.447	37.497	14.168

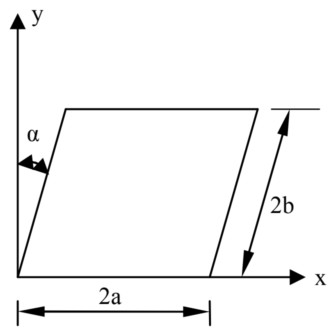

Figure 1. Skew plate.

Table 5. Central deflection ($w^* = w100D/Pa^2$) of skew plate/ shell panel under concentrated load at the centre of the panel. h/a = 0.01, b/a = 1.0 ν = 0.3.

Boundary Condition	a/R	Skewed angle	Sources	w^*
SSSS	0	15^0	Present	43.907
			Butalia et al. [10]	43.555
		30^0	Present	36.402
			Agarwal [14]	36.000
		45^0	Present	25.521
			Agarwal [14]	25.094
FFSC	0.5	15^0	Present	0.4894
FFCC		30^0	Present	0.2850
CFFF		45^0	Present	0.2189
		15^0	Present	0.1239
		30^0	Present	0.1153
		45^0	Present	0.1057
		15^0	Present	0.3374
		30^0	Present	0.3947
		45^0	Present	0.5388

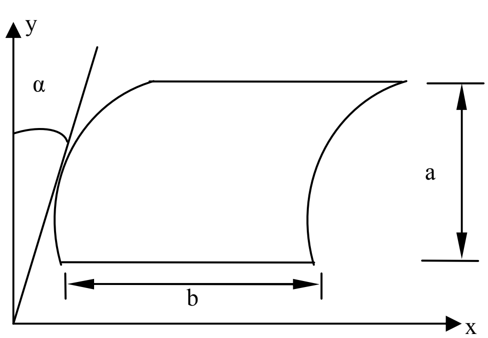

Fig. 3. Skew cylindrical shell panel

4.REFERENCES

[1] Zienkiewics, O. C., and Tailor, R. L. 1988. The finite element (two volumes). New York, McGraw- Hill.

[2] Mizusawa, T. 1994. Application of the spline element method to analyze the bending of skew plates. Comp. Struct. 53 (1994). 439-448.

[3] Belyanovski, E. S., and Pukhlii, V. A. 1981. Nature vibration of parallelogram cylindrical panels. Strength of Materials. 13 9 (September 1981). 1143-1147.

[4] Mizusawa, T., and Singh, A. V. 2004. A p-type solution for the bending of rectangular, circular, elliptic and skew plates. Int J Solids Struct. 41 (July 2004). 3977-3997.

[5] Haldar, S. 2008. Free vibration of composite skewed cylindrical shell panel by finite element method. J Sound and Vib. 311 (2008). 9-19.

[6] Selmane, A., and Lakis, A. A. 1997. Dynamic analysis of anisotropic open cylindrical shells. Comp. Struct. 62 (1997). 1-12.

[7] Hossain, S. J., Shina, P. K., and Sheikh, A. H. 2004. A finite element formulation for the analysis of laminated composite shells. Compt. Struct. 82 (2004). 1623-1638.

[8] Kandasamy, S., and Singh, A. V. 2006. Free vibration analysis of skewed open circular cylindrical shells. J Sound and Vib. 290 (2006). 1100-1118.

[9] Zienkiewicz, O. C. 1977. The finite element method, London. McGraw- Hill. 3th Edition.

[10] Butilia, T. S., Kant, T., and Dixit, V. D. 1990. Performance of heterosis element for bending of skew rhombic plates, Comp. Struct. 134 (1990). 23-49.

[11] Sengupta, D. 1995. Performance study of a simple finite element in the analysis of skewed rhombic plates. Comp. Struct. 54 (1995). 1173-1182.

[12] Qatu, M. S., and Algothani, A. 1994. Mending analysis of laminated composite plates and shells by different method. Comp. Struct. 52 (1994). 529-539.

[13] Timoshenko S. P., and Woinowsky, Krieger, S. 1970. Theory of plates and shells. Singapore. McGraw-Hill. 2nd. Edition.

[14] Agarwal B.D. 1996. Bending of rhombic plates, Quart. J. Mech. Appl. Math. 19 (1996). 79-82.