Multiresolution Transform Techniques in Digital Image Processing

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ABSTRACT
Transform analysis deeply concern in the development of digital image processing, from the part of transform analysis, multiresolution transform are associated to image processing, signal processing and processor vision, The curvelet transform is a multiresolution directional transform, which deals with an practically ideal non adaptive scant depiction of objects with edges. Although the statement those wavelets transform ensure a wide-ranging influence in image processing, they miscarry to proficiently signify objects with extremely anisotropic basics such as lines or curvilinear constructions. But the reason is that wavelets are non-geometrical and do not exploit the regularity of the edge curve. The curvelet transforms were developing as a response to the strength of the wavelet transform. curvelets yield the usage of base features which show high directional capability. Multiresolution transform are current linear image depictions. This paper present the curvelet transform analysis, with its present beginning and relationship to other multiresolution multidirectional transform like Radon transform, Ridgelet transform for image denoising and reconstruction on the basis of varying parameter.

Keywords
Multiresolution, Image processing, Curvelet transform, Wavelet Transform

1. INTRODUCTION
For the analysis of image processing methods we deliberate block transform methods. From the block transform methods, we introduced the generation of curvelet transform and its representation, Curvelet transform is a multiresolution transform[1] generation of curvelet is based on the Ridgelet transforms, but the ridgelet analysis is generated from the Radon transform and one dimensional wavelet transform. A new multiscale/multiresolution ideas in the field of image processing is introduce for the analysis of image in the extensive field of image denoising, Image Compression and feature extraction, the expansion of wavelets and associated ideas led to appropriate tools to circumnavigate through big datasets, to communicate compacted data quickly, to eliminate noise from signals and images, and to classify dynamic passing features in such datasets. In this research paper curvelet first generation analysis is summarized.

2. RADON TRANSFORM
Radon transform is a higher example of Trace transform, it is integral transform containing of the integral of a function or object over straight lines, are able to convert two dimensional images and data with lines into a domain of possible line parameters [2], where each line in the image and data will give a ultimate positioned at the resultant line parameters. Fig. 1 shows the representation of Radon transform [3].

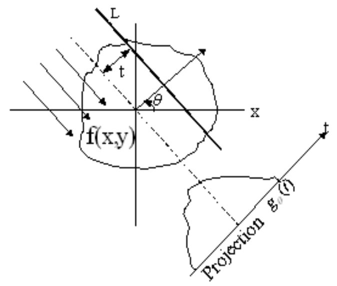

Fig 1: Radon Transform Projection

For the generation of curvelet transform [4,5], ridgelet transform and wavelet transform analysis used in the radon domain. The Radon transform of an function or object is defined by the group of line integrals in range $(\theta, \mathrm{t}) \in [0, 2\pi) \times \mathrm{R}$ given by

$$\mathrm{R}\, f(\theta, \mathrm{t}) = \int f(x_1, x_2)\delta(x_1 \cos\theta + x_2 \sin\theta - t)\, \mathrm{d}\, x_1 dx_2 \tag{1}$$

Where δ is denoted the Dirac distribution. For the ridgelet coefficients $Rf(\mathrm{a}, \mathrm{b}, \theta)$ of an object 'f' are known by study of the Radon transform through

$$\mathrm{R}\ f(\mathrm{a}, \mathrm{b}, \theta) = \int Rf(\theta, \mathrm{t}) a^{-1/2} \psi((\mathrm{t} - \mathrm{b}) / \mathrm{a})\, \mathrm{dt}$$
(2)

Equation (2) shows that the ridgelet transform analysis is related with one-dimensional (1-D) wavelet transform to the wedges of the Radon transform [6].

Finite Radon Transform (FRAT) analysis is based on the image pixels over a specified set of "lines" [7, 8]. Euclidean geometry analysis of Radon transform also based on finite geometry in a related mode, for finite field $Z_n = \{0, 1, ..., n - 1\}$, where 'n' is a prime number and Z_n is a finite field with modulo 'n' operations [9]. For analysis, we signify $Z_n* = \{0, 1, ..., n\}$.

Finite Radon analysis of a real function 'f' on the finite grid Z_n^2 is defined as

$$rk[l] = FRATf(k, l) = \frac{1}{n} \sum_{(i,j) \in \mathrm{L}, k, l} f[i, j]$$
(3)

Where $L_{k,l}$ is the set of rules that score a line on the lattice Z_n^2, $L_{k,l}$ is shows

$$L_{k,l} = \{(i, j) : j = ki + l(\bmod n),\ i \in Z_n\}, 0 \le k < \text{n},$$
$$L_{\text{n},l} = \{(l, j) : j \in Z_n\}$$
(4)

Set of eight parallel lines from the range of 0 to 7 selected in eight possible order for the analysis of finite radon transform. Image is situated from top to bottom, left to right are equivalent to the values of parallel lines. Each image have a distinct value of image point or (pixels) with dissimilar gray-scales, to find the energy of in Radon domain is that the mean is subtracted from the original image f[i, j], In the Euclidean geometry analysis, a line $L_{k,l}$ on the plane Z_n^2 is exclusively denoted by its fall of direction $k \in Zn^*$ (k=n infinite slope or upright lines) and its impose $l \in Z_n$, and can be evaluated that there are $n^2 + n$ lines distinct in this way and every line covers n points. Furthermore, any two separate points on Z_n^2 acceptable to objective one line. Also, two lines of dissimilar drops cross at just one point. There are n parallel lines that deliver a whole cover of the lane Z_n^2. This means that for an input image f[i,j] by zero-mean, we have

$$\sum_{l=0}^{n-1} rk[l] = \frac{1}{\sqrt{n}} \sum_{(i,j)\in zn^2} f[i, j] = 0$$
(5)

Thus, (5) clearly show that the redundancy of the radon transform in each direction, there are only n − 1 independent Radon coefficients. Those coefficients at n + 1 directions coefficient generate the mean value by n^2 self-determining coefficients and shows degrees of freedom in the predetermined Radon domain. For the continuous case, the finite back projection (FBP) is defined the combined analysis of Radon coefficients of all the lines that drive through a given point and position, that is

$$FB\,\text{Pr(i, j)} = \frac{1}{\sqrt{n}} \sum_{(k,\text{l})\in ni,j} rk[l], i, j \in Zn^2 \tag{6}$$

where $P_{i,j}$ signifies the set of solutions of all the lines that use a point (i,j) ∈ Zn^2. More precisely from (4) we can write.

$$ni, j = \{(k,l) : l = j - ki(\bmod n), \text{k} \in \text{Zn}\} \cup \{(\text{n, j})\}$$
(7)

From the mathematical analysis of (3) and (6) we obtain.

$$\begin{aligned} FB\,\text{Pr(i, j)} &= \frac{1}{n} \sum_{(k,\text{l})\in ni,j}\ \sum_{(i',j')\in \text{L}k,l} f[i', j'] \\ &= \frac{1}{n}\Big(\sum_{(i',j')\in \text{Z}_n^2} f[i', j'] + n.f[\text{i, j}]\Big) \\ &= f[i, j] \end{aligned} \tag{8}$$

This (8) represent the back-projection operator and used in the analysis of the inverse Radon transform for zero-mean images. So we have an effective and precise reform algorithm for the Radon transform. Radon transform analysis requires n^3 additions and n^2 multiplications for computation, for efficiency, each pixel of original image pass once for histogram analysis in Radon transform [10, 11].

3. RIDGELET TRANSFORM

A newly developed multiresolution analysis is Ridgelet analysis [12]. Its result available graphics by super sites of ridge functions or by simple elements that are in some way related to ridge functions $r(a_1x_1+\ldots+a_nx_n)$; these are functions of n variables, constant along hyper planes $a_1x_1+\ldots+a_nx_n = c$; the graph of such a function in dimension two looks like a ‘ridge’.

From Radon transform we can see that the ridgelet transform related with other transforms in the continuous domain [13, 14]. For the analysis of any function or object f(x) continuous ridgelet transform (CRT) in R^2 is distinct via [15].

$$CRTf\,(\text{a, b}, \theta) = \int_{R^2} \psi a, b, \theta(x) f(x)\,\text{d}\,x,$$
(9)

Where the ridgelets $\psi_{a,b,\theta}(x)$ in two dimensional analyses are defined using one dimensional analysis 1-D

$$\psi(x)\,\text{as}\,\psi_{a,b,\theta}(x) = a^{-1/2}\psi((x_1 \cos\theta + x_2 \sin\theta - b)/a)$$
(10)

Ridgelet function is concerned with an angle θ and is constant along the lines which represent by $x_1 \cos\theta + x_2 \sin\theta = \text{const.}$ Continuous wavelet transform of any function f(x) in R^2 domain ‘f’ can be defined as

$$\text{CWT}\, f(a_1, a_2, b_1, b_2) = \int_{R^2} \psi a_1, a_2, b_1, b_2(x) f(x)\,\text{d}\,x$$
(11)

In this the wavelets in 2-D are the product which is defined as

$$\psi a_1, a_2, b_1, b_2(x) = \psi a_1, b_1(x1)\psi a_2, b_2(x2)$$
(12)

From one dimensional analysis of wavelet transform we know

$$\psi_{a,b}(\text{t}) = a^{-1/2}\psi((t-b)/a)$$

From this it is clear that the ridgelet and wavelet analysis are same except the difference of parameter, in wavelet transform point parameters (b_1,b_2) are changed by the line parameters (b,θ) for ridgelet transform [16]. Representation of both these multiscale transform are defined as

Wavelets:ψ scale, point−position,

Ridgelets: ψ scale, line−position.

Representation of ridgelet transform are based on the point singularity of object two dimensional analysis of wavelet and ridgelet are concern via the Radon transform analysis that’s shows in (13) from the representation of Radon transform [17].

$$R_f(\theta,t) = \int_{R^2} f(x)\delta\left(x_1 cos\theta + x_2 sin\theta - t\right)dx,$$
(13)

Similarly the representation of ridgelet transform in term of wavelet and Radon is,

$$CRT_f(a,b,\theta) = \int_R \psi_{a,b}(t) R_f(\theta,t)\,dt$$
(14)

Another analysis of ridgelet transform is based on the projection slice theorem that shows fourier analysis of any function f(x) and its its two dimensional fourier transform is $F_f(\omega)$.

$$Ff(\xi\cos\theta, \xi\sin\theta) = \int_R e^{-j\xi t} Rf(\theta,t)\,dt$$
(15)

This is commonly used in image reconstruction from projection methods. The relationship between Radon transform [18][19], Wavelet transform and Ridgelet ransform are shown in fig.

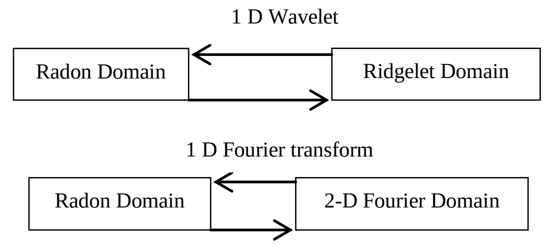

Fig 2: Radon Transform Relation with Ridgelet and Fourier transform

It is clear that we can obtain an invertible discrete ridgelet transform by taking the discrete wavelet transform (DWT) on each one FRAT plan system, ($r_k[0],r_k[1],...,r_k[p-1]$), where the path k is stable. Fig 3 shows implementation of ridgelet transforms.

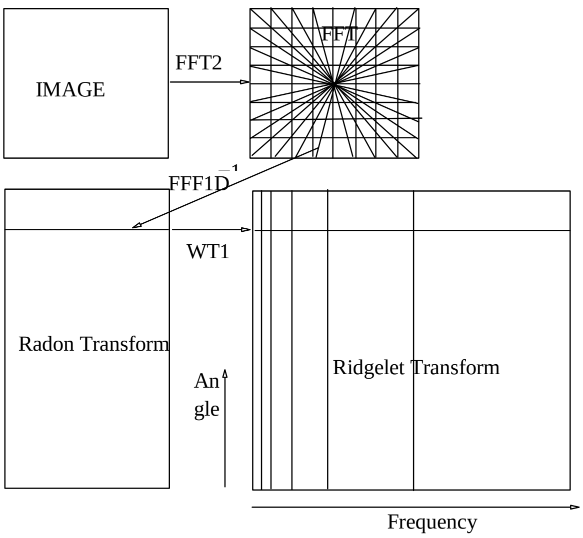

Fig 3: Ridgelet Transform implementation

4. CURVELET TRANSFORM

Overcome the drawback of Wavelet Transform. Curvelet Transform is developed, Curvelet Transform is multilevel transform that not only used for a multi scale Time – Frequency analysis [20, 21], it is also used for the analysis of directional features. Curvelet concept is by Candes and Donoha, this transform is based on the multiresolution analysis, length and width are related anisotropic scaling law. Furthermore, edge fundamental in curvelet is defined by scaling, position and orientation parameters but in wavelets there are only scale and location parameter. Curvelet transform are used in both domain analysisis frequency domain and time domain. All analysis of curvelet transform is based on the first generation (DCTG1) and curvelet second generation (DCTG2).

4.1 First Generation Curvelets Construction

The first generation CurveletG1 transform [22] are based on the possibility to analyse an image with different block sizes curvelet G1 analysis based on the flow graph shown in fig 4.

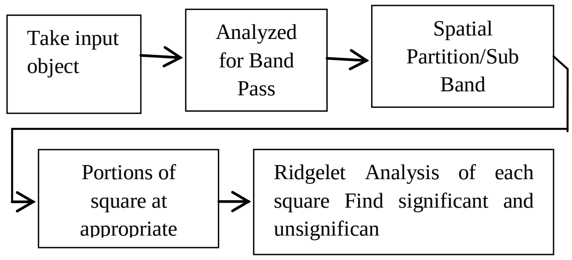

Fig 4: Ridgelet Transform Analysis for Curvelet Generation 1

Fig. 4 represent the generation of curvelet transform from ridgelet analysis [23, 24, 25], in this any object selected as a input and process this selected input for band pass filtering a, sub band decomposition, parameter analysis and ridgelet analysis of each square.. The First Generation Discrete Curvelet Transform (DCTG1) of a continuous function f(x) creates use a sequence of scales, and a filters bank property, in this property the band pass filter Δj is in the frequencies [2^{2j},2^{2j+2}], e.g.

$$\Delta_j(f) = \Psi 2j * f, \qquad \hat{\Psi} 2j(v) = \hat{\Psi}(2^{-2j} v)$$

Decomposition of curvelet transform are based on sub band decomposition [26, 27], smooth portioning and ridgelet analysis, this produce that the curvelet decomposition of function in the range [2^{2j},2^{2j+2}].

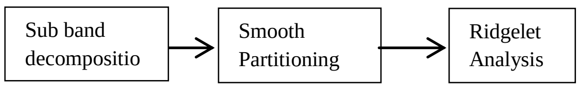

Fig 5: Decomposition of Curvelet generation 1

Before the final analysis (ridgelet analysis) [28, 29, 30] of curvelet decomposition two dyadic sub band in the range 2^n and 2^{n+1}, for this a isotropic wavelet transform are required, algorithm representation of curvelet decomposition as a superposition is in the form of any image $f[i_1,i_2]$ n×n is.

$$f[i_1,i_2] = Z_J[i_1,i_2] + \sum_{j=1}^{J} wj[i_1,i_2]$$

(16)

Where Z_J is a coarse or flat form of the original image f and wj signifies ‘the details of f at scale 2^{-j}. Thus, the algorithm outputs J + 1 sub-band arrays of size n × n. algorithm representation are as follows:

Algorithm 1 for curvelet generation 1

Select n × n image $f[i_1,i_2]$,

1: Apply two dimensional wavelet transform with J scales,

2: Set $Z_1 = Z_{min}$,

3: for j = 1,...,J do

4: Partition the sub-band with a block size Cj and apply the DRT to each block,

5: if j modulo 2 = 1 then

6: $Z_{j+1} = 2Z_j$,

7: else

8: $Z_{j+1} = Z_j$.

9: end if

10: end for

In this the side-length of the containing windows is doubled at every other dyadic sub-band, hence recalling the essential property of the curvelet transform at j^{th} for the features of length about $2^{-j/2}$.

5. RESULT AND DISCUSSION

For the implementation of multiresolution analysis , there are two box ridgelet transform and curvelet transform toolbox, Implementation is based on the MATLAB software. From ridgelet toolbox original image are compared for the value of signal to noise ratio (SNR), comparison of different multiresolution is shown in fig 6, 7 and 8. In other multiresolution analysis representation of curvelet transform are produce varying different parameter.

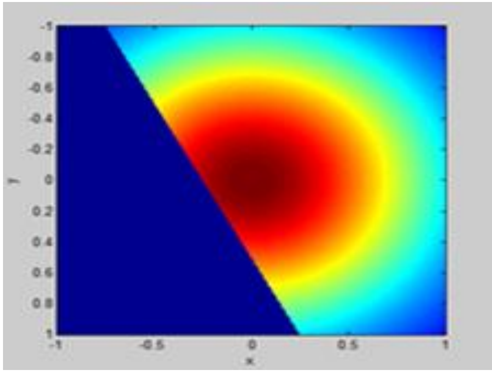

Fig 6: Original Image

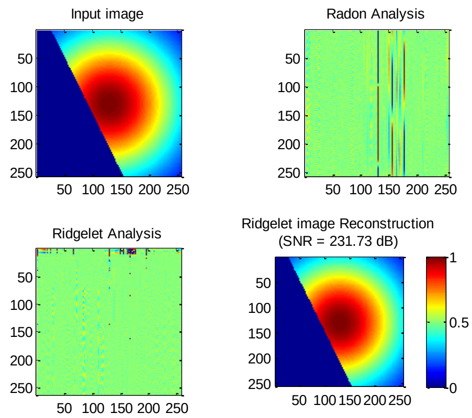

Fig 7: Image denoising by FRAT and FRIT

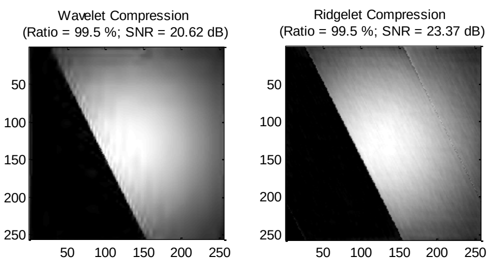

Fig 8: Analysis of compressed image by FRIT

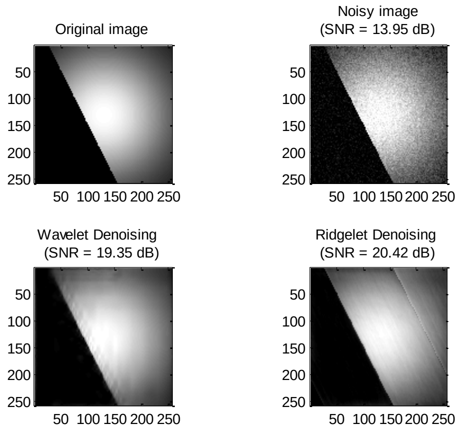

Fig 9: Image Denoising by Wavelet and Ridgelet Transform

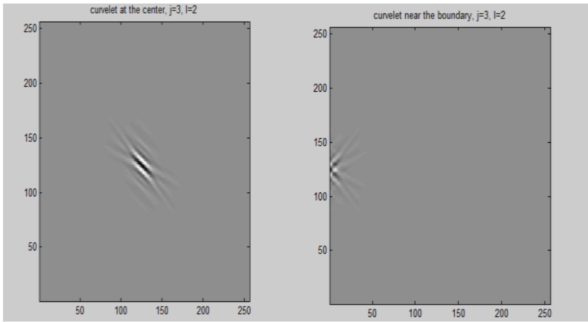

Fig 10: Analysis of curvelet for j=3,l=2 near the boundary and at the center

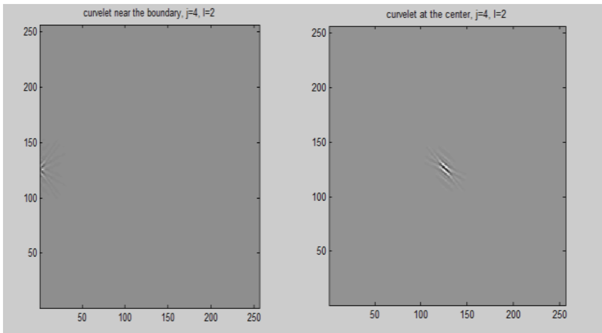

Fig 11: Analysis of curvelet for j=4,l=2 near the boundary and at the Centre

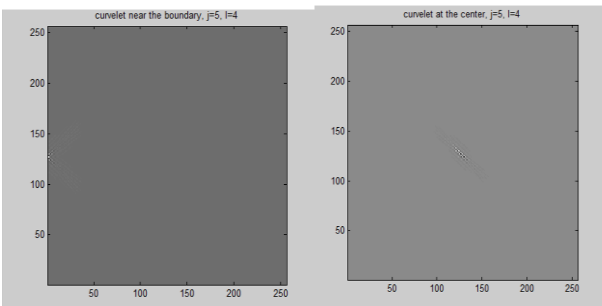

Fig 12: Analysis of curvelet for j=5,l=4 near the boundary and at the Centre

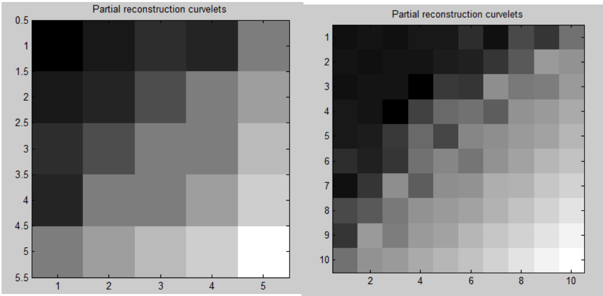

Fig 13: Partial reconstruction of curvelet for N=5 and N=10.

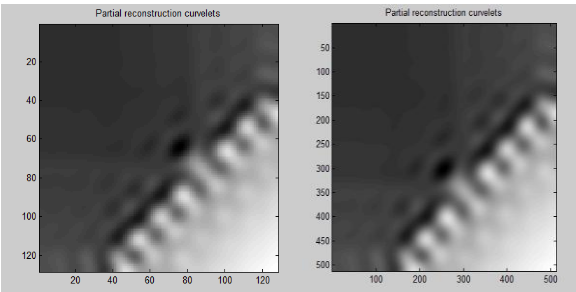

Fig 14: Partial reconstruction of curvelet for N=128 and N=256

6. CONCLUSION

Multiresolution transform analysis is produced for the performance comparison. In this primary result analysis is based on the selection of Guass half circle image and its analysis for comparison of signal to noise ratio for FRAT, FRIT Wavelet transform, this shows that the FRIT produce better result than other methods. Secondary analysis is based on the implementation of curvelet transform generation, Curvelet representation is based on the variation of i, j and N, according the variation of these parameter different types of representation are generated. Implementation of algorithms is based on MATLAB transform using curvelab and ridgeleler toolbox.

7. REFERENCES

[1] E. Cand`es, L. Demanet, D. Donoho, and L. Ying, “Fast discrete curvelet transforms,” Multiscale Modeling and Simulation, vol.5, no. 3, pp. 861–899, 2006.

[2] J. Ma, M. Fenn, “Combined complex ridgelet shrinkage and total variation minimization,” SIAM J. Sci. Comput., 28 (3), 984-1000 (2006).

[3] Sigurdur Helgason, “Radon Transform,” Second Edition.

[4] F. Matus and Jan Flusser, “Image Representation via Finite Radon Transform,” IEEE Transaction on pattern analysis and machine intelligence, Vol. 15, No. 10, 1993.

[5] YVES NIEVERGELT, “Exact Reconstruction Filters to Invert Radon Transforms with Finite Elements,” JOURNAL OF MATHEMATICAL ANALYSIS AND APPLICATIONS 120, 288314 (1986).

[6] MICHAEL E. ORRISON, “RADON TRANSFORMS AND THE FINITE GENERAL LINEAR GROUPS,” This article appeared in Forum Mathematicum, no. 1, 97-107, 2004.

[7] Lauren A. Ostridge and John C. Bancroft, “An investigation of the Radon transform to attenuate noise after Migration,” CREWES Research Report, Vol. 19, 2007.

[8] M. Hasan, AL.Jouhar and Majid A. Alwan, “Face Recognition Using Improved FFT Based Radon by PSO and PCA Techniques,” International Journal of Image Processing (IJIP), Vol. 6, 2012.

[9] E. Candes, F. Guo, “New multiscale transforms, minimum total variation synthesis: applications to edge-preserving image reconstruction,” Signal Process., 82 (11), 1519-1543 (2002).

[10] Alexander Kadyrov and Maria Petroa, “The Trace Transform and its application,” IEEE Trans.Pattern Analysis and Machine Intelligence, Vol.23, No.8, 2001.

[11] Y. Lu and M. N. Do, “Multidimensional directional filter banks and surfacelets,” IEEE Trans. Image Process., vol. 16, no. 4, pp. 918–931, 2007.

[12] Minh N. Do, Martin Vetterli, “The Finite Ridgelet Transform for Image Representation,” IEEE TRANSACTIONS ON IMAGE PROCESSING.

[13] H. Douma and M. de Hoop, "Leading-Order Seismic Imaging Using Curvelets," Geophysics, vol. 72, no. 6, pp. S231–S248, 2007.

[14] S. Foucher, V. Gouaillier and L. Gagnon, “Global Semantic Classification of Scenes using Ridgelet Transform,” R&D Department, Computer Research Institute of Montreal, 2004.

[15] Zongjian LI, Li ZENG and Rifeng ZHOU, “Ridgelet Compression Method of Luggage X-ray Inspection Images,” 17th World Conference on Nondestructive Testing, Shanghai, China, 2008.

[16] S. Etter and P. Grohs, “A Fast Finite Ridgelet Transform for Radiative Transport,” Research Report No. 2014-11, March 2014.

[17] Lenina Birgale and Manesh Kokare, “Iris Recognition Using Ridgelets,” Journal of Information Processing Systems, Vol.8, No.3, September 2012.

[18] Philippe Carre and Eric Andres, “Discrete Analytical Ridgelet Transform,” Signal processing 84-11, Singpro. 2004.

[19] Vinay Mishra and Pallavi, “Image enhancement by curvelet, ridgelet, and wavelet transform,” Second International Conference on Digital Image Processing, 2010.

[20] M. Sambasivudu and V. G. Akula, “Watermarking algorithm for Image coding using Ridgelet Transformation,” Scholars Journal of Engineering and Technology (SJET), 2(1), 2014.

[21] G.Y. Chen and B. Kegl, “Image denoising with complex ridgelets,” Pattern Recognition, 578 – 585, 2007.

[22] M.J. Fadili, J.-L. Starck, “Curvelets and Ridgelets,” October 24, 2007.

[23] E. Candès and L. Demanet, “The curvelet representation of wave propagators is optimally sparse,” Commun. Pure Appl.Math. pp. 1472–1528, 2005, vol. 58, no. 11.

[24] B. S. Manjunath et al, “Color and Texture Descriptors,” IEEE Transactions CSVT, 703-715, 2001, 11(6).

[25] Raman Maini and Himanshu Aggarwal, “A Comprehensive Review of Image Enhancement

Techniques," JOURNAL OF COMPUTING, VOL. 2, ISSUE 3, 2010.

[26] Jianwei Ma and Gerlind Plonka, "The Curvelet Transform," IEEE SIGNAL PROCESSING MAGAZINE, 118-133, 2010.

[27] Akash Tayal and Dhruv Arya, "Curvelets and their Future Applications," Proceedings of the National Conference; INDIACom-2011 Computing For Nation Development, March 10 – 11, 2011.

[28] S. K. Bandyopadhyay, T. U. Paul, A. Raychoudhury, "Image Compression using Approximate Matching and Run Length," International Journal of Advanced Computer Science and Applications, Vol. 2, No. 6, 2011.

[29] E. Gomathi and K. Baskaran "Face Recognition Fusion Algorithm Based on Wavelet," European Journal of Scientific Research ISSN 1450-216X Vol.74 No.3, pp. 450-455, 2012

[30] M. Hasan, AL.Jouhar and Majid A. Alwan, "Face Recognition Using Improved FFT Based Radon by PSO and PCA Techniques," International Journal of Image Processing (IJIP), Vol. 6, 2012.