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Robust Sliding Mode Control for Systems with Noise and Unmodeled Dynamics based on Uncertainty and Disturbance Estimation (UDE)

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ABSTRACT
This paper relates generally to sliding mode control (SMC) system based on uncertainty and disturbance estimation (UDE) and more particularly to a system, with the presence of high frequency measurement noise and unmodeled dynamics. Higher order dynamics are difficult to identify and remain excluded from theoretical model of the system, generally the main cause of instability. The presence of unmodeled dynamics results in undesirable oscillations, affects overall stability and leads to limited control performance. Uncertainty and disturbance estimator is presented for estimating the perturbation. External low-pass filter is used to filter out the high frequency noise and UDE to reduce sensitivity to sensor noise and instability effect due to unmodeled dynamics up to certain limit. With the help of low-pass filter and UDE, the undesirable oscillations can be suppressed and the system stability can also be improved by proper selection of estimation filter time constant of uncertainty and disturbance estimator.

Categories and Subject Descriptors
J.7 Computers in other Systems – *Command and control*, *Industrial control*

General Terms
Algorithms, Design, Theory

Keywords
sliding mode control, uncertainty and disturbance estimation, noise, unmodeled dynamics.

1. INTRODUCTION
Control design of linear systems with noise and unmodeled constitutes one of the important design problem to be faced in practice. Considerable theoretical work has been devoted to systems with attention mainly focused on improvement in stability and robustness. Kutay and others [1] addressed adaptive feedback control of uncertain nonlinear systems with noisy output measurements, while Wang et al. [2] developed sensor noise model for Advanced Vehicle Control Systems (AVCS). How and Tillerson [3] analyzed the impact of sensor noise on formation flying control. Systematic procedure for design of robust model following sliding mode load frequency controller for single area power system, with parametric uncertainties and external disturbance based on UDE has been presented [4, 5, 6]. Despite the current state of technology, when it comes to sensing devices; information transmitted by the instrumentation and control equipments is often noisy i.e. sensor will have certain range of inaccuracy and propagations of the signal from sensing device to the control panel is likely to contain some noise. In that sense, data about the state of world will be uncertain; affecting both the interface contents and the ways operators will cope with the noise equipment. Shendge and Patre [20] investigated the impact of sensor noise with uncertainty and disturbance estimation for continuous sliding mode control, where UDE filter time constant play vital role. Dynamic systems often exhibit resonance properties, which are associated with higher order dynamics that are unnecessary and undesirable for proper operation. In many practical situations, the higher order dynamics are difficult to identify and remain excluded from the theoretical model of the system. When feedback control is applied to enhance operation of a dynamic system, the presence of higher order dynamics results in undesirable oscillations, affects overall stability and leads to limited control performance.

Different authors have handled unmodeled dynamics problem. Fu, Costa and Hsu [7, 8] developed some variable structure or switching control schemes to deal with a class of plants of unmodeled dynamics. Motivated with this work, Yan et al. [9] proposed a new variable structure robust model reference adaptive control scheme. For linear unmodeled dynamics, Krstic et al. [10] presented a redesign that ensures boundedness of the closed loop situations. Extensions to nonlinear unmodeled dynamics were made by Krstic and Kokotovic [11], Jiang and Marcels [12] where all these design require small gain condition on unmodeled dynamics. Considering the level of higher order dynamics contribution to the output of a dynamic system, two categories of control applications can be identified. In the first category, the effect of higher order dynamics under given operating condition exceeds acceptable errors in the output of the system; while the second category comprises dynamic output subject to control, remain within acceptable limits and therefore can be tolerated without sacrificing desired accuracy. Hosek et al. [13] proposed a patented observer corrector control system for system with unmodeled dynamics; deals with the first category. Canale and Milanese [14] gave necessary and sufficient condition on internal mode controllers for robust closed loop stability in presence of unmodeled dynamics and actuator saturation. This paper deals with the first category. It is directed to a method of reducing destabilizing effects of higher order dynamics in a controlled system. Feedback signal includes a signal component that represents the dominant dynamics in the output signal of the controlled system, where in destabilizing effect of unmodeled dynamics in the dynamic system output is reduced. Stability, a preliminary requirement for the closed loop design, a effort has been devoted to give conditions for a linear system in presence of unmodeled dynamics.

This paper is organized as follows: in the following section problem statement is defined. Section III presents analysis of system with noise and UDE, while controller transfer function is analyzed in Section IV. Controller transfer function and stability analysis is derived in Section IV and V. Numerical example is discussed in section VI, while section VII deals with simulation of second order linear system. Section VIII summarizes the conclusions.

2. PROBLEM STATEMENT

Figure 1 is schematic of a system with external disturbance, unmodeled dynamics and noise. As shown u_m denotes reference signal, signal u refers to control action, d is external disturbance, N stands for measurement noise.

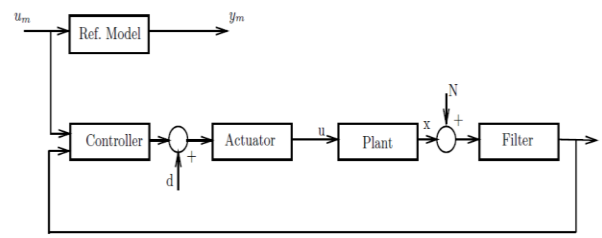

Figure 1. System with noise and unmodeled dynamics

Consider a single input, single output plant;

$$\dot{x} = A\,x(t) + b\,u(t) + \Delta A\,x(t) + \Delta b\,u(t) + d(x,t) \qquad (1)$$

$$y = C\,x(t) \qquad (2)$$

where A, b and C are known matrices, $\Delta A, \Delta b$ are uncertainties, $d(x,t)$ is an unmeasurable disturbance signal, $x(t)$ is the state vector and $u(t)$ is control input.

Assumption 1 *The uncertainties* $\Delta A, \Delta b$ *and the disturbance* $d(x,t)$ *satisfy the matching conditions given by:*

$$\Delta A = b\,D, \qquad \Delta b = b\,E, \qquad d(x,t) = b\,v(x,t)$$

where D and E are unknown matrices of appropriate dimensions and $v(x,t)$ *is an unknown function.*

The system (1) can now be written as described by,

$$\dot{x}(t) = A\,x(t) + b\,u(t) + b\,e(x,t) \qquad (3)$$

where $e(x,t) = D\,x + E\,u + v(x,t)$ and referred as lumped uncertainty in the system.

This system (3) with states noise is represented as,

$$\dot{x}(t) = A(x(t) + N) + b\,u(t) + b\,e(x,t) \qquad (4)$$

Let,

$$\dot{x}_m = A_m x_m + b_m u_m \qquad (5)$$

be a stable model satisfying the following conditions.

Assumption 2

$$A - A_m = b\,L, \qquad b_m = b\,M$$

where L and M are suitable known matrices. The objective is to design a control $'u'$ so as to force the plant (1) to follow the model (5) inspite of the parameter variations. The assumptions 1 and 2 are well known matching conditions required to guarantee invariance and are explicit statements of the structural constraints stated in [15].

Practical sensors itself have some noisy characteristics that carry with the measured signal causing uncertainty in the initial conditions. So the states of the plant, is represented as $x_n = x + N$ (instead of $'x'$) where $'N'$ is noise presented in the states i.e. sensor noise is considered for analysis.

3. UNCERTAINTY AND DISTURBANCE ESTIMATION(UDE) WITH SENSOR NOISE

Defining sliding plane equation [4, 6, 16, 17, 18]

$$\sigma = b^T x + z \qquad (6)$$

where $'z'$ is an auxiliary variable to be defined. The surface can be used for model following; if the auxiliary variable is defined as

$$\dot{z} = -b^T A_m x - b^T b_m u_m; \qquad z(0) = -b^T x(0) \qquad (7)$$

If the states are available, the control that ensures model following is derived in [4, 17]. Let the control be,

$$u = u_{eq} + u_n \qquad (8)$$

where u_{eq} caters to the known part of the system while u_n caters to unknown disturbances and structured uncertainties in the system.

$$\dot{\sigma} = b^T(Ax + AN + bu + be(x,t) - A_m(x+N) - b_m u_m) \qquad (9)$$

$$= b^T(bLx - bM\,u_m + b\,u_{eq} + b\,u_n + be(x,t) + bLN)$$

Selecting $\qquad u_{eq} = -(Lx - M\,u_m) - (b^T b)^{-1} K\sigma$

(10)where K is a positive constant

3.1 Estimation of uncertainties and disturbances with noise

$$\dot{\sigma} = b^T b\,u_n + b^T bLN + b^T be(x,t) - K\sigma$$

(11)

Next we use the idea of UDE to estimate the uncertainty $e(x,t)$ and use the estimate in u_n to negate $e(x,t)$

$$e(x,t) = (b^T b)^{-1}(\dot{\sigma} + K\sigma) - u_n - LN$$

(12)

The estimate of $e(x,t)$ can be obtained as explained before;

$$e_{est}(x,t) = e(x,t)\, G_f(s)$$

(13)

where $G_f(s) = 1/\tau s{+}1$

$$e_{est}(x,t) = [(b^T b)^{-1}(\dot{\sigma} + K\sigma) - u_n - LN]\, G_f(s)$$

(14)

As the controller is defined as,

$$u_n = -e_{est}(x,t)$$

(15)

Then, some simple calculation gives,

$$u_n = -\frac{(b^T b)^{-1}}{\tau}\left(\sigma + \frac{K\sigma}{s}\right) + \frac{LN}{\tau s}$$

(16)

4. TRANSFER FUNCTION OF THE CONTROLLER WITH UNMODELED DYNAMICS

Let the control be

$$u(s) = u_{eq}(s) + u_n(s)$$

(17)

where u_{eq} takes care of known terms and u_n caters for the uncertainty.

$$u_{eq}(s) = -Lx(s) + M u_m(s) - (b^T b)^{-1} K\sigma(s)$$

(18)

$$u_n(s) = \frac{(b^T b)^{-1}}{\tau}\left(\sigma(s) + \frac{K\sigma(s)}{s}\right)$$

(19)

As u_m does not contribute to this transfer function and from Eqn. (18) and (19)

$$u(s) = -Lx(s) - (b^T b)^{-1} K\sigma(s) + \frac{(b^T b)^{-1}}{\tau}\left(\sigma(s) + \frac{K\sigma(s)}{s}\right)$$

(20)

$$u(s) = -Lx(s) + (b^T b)^{-1}\left(K + \frac{1}{\tau} + \frac{K}{\tau s}\right)\sigma(s)$$

(21)

From (7)

$$Z(s)s \;=\; -b^T A_m\, x(s)$$

(22)

So from Eqn. (6) and (22)

$$\sigma(s) = b^T x(s) - \frac{b^T}{s} A_m\, x(s) = b^T\left(I_{nxn} - \frac{A_m}{s}\right)x(s)$$

(23)

Inserting (23) in (21) and correctly writing of scalar matrix multiplication

$$u(s) = \left[-L - \left(K + \frac{1}{\tau} + \frac{K}{\tau s}\right)(b^T b)^{-1}\, b^T\left(I_{nxn} - \frac{A_m}{s}\right)\right]x(s)$$

(24)

So the transfer function is

$$G_{ux}(s) = \left[-L - \frac{(K\tau s + s + 1)}{\tau s}(b^T b)^{-1}\, b^T\left(I_{nxn} - \frac{1}{s}A_m\right)\right]$$

(25)

where

$$A_m = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & & & & \vdots \\ -a_{m1} & -a_{m2} & \cdots & \cdots & -a_{mn} \end{bmatrix}; \quad L = [l_1 \quad l_2 \quad \cdots \quad l_n]$$

4.1 Design of Control

Let the control be

$$u(s) = u_{eq} + u_n$$

(26)

From (24) total control required is,

$$u(s) = \left[-L - \left(K + \frac{1}{\tau} + \frac{K}{\tau s}\right)(b^T b)^{-1}\, b^T\left(I_{nxn} - \frac{A_m}{s}\right)\right]x(s)$$

(27)

5. STABILITY ANALYSIS

The total resultant transfer function of Figure 1 can be written as,

$$G(s) = \frac{Z_1 Z_2 Z_4}{1 + Z_1 Z_2 Z_3 Z_4}$$

(28)

The transfer function of the actuator is $Z_1(s) = 1/\tau_1 s{+}1$, plant is $Z_2(s) = x_n(s)/u(s)$, while for low pass filter is $Z_3(s) = 1/\tau_2 s{+}1$. $Z_4(s)$ is the controller transfer function. As the stability of resulting system consisting of plant, actuator, filter, controller the following theorem will be presented as a sufficient condition for the stability of the overall system.

Theorem 1 *The overall system with uncertainty and disturbance estimator and unmodeled dynamics is stable; if the eigenvalues λ, of the following characteristic equation, lies in the left half of the s-plane.*

$$\left|\lambda_i(\phi_n)\right| < 0; \qquad \textit{where } \phi_n = (1 + Z_1 Z_2 Z_3 Z_4)$$

(29)

where $\lambda_i\,(i = 1,2,...n)$ are eigenvalues of the matrix ϕ_n. From $det(\lambda_i - \phi_n)$ is the polynomial.

6. NUMERICAL EXAMPLE

The plant in continuous time form is;

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \qquad \Delta A = \begin{bmatrix} 0 & 0 \\ -1 & -2 \end{bmatrix}, \quad \Delta b = \begin{bmatrix} 0 \\ -0.4 \end{bmatrix}$$

Disturbance is $d = 2\sin(t)x_1^2 + \cos(t)x_2 + 1$. The model to be followed is, $A_m = \begin{bmatrix} 0 & 1 \\ -4 & -2.8 \end{bmatrix}$, $b_m = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$

The initial conditions for plant and model is $x_0 = [1 \quad 0]^T$, $x_{m0} = [0 \quad 0]^T$ respectively and high frequency noise is added in the system states. The control gain K = 5. The reference input is a square wave with unity amplitude.

7. SIMULATIONS

Figure 2 – Figure 5 shows the different cases for the unmodeled dynamics effect on a system response in continuous sliding mode control. Figure 2 shows system with low unmodeled dynamics, which is stable. As dynamics increases system becomes unstable; can be easily verified with Figure 3. System can be made stable by decreasing control gain or increasing UDE filter time constant.

Case 1: For low unmodeled dynamics system is stable. UDE filter time constant τ = 1 msec. Actuator time constant τ_1 = 0.02 sec, filter time constant τ_2 = 1 msec, control gain K = 4.

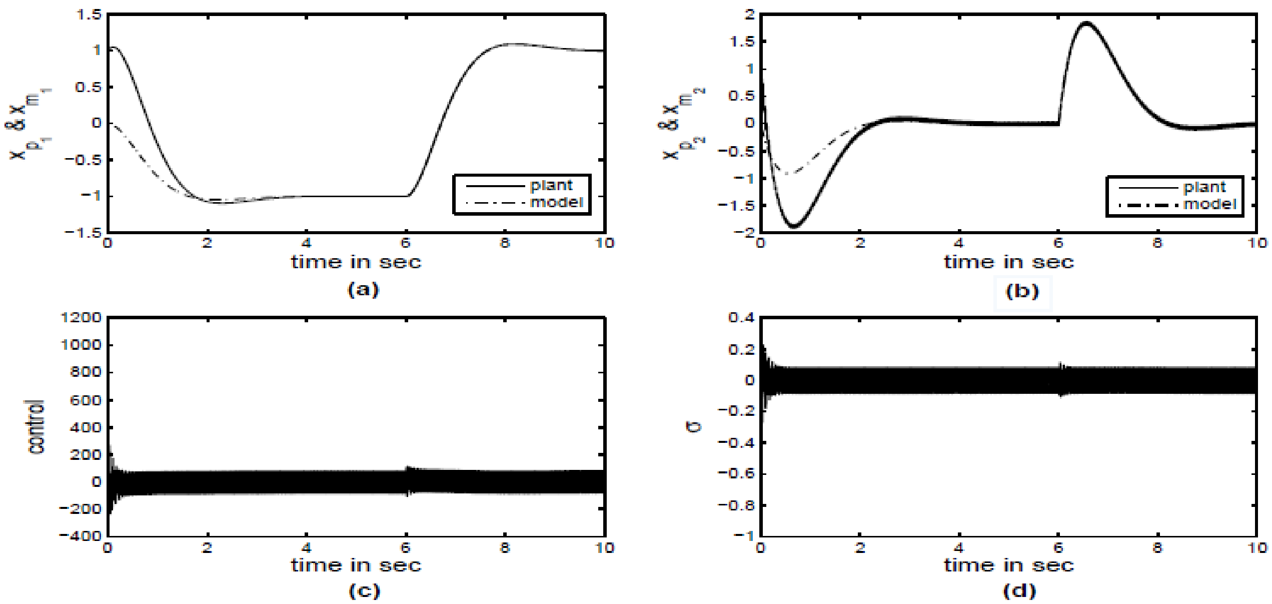

Figure 2. Response for Case 1 (a) plant and model state x_{p1} and x_{m1} (b) plant and model state x_{p2} and x_{m2} (c) system control (d) sliding variable

Case 2: System becomes unstable for high-unmodeled dynamics. UDE filter time constant τ = 1 msec. Actuator time constant τ_1 = 0.2 sec, filter time constant τ_2 = 1 msec, control gain K = 4.

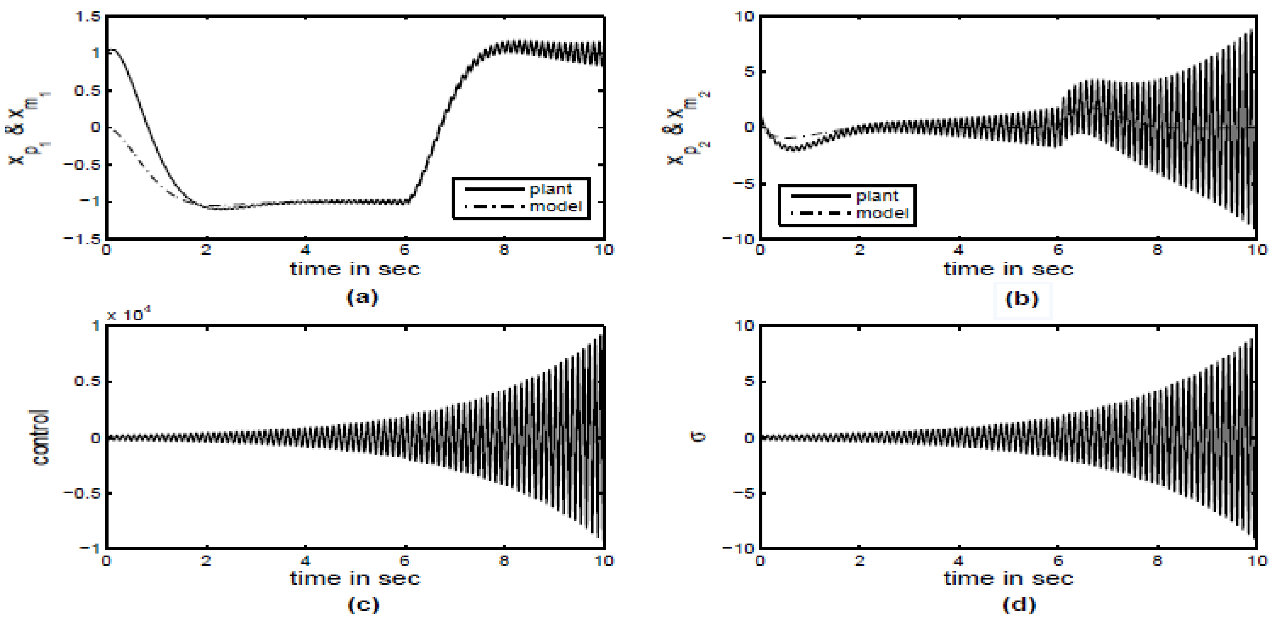

Figure 3. Response for Case 2 (a) plant and model state x_{p1} and x_{m1} (b) plant and model state x_{p2} and x_{m2} (c) system control (d) sliding variable

Case 3: Unstable system due to unmodeled dynamics can be make stable by decreasing control gain. UDE filter time constant $\tau = 1$ msec. Actuator time constant $\tau_1 = 0.2$ sec, filter time constant $\tau_2 = 1$ msec, control gain K = 1.

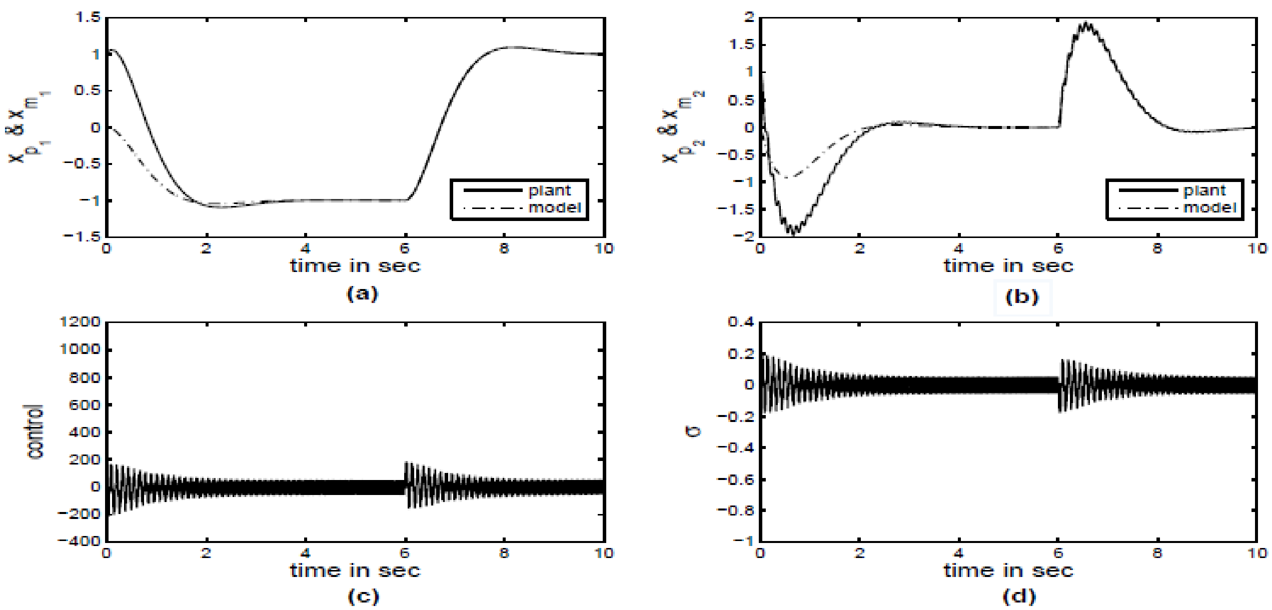

Figure 4. Response for Case 2 (a) plant and model state x_{p1} and x_{m1} (b) plant and model state x_{p2} and x_{m2} (c) system control (d) sliding variable

Case 4: Unstable system due to unmodeled dynamics can be make stable by increasing UDE filter time constant. UDE filter time constant $\tau = 2$ msec. Actuator time constant $\tau_1 = 0.2$ sec, filter time constant $\tau_2 = 1$ msec, control gain K = 4.

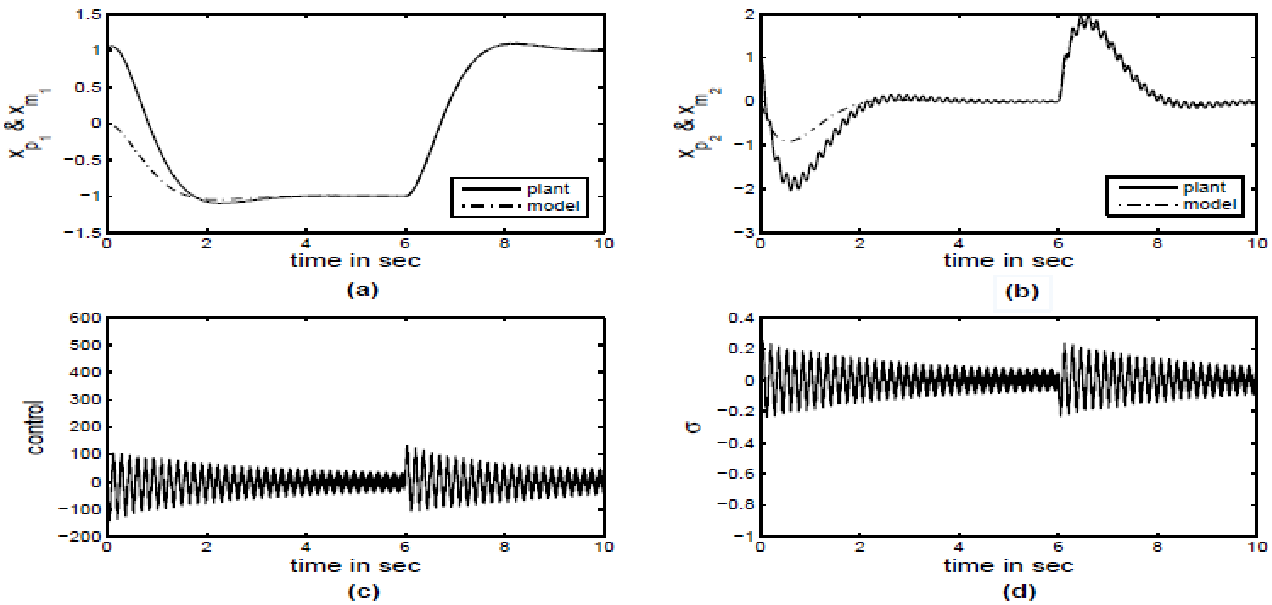

Figure 5. Response for Case 2 (a) plant and model state x_{p1} and x_{m1} (b) plant and model state x_{p2} and x_{m2} (c) system control (d) sliding variable

Table 1. Pole location with unmodeled dynamics for Case 1-4

Sr. No.	Case 1 10^{-3} x	Case 2 10^{-3} x	Case 3 10^{-3} x	Case 4 10^{-3} x
1	0	0	0	0
2	0	0	0	0
3	– 1.0298	– 1.0030	– 1.0030	– 1.0015
4	– 1.0000	– 1.0000	– 1.0000	– 1.0000
5	– 0.0087 + 0.1704i	0.0004 + 0.0547i	– 0.0011 + 0.0547i	– 0.0004 + 0.0387i
6	– 0.0087 – 0.1704i	0.0004 – 0.0547i	– 0.0011 – 0.0547i	– 0.0004 – 0.0387i
7	– 0.0500	– 0.0050	– 0.0005	– 0.0050
8	– 0.0040	– 0.0040	0.0037	– 0.0040
9	– 0.0037	– 0.0037	– 0.0014 + 0.0014i	0.0037
10	– 0.0014 + 0.0014i	– 0.0014 – 0.0014i	– 0.0014 – 0.0014i	– 0.0014 + 0.0014i
11	– 0.0014 – 0.0014i	– 0.001	– 0.0010	– 0.0014 – 0.0014i
12	– 0.0003	– 0.0003	– 0.0003	– 0.0003

8. CONCLUSION

In this paper, the problem of robust sliding mode control design, for a nonlinear system, in presence of noise and unmodeled dynamics has been investigated. It ensures better transient performance and robustness with respect to unmodeled dynamics. Operation of a dynamic system in the presence of unmodeled dynamics results in undesirable oscillations affects overall stability and leads to limited control performance. With the help of uncertainty and disturbance estimation; it is possible to overcome this dynamic effect with the help of UDE filter time constant. By increasing estimation filter time constant, dynamic effect can be minimized. The results presented here are for a linear control structure in presence of nonlinear perturbation and unmodeled dynamics. The error converges towards zero in finite time; even in the presence of unmodeled dynamics, with the UDE.

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