Optimizing Spectrum Sensing for Cognitive Radio Networks to Limit Interference on Primary User

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ABSTRACT
Cognitive radio (CR) is an effective way to improve the utilization of spectrum resource. Spectrum sensing is playing an important role for finding free channels to be used by CR. One of the most critical issues in spectrum sensing is the interference on primary user (PU). In this work, a general optimal voting rule is studied to minimize the Bayes risk function in cooperative spectrum sensing. Furthermore, an algorithm to optimize the energy detection threshold for the CR users for any fusion rule is presented. On the other hand, an algorithm that determines the optimum fusion rule and optimum threshold that minimizes the false alarm probability while the missing probability is under constraint (bounded) is suggested.

Keywords
Cognitive radio, cooperative spectrum sensing, Bayes risk function, optimization, energy detector.

1. INTRODUCTION
Recent years, with the rapid development of wireless communication technology, more and more spectrum resources are needed to support the high data rate. Spectrum scarcity becomes a problem. The Federal Communications Commission (FCC) Spectrum Policy Task Force (SPTF) have demonstrated that the actual licensed spectrum is unoccupied most of the time [1]. Another work on spectrum occupancy measurements showed that the average spectrum occupancy from 30 MHz to 3 GHz over six cities is 5.2% [2]. The cognitive radio CR has been proposed [3], [4] to mitigate the conflict between spectrum scarcity and low spectrum efficiency.

One of the most challenging tasks in CR networks is spectrum sensing, which is required to opportunistically access the idle radio spectrum. Generally, the spectrum sensing techniques can be classified as energy detection, matched filter detection, and cyclostationary feature detection [5]. Among these techniques, energy detection has low complexity, low implementation cost, and demands no apriori knowledge about the primary user signal [6]. In this paper, the energy detection is considered as the local spectrum sensing scheme.

One of the great challenges of implementing spectrum sensing is the hidden terminal problem, this problem can be caused by many factors including severe multipath fading, shadowing, or inside buildings with high penetration loss, while a primary user (PU) is operating in the vicinity [7].

Due to the hidden terminal problem, a CR device causes unwanted interference to the PU (receiver) as the primary transmitters signal. The cooperative spectrum sensing can solve the hidden terminal problem in cognitive radio networks (CRNs). It has been shown that spectrum sensing performance can be greatly improved with an increase of the number of cooperative partners [8].

In [9] the optimal number of SUs for the majority voting rule and the optimal detection threshold were derived to minimize the sum of false alarm probability and missed detection probability.

The rest of the paper is organized as follows. Section 2 presents the system model and defines the symbols that will be used through the paper. Section 3 presents the Bayes risk function and studies the optimal voting rule analytically and through simulations. Section 4 determines the optimum fusion rule and optimum threshold that minimizes the false alarm probability while the missing probability is bounded. Finally, section 5 concludes the paper.

2. SYSTEM MODEL
A CRN with K Secondary Users (SUs) and one central node are considered. Assume that each SU performs spectrum sensing independently and takes its local binary decision (the binary decision 0 or 1 which represent the channel is occupied or the channel is unoccupied respectively). The central node collects the K binary decisions from the K SUs to decide whether the PU is active or not.

The spectrum sensing can be modeled as a binary hypothesis testing problem with hypothesis H_0 and H_1 denoting the absence and presence of a PU, respectively. In the proposed model, the low-pass equivalent of the ith sample of the received signal at the kth radio is written as:

$$r_k(i) = \begin{cases} w_k(i), & H_0 \\ h_k(i)s(i) + w_k(i) & H_1 \end{cases}$$

Where $r_k(i)$ is the received signal at the kth CR at the ith sample, $w_k(i)$ is the additive white Gaussian noise (AWGN) of the kth CR at the ith sample, $h_k(i)$ is the complex gain of the sensing channel between the PU and the kth CR at the ith sample, and $s(i)$ is the PU signal at the ith sample. Assume that the observation window of each radio has I samples, and sensing is performed with K radios. Then $i = 1, 2, \ldots I$ and $k = 1, 2, \ldots K$. Also assume sampling is done at the symbol rate, the sensing channel is time-invariant during the sensing process and error free.

For the kth CR with the energy detector, the average probability of false alarm, the average probability of detection, and the average probability of missed detection over AWGN channels are given, respectively, by [10].

$$P_{f,k} = \frac{\Gamma\left(u, \frac{\lambda_k}{2}\right)}{\Gamma(u)} \tag{1}$$

$$P_{d,k} = Q_u\left(\sqrt{2\gamma_k}, \sqrt{\lambda_k}\right) \tag{2}$$

and
$$P_{m,k} = 1 - P_{d,k} \qquad (3)$$
Where γ_k and λ_k denote the instantaneous signal-to-noise ratio (*SNR*) and the energy detection threshold of the kth CR respectively, u is time-bandwidth product of the energy detector, $\Gamma(a, x)$ is the incomplete gamma function, $\Gamma(a)$ is the gamma function, and $Q_u(a,b)$ is the generalized Marcum Q-function.

In cooperative spectrum sensing each CR device make a binary decision depending on its local observation and then forwards one bit of the decision D_k (1 standing for the presence of the PU, 0 for the absence of the PU) to the central node through an error-free channel. At the central node, all 1-bit decisions are fused together according to the following logic rule

$$Z = \begin{cases} \sum_{k=1}^{K} D_k \geq n & , & H_1 \\ \sum_{k=1}^{K} D_k < n & , & H_0 \end{cases} \qquad (4)$$

Where the threshold n is an integer, representing the "*n*-out-of-*K*" voting rule.

Assume the distance between the PU and any CR device is very large comparing with the distance between any two CRs, then the received signal at all the CRs have almost identical path losses. Therefore, in the case of an AWGN environment, it can assume that $\gamma_1 = \cdots = \gamma_K = \gamma$. These assumptions result in $P_{f,k}$ being independent of k, then it can be denoted as P_f. In the case of an AWGN channel, $P_{d,k}$ is independent of k (P_d).

In the case of a Rayleigh fading environment, it is reasonable to assume there are have independent and identically distributed (i.i.d.) Rayleigh fading with the instantaneous *SNR*s $\gamma_1, \ldots \ldots, \gamma_K$ being i.i.d. exponentially distributed random variables with the same mean γ. Furthermore, assume that all CRs use the same threshold λ. In the case of fading channel, let P_d be $P_{d,k}$ averaged over the statistics of γ_k. Where $P_d = \int_\gamma Q_u(\sqrt{2\gamma_k}, \sqrt{\lambda_k}) f_\gamma(x) dx$

In a Rayleigh fading channel P_d is given by [11] as

$$P_d^{Ray} = \frac{\Gamma\left(u-1, \frac{\lambda_k}{2}\right)}{(u-2)!} + \left(\frac{1+\overline{\gamma_k}}{\overline{\gamma_k}}\right)^{u-1} \exp\left(\frac{-\lambda_k}{2(1+\overline{\gamma_k})}\right) \left[1 - \frac{\Gamma\left(u-1, \frac{\overline{\gamma_k}\lambda_k}{2(1+\overline{\gamma_k})}\right)}{(u-2)!}\right] \quad (5)$$

Where $\overline{\gamma_k}$ is the average *SNR* of the kth CR user. Moreover $P_{f,k}$ is independent on instantaneous *SNR* γ_k, and $P_{f,k}$ is given by (1).

Therefore, the false alarm probability of cooperative spectrum sensing is given by
$$Q_f = Prob\{H_1|H_0\} = \sum_{l=n}^{K} \binom{K}{l} P_f^l \left(1 - P_f\right)^{K-l} \quad (6)$$

And the missed detection probability of cooperative spectrum sensing is given by

$$Q_m = Prob\{H_0|H_1\} = 1 - \sum_{l=n}^{K} \binom{K}{l} P_d^l \left(1 - P_d\right)^{K-l} \quad (7)$$

3. BAYES RISK FUNCTION OPTIMIZATION
3.1 Optimum Fusion Rule with Minimum Risk function
In [9] the author defined, the total error as equal $Q_f + Q_m$ (i.e. he gave Q_f and Q_m equal weights), then he got an expression for the optimal value of n for the "*n*-out-of-*K*" rule.

In this work, the more general case where Q_f and Q_m have unequal weights is considered because of in CR the missing probability Q_m causes a harmful interference on the PU, while

Q_f decreases the spectrum utilization. So, it assume that Q_m is more "dangerous" than Q_f. Therefore, an expression for the optimal voting rule that minimizes the Bayes risk function in case of unequal weights for Q_m and Q_f is needed to derive.

Let's now, define the Bayes risk function $\mathcal{R}$ as:

$$\mathcal{R} = c_{00}P_{00}P_0 + c_{11}P_{11}P_1 + c_{10}P_{10}P_0 + c_{01}P_{01}P_1 \qquad (8)$$

Where,
- c_{ij} is the cost of deciding the channel is H_i while the channel is H_j.

- P_{ij} is the probability of deciding the channel is H_i while the channel is H_j,

- P_i is the probability that the channel is H_i

In this case, it can assume that $c_{00} = c_{11} = 0$ (i.e., the correct decision costs are zero), $P_{10}=Q_f$, and $P_{01}=Q_m$ then equation (8) becomes:

$$\mathcal{R} = c_{10}P_0Q_f + c_{01}P_1Q_m \qquad (9)$$
Without loss of generality assume $\mathcal{R}$ is normalized such that, $c_{10}P_0 + c_{01}P_1 = 1$

Then it can be rewritten as

$$\mathcal{R} = \beta Q_f + (1 - \beta)Q_m \qquad (10)$$
Where $\beta = c_{10}P_0$, which represents the weight of Q_f and $(1 - \beta)$ represents the weight of Q_m. So that, the normalized risk cost function $\mathcal{R}$ represents the error rate probability.

Therefore the normalized cost function $\mathcal{R}$ is a function of n (voting rule), λ (energy detection threshold), K (total number of cooperated secondary users), and β (weight of false alarm probability). It should be noted that there is no optimum value for K that minimizes the normalized cost function $\mathcal{R}$, as K increases the cost function decreases. Through this paper, studying the optimum voting rule n_{op} is concentrated, and the optimum energy detection threshold λ_{op} that minimize the cost function $\mathcal{R}$ for different weights β, and constant K.

By substituting (6) and (7) in (10), the normalized cost function $\mathcal{R}$ can rewrite as

$$\mathcal{R}(n) = \sum_{l=n}^{K} \binom{K}{l} \left[\beta P_f^l (1 - P_f)^{K-l} - (1 - \beta)(1 - P_m)^l P_m^{K-l} \right] - 1$$

To get n that minimizes $\mathcal{R}$, differentiate $\mathcal{R}$ with respect to n and equating it to zero:

$$\frac{\partial \mathcal{R}}{\partial n} \cong [\mathcal{R}(n+1) - \mathcal{R}(n)] = 0$$

By subtracting $\mathcal{R}(n)$ from $\mathcal{R}(n+1)$,

$$\beta P_f^{n_{op}} \left(1 - P_f\right)^{K-n_{op}} - (1 - \beta)(1 - P_m)^{n_{op}} (P_m)^{K-n_{op}} = 0$$

After some mathematical manipulations, it get the following as a value for n_{op}

$$n_{op} = \frac{Kln\left(\frac{P_m}{1-P_f}\right) - ln\left(\frac{\beta}{1-\beta}\right)}{\left(ln\left(\frac{P_f}{1-P_m}\right) + ln\left(\frac{P_m}{1-P_f}\right)\right)} \qquad (11)$$

n_{op} Should be an integer value and always less than or equal to the number of SUs, then n_{op} can be rewritten as,

$$n_{op} = min\left(K, \left\lceil \frac{K \; ln\left(\frac{P_m}{1-P_f}\right) - ln\left(\frac{\beta}{1-\beta}\right)}{ln\left(\frac{P_f}{1-P_m}\right) + ln\left(\frac{P_m}{1-P_f}\right)} \right\rceil \right) \quad (12)$$

This is the optimal value of n (for the fusion rule "n-out-of-K") in case of Q_f and Q_m have unequal weights. Then n_{op} depends on the total number of users K, the energy detection threshold λ, the weight β, and the SNR. The behavior of n_{op} with these parameters is illustrated in Fig. 1 where n_{op} is plotted versus K at constant λ in Rayleigh fading channel for different SNR and different β.

Fig. 2 illustrates the n_{op} dependency on λ, for each value of λ there is a different value for n_{op} at K=15 for different β. The figure shows that as λ increases n_{op} decreases and also when β decreases, n_{op} decreases.
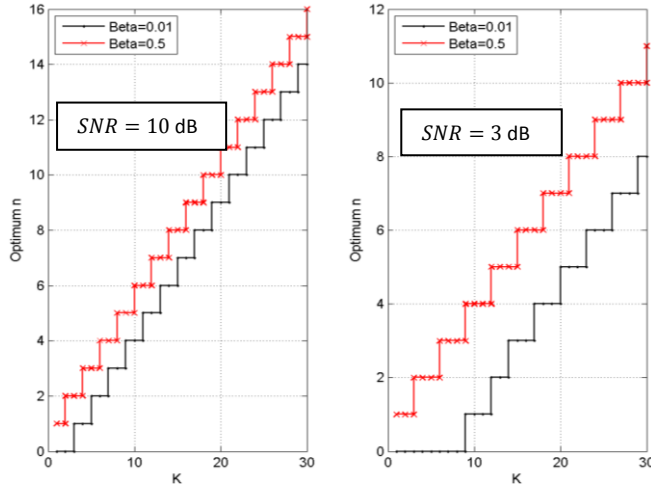

Fig. 1 n_{op} versus number of users K of cooperative spectrum sensing in Rayleigh fading channel for different SNR, at constant energy detection threshold $\lambda = 8$ for varies values of $\beta = \{0.01, 0.5\}$.

In Fig. 3, Q_m, Q_f, and $\mathcal{R}$ are plotted versus different K at $\lambda = 8$ and $SNR = 10$ dB, from the figure at $\beta = 0.5$, Q_m is almost equal to Q_f, and as β decreases Q_m decreases and Q_f is increases. The improvement gain between cooperative system (K > 1) and none cooperative (K=1) can be noted.

The optimum value n_{op} in equation (11) can be rewritten as

$$n_{op} = \frac{Kln\left(\frac{P_m}{1-P_f}\right)}{\left(ln\left(\frac{P_f}{1-P_m}\right) + ln\left(\frac{P_m}{1-P_f}\right)\right)} - \frac{ln\left(\frac{\beta}{1-\beta}\right)}{\left(ln\left(\frac{P_f}{1-P_m}\right) + ln\left(\frac{P_m}{1-P_f}\right)\right)} \quad (13)$$

Where the first term in (13) represents n_{op} when the weight β is neglected (i.e. $\beta = 0.5$) as in [9] or

$$n_{op} = \frac{Kln\left(\frac{P_m}{1-P_f}\right)}{\left(ln\left(\frac{P_f}{1-P_m}\right) + ln\left(\frac{P_m}{1-P_f}\right)\right)} \quad (14)$$

Fig. 4 illustrates that the normalized risk $\mathcal{R}$ of (10) when n_{op} is calculated using (13) and using (14). In the figure, it is clear that the performance is degraded if the effect of β is neglected.
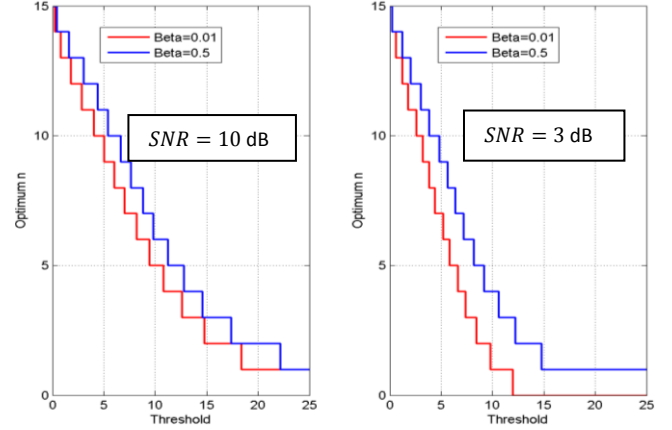

Fig. 2 Optimum fusion rule n_{op} versus detection threshold λ of cooperative spectrum sensing at K =15 in Rayleigh fading channel for different SNR, for varies values of $\beta = \{0.01, 0.5\}$.
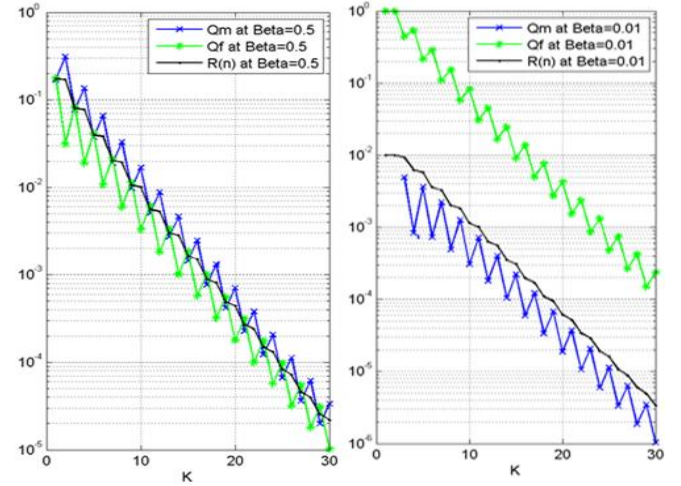

Fig. 3 Q_m, Q_f, and $\mathcal{R}$ versus K in Rayleigh fading channel at $SNR = 10$ dB and $\lambda = 8$ for varies values of $\beta = \{0.5, 0.01\}$.
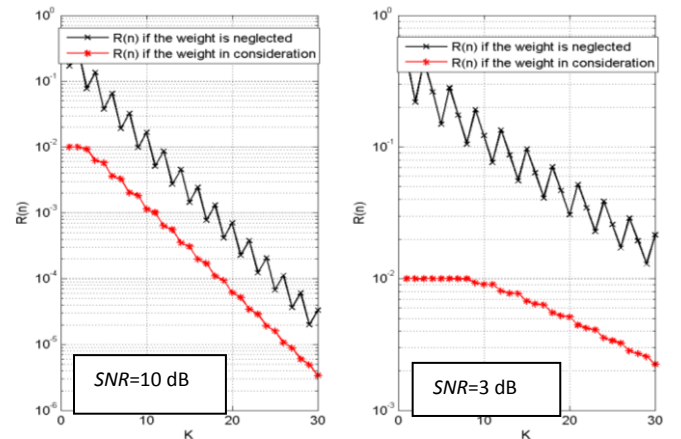

Fig. 4 $\mathcal{R}(n)$ versus K for $\beta = 0.01$ if β is neglected and if β in consideration in Rayleigh fading channel for different SNR, and $\lambda = 8$.

3.2 Optimal Energy Detection Threshold

If K and $SNR\ \gamma$ are known and n_{op} is expressed as in (12) then, to determine the optimum energy detection threshold λ_{op} that minimizes $\mathcal{R}\ (n)$ such that

$$\lambda_{op} = \min_{\lambda}[\mathcal{R}\ (n)] = \min_{\lambda}(\beta\ Q_f + (1-\beta)Q_m) \qquad (15)$$

The total error rate in terms of λ has a global minimum in λ for each n [9]. This means that there exist only one value of λ that minimizes $\mathcal{R}\ (n)$.

Then, to get an expression for λ_{op} ,differentiate $\mathcal{R}$ with respect to λ and equating it to zero.

$$\beta\ \frac{\partial Q_f}{\partial \lambda} + (1-\beta\ )\frac{\partial Q_m}{\partial \lambda} = 0 \qquad (16)$$

The expressions for $\frac{\partial Q_m}{\partial \lambda}$ and $\frac{\partial Q_f}{\partial \lambda}$ were derived in [9], however, it is quite difficult to get a closed-form solution for λ. Because there exists one solution, it can employ any one dimensional optimization algorithms (e.g., Bisection search, Fibonacci search, Golden section search, Newton search) to find λ_{op}. An example to find λ_{op} is given in algorithm 1.

Algorithm 1

Find λ_{op} assume K, n , and $SNR\ \gamma$ are known

Input K, n, $SNR\ \gamma$, and Δ(Δ is the tolerance of accuracy of λ_{op}).

Initialize $\lambda^{(-1)} = 0$, $\lambda^{(0)} = 1$, $l = 0$

while $\lambda^{(l)} - \lambda^{(l-1)} > \Delta$

calculate$\lambda^{(l+1)}$;$\lambda^{(l+1)} = \dfrac{\frac{\partial \mathcal{R}(\lambda^{(l)})}{\partial \lambda}\lambda^{(l-1)} - \frac{\partial \mathcal{R}(\lambda^{(l-1)})}{\partial \lambda}\lambda^{(l)}}{\frac{\partial \mathcal{R}(\lambda^{(l)})}{\partial \lambda} - \frac{\partial \mathcal{R}(\lambda^{(l-1)})}{\partial \lambda}}$

$l = l + 1$

end while

Output $\lambda_{op} = \lambda^{(l)}$

3.3 Optimum Fusion Rule and Energy Threshold

In this subsection the objective is determination of (λ_{op}, n_{op}) that yields the lowest error rate probability (minimum $\mathcal{R}$). This process is proposed in algorithm 2 [12].
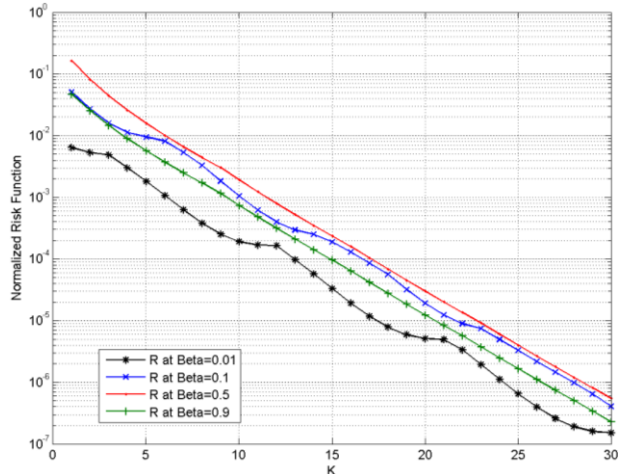
Fig. 5 illustrates $\mathcal{R}(n_{op}, \lambda_{op})$ versus K for different values of β, from the figure, the improvement on $\mathcal{R}(n_{op}, \lambda_{op})$ can be noted comparing with the $\mathcal{R}(n_{op})$ of Fig. 3 which is evaluated for n_{op} and a fixed value of λ.

Fig. 6 illustrates the optimum voting rule n_{opt} versus the number of users 'K' of cooperative spectrum sensing in Rayleigh fading channel with $SNR = 10$ dB for $\beta = \{0.01, 0.5\}$. From the figure there is a very large difference between n_{op} at $\beta = 0.01$ and at $\beta = 0.5$ can be noted. If the weight β is neglected, degradation in the performance of the system will occur.
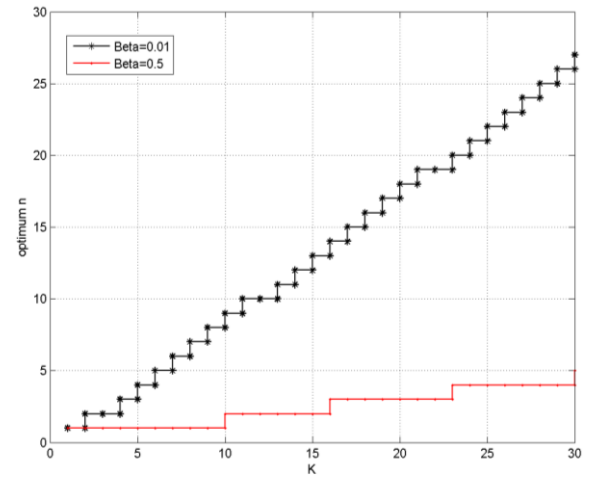

Fig. 6 n_{op} **versus K in Rayleigh fading channel with** $SNR = 10$ **dB and for** $\beta = \{0.01, 0.5\}$.

4. OBJECTIVE-CONSTRAINT OPTIMIZATION

So far, the case of unequal weights for the missing and false alarm probabilities is considered. In this section, a technique that optimize n and λ, which denoted as n_{op} and λ_{op} that minimizes the false alarm probability Q_f with constraint on Q_m and considering a fixed value of K. Then this problem is an optimization problem with constraint.

$$F = Q_f + \Psi\ (\ Q_m - \varepsilon) \qquad (17)$$

Where Q_f is given as in (6), $Q_m \leq \varepsilon$ is the constraint and Ψ is the Lagrangian multiplier.

$$\frac{\left(\frac{\partial Q_f}{\partial n}\right)}{\left(\frac{\partial Q_m}{\partial n}\right)} = \frac{\left(\frac{\partial Q_f}{\partial \lambda}\right)}{\left(\frac{\partial Q_m}{\partial \lambda}\right)} = \Psi \qquad (18)$$

This optimization problem is solved to get n_{op} and λ_{op}. It is quite difficult to get the closed-form solution of n_{op} and λ_{op}, therefore, an algorithm is proposed to determine them.

Fig. 8 Q_m, Q_f, and $\mathcal{R}$ versus n at K=15, $Q_m \leq 10^{-4}$ in Rayleigh fading channel with $SNR = 10$ dB.

Algorithm 2

Find λ and n to get minimum Q_f such that $Q_m \leq \mathcal{E}$ assume K and SNR γ are known.

Input K, SNR γ, $\mathcal{E}$, and Δ(the tolerance of accuracy of λ)

Initialize $Q_f min = 1, n_{op} = 1, \lambda_{op} = 0$

for $n = 1$ to K

   step $= 2, \lambda = 1$

   while step $> \Delta$

      find $Q_m(\lambda, n)$

      if $Q_m(\lambda, n) \leq \mathcal{E}$

      $\lambda = \lambda + $ step

     else if step $= 2$

      at the value of n there is no λ grantees the constraint

      step $= \frac{step}{2}, \lambda = \lambda - $ step

     end if

   end while

   find $Q_f(\lambda, n)$

   if $Q_f(\lambda, n) \leq Q_f min$

     $n_{op} = n, \lambda_{op} = \lambda$

   end if

end for

Output $\left(n_{op}, \lambda_{op} \right)$

Fig. 7 illustrates λ_{op} versus n when using algorithm 2 at $\mathcal{E} = 10^{-4}$, K=15, and $SNR = 10$ dB. Fig. 8 illustrates Q_m, Q_f and $\mathcal{R}$ versus n, in the figure $Q_m \leq 10^{-4}$, $n_{op} = 2$ the value that minimizes Q_f and the optimum energy detection threshold $\lambda_{op} = 15$ (from Fig. 7).
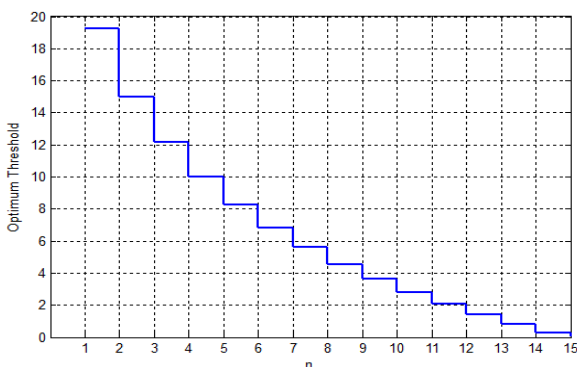

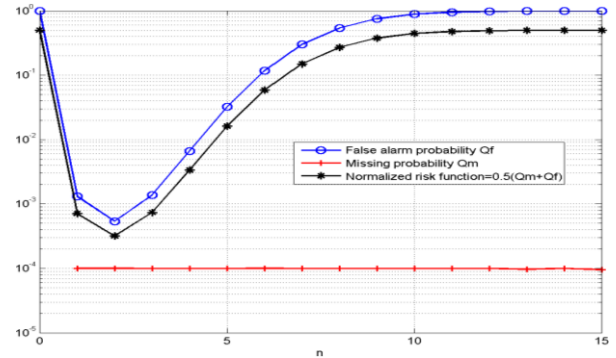**Fig. 7 λ_{op} versus n at K=15, $Q_m \leq 10^{-4}$ in Rayleigh fading channel with $SNR = 10$ dB.**

5. CONCLUSION

In this paper the two parameters: the optimal fusion rule and the optimum threshold to minimize the interference on the PU are studied. An unequal weight Bayes risk function is defined, derive an expression to determine the optimum fusion rule, and an algorithm to evaluate the optimum threshold is proposed. In addition, an optimal spectrum sensing problem was formulated by optimizing both fusion rule and threshold. Another problem where the missing probability is guarantee to be less than a certain threshold and minimizes the false alarm probability is proposed, the optimum fusion rule and the optimum threshold is evaluated for this problem.

6. ACKNOWLEDGMENTS

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