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Square Difference 3-Equitable Labeling of Paths and Cycles

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ABSTRACT

A square difference 3-equitable labeling of a graph G with vertex set V is a bijection f from V to $\{1, 2, \ldots, |V|\}$ such that if each edge uv is assigned the label -1 if $|[f(u)]^2 - [f(v)]^2| \equiv -1(mod\,4)$, the label 0 if $|[f(u)]^2 - [f(v)]^2| \equiv 0(mod\,4)$ and the label 1 if $|[f(u)]^2 - [f(v)]^2| \equiv 1(mod\,4)$, then the number of edges labeled with i and the number of edges labeled with j differ by atmost 1 for $-1 \leq i, j \leq 1$. If a graph has a square difference 3-equitable labeling, then it is called square difference 3-equitable graph. In this paper, we investigate the square difference 3-equitable labeling behaviour of paths and cycles.

Keywords

Square difference 3-equitable labeling, square difference 3-equitable graphs

1. INTRODUCTION

DEFINITION 1. *Let $G = (V, E)$ be a graph. A mapping $f : V(G) \to \{-1, 0, 1\}$ is called ternary vertex labeling of G and $f(v)$ is called the label of the vertex v of G under f.*

For an edge $e = uv$, the induced edge labeling is given by $f^* : E(G) \to \{-1, 0, 1\}$. Let $v_f(-1)$, $v_f(0)$, $v_f(1)$be the number of vertices of G having labels -1, 0, 1 respectively under f and $e_f(-1)$, $e_f(0)$, $e_f(1)$ be the number of edges having labels -1, 0, 1 respectively under f^*.

DEFINITION 2. *A ternary vertex labeling of a graph G is called a* **3-equitable labeling** *if $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$ for all $-1 \leq i, j \leq 1$. A graph G is 3-equitable if it admits 3-equitable labeling.*

DEFINITION 3. *A square difference 3-equitable labeling of a graph G with vertex set $V(G)$ is a bijection $f : V(G) \to \{1, 2, 3, ..., |V|\}$ such that the induced edge labeling $f^* : E(G) \to \{-1, 0, 1\}$ is defined by*

$$f^*(e = uv) = \begin{cases} -1 & if\ |[f(u)]^2 - [f(v)]^2| \equiv -1(mod\,4) \\ 0 & if\ |[f(u)]^2 - [f(v)]^2| \equiv 0(mod\,4) \\ 1 & if\ |[f(u)]^2 - [f(v)]^2| \equiv 1(mod\,4) \end{cases}$$

and $|e_f(i) - e_f(j)| \leq 1$ for all $-1 \leq i, j \leq 1$. A graph which admits square difference 3-equitable labeling is called square difference 3-equitable graph.

EXAMPLE 1. *Consider the following graph G.*

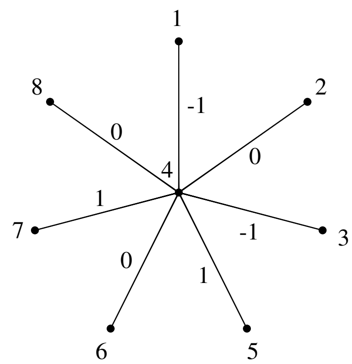

Fig 1. A square difference 3-equitable graph

We see that $e_f(-1) = e_f(1) = 2$ and $e_f(0) = 3$.
Thus $|e_f(i) - e_f(j)| \leq 1$ for all $-1 \leq i, j \leq 1$ and hence G is square difference 3-equitable.

2. MAIN RESULTS

THEOREM 1. *The path P_n admits square difference 3-equitable labeling.*

PROOF. Let $P_n : u_1 u_2 ... u_n$ be the path.
If $n \leq 4$, the following table gives the square difference 3-equitable labeling of P_n.

n	u_1	u_2	u_3	u_4
1	1			
2	2	1		
3	2	3	1	
4	2	3	1	4

Table. Square difference 3-equitable labeling of $P_n, n \leq 4$

If $n \geq 5$, we consider the following cases.

Case(i): $n \equiv 1(mod\,6)$
Define

$$f(u_1) = 2$$
$$f(u_2) = 3$$
$$f(u_3) = 1$$
$$f(u_4) = 4,$$

for $1 \leq i \leq \frac{n-7}{6}$,

$$f(u_{6i-1}) = 6i+1$$
$$f(u_{6i}) = 6i-1$$
$$f(u_{6i+1}) = 6i$$
$$f(u_{6i+2}) = 6i+2$$
$$f(u_{6i+3}) = 6i+3$$
$$f(u_{6i+4}) = 6i+4$$

and

$$f(u_{n-2}) = n$$
$$f(u_{n-1}) = n-2$$
$$f(u_n) = n-1.$$

Then

$$|[f\,(u_1)]^2 - [f\,(u_2)]^2\,| \equiv 1(mod\,4)$$
$$\Rightarrow f^*(u_1u_2) = 1$$
$$|[f\,(u_2)]^2 - [f\,(u_3)]^2\,| \equiv 0(mod\,4)$$
$$\Rightarrow f^*(u_2u_3) = 0$$
$$|[f\,(u_3)]^2 - [f\,(u_4)]^2\,| \equiv -1(mod\,4)$$
$$\Rightarrow f^*(u_3u_4) = -1,$$

for $1 \leq i \leq \frac{n-7}{6}$,

$$|[f\,(u_{6i-2})]^2 - [f\,(u_{6i-1})]^2\,| \equiv 1(mod\,4)$$
$$\Rightarrow f^*(u_{6i-2}u_{6i-1}) = 1$$
$$|[f\,(u_{6i-1})]^2 - [f\,(u_{6i})]^2\,| \equiv 0(mod\,4)$$
$$\Rightarrow f^*(u_{6i-1}u_{6i}) = 0$$
$$|[f\,(u_{6i})]^2 - [f\,(u_{6i+1})]^2\,| \equiv -1(mod\,4)$$
$$\Rightarrow f^*(u_{6i}u_{6i+1}) = -1$$
$$|[f\,(u_{6i+1})]^2 - [f\,(u_{6i+2})]^2\,| \equiv 0(mod\,4)$$
$$\Rightarrow f^*(u_{6i+1}u_{6i+2}) = 0$$
$$|[f\,(u_{6i+2})]^2 - [f\,(u_{6i+3})]^2\,| \equiv 1(mod\,4)$$
$$\Rightarrow f^*(u_{6i+2}u_{6i+3}) = 1$$
$$|[f\,(u_{6i+3})]^2 - [f\,(u_{6i+4})]^2\,| \equiv -1(mod\,4)$$
$$\Rightarrow f^*(u_{6i+3}u_{6i+4}) = -1$$

and

$$|[f\,(u_{n-3})]^2 - [f\,(u_{n-2})]^2\,| \equiv 1(mod\,4)$$
$$\Rightarrow f^*(u_{n-3}u_{n-2}) = 1$$
$$|[f\,(u_{n-2})]^2 - [f\,(u_{n-1})]^2\,| \equiv 0(mod\,4)$$
$$\Rightarrow f^*(u_{n-2}u_{n-1}) = 0$$
$$|[f\,(u_{n-1})]^2 - [f\,(u_n)]^2\,| \equiv -1(mod\,4)$$
$$\Rightarrow f^*(u_{n-1}u_n) = -1.$$

Thus $e_f(-1) = e_f(0) = e_f(1) = \frac{n-1}{3}$.

Case(ii): $n \equiv 2(mod\,6)$
Define

$$f(u_1) = 2$$
$$f(u_2) = 3$$
$$f(u_3) = 1$$
$$f(u_4) = 4,$$

for $1 \leq i \leq \frac{n-8}{6}$,

$$f(u_{6i-1}) = 6i+1$$
$$f(u_{6i}) = 6i-1$$
$$f(u_{6i+1}) = 6i$$
$$f(u_{6i+2}) = 6i+2$$
$$f(u_{6i+3}) = 6i+3$$
$$f(u_{6i+4}) = 6i+4$$

and

$$f(u_{n-3}) = n-1$$
$$f(u_{n-2}) = n-3$$
$$f(u_{n-1}) = n-2$$
$$f(u_n) = n.$$

Then $e_f(-1) = e_f(1) = \frac{n-2}{3}$ and $e_f(0) = \frac{n+1}{3}$.

Case(iii): $n \equiv 3(mod\,6)$
Define

$$f(u_1) = 2$$
$$f(u_2) = 3$$
$$f(u_3) = 1$$
$$f(u_4) = 4,$$

for $1 \leq i \leq \frac{n-9}{6}$,

$$f(u_{6i-1}) = 6i+1$$
$$f(u_{6i}) = 6i-1$$
$$f(u_{6i+1}) = 6i$$
$$f(u_{6i+2}) = 6i+2$$
$$f(u_{6i+3}) = 6i+3$$
$$f(u_{6i+4}) = 6i+4$$

and

$$f(u_{n-4}) = n-2$$
$$f(u_{n-3}) = n-4$$
$$f(u_{n-2}) = n-3$$
$$f(u_{n-1}) = n-1$$
$$f(u_n) = n.$$

Then $e_f(-1) = \frac{n-3}{3}$ and $e_f(0) = e_f(1) = \frac{n}{3}$.

Case(iv): $n \equiv 4(mod\,6)$
Define

$$f(u_1) = 2$$
$$f(u_2) = 3$$
$$f(u_3) = 1$$
$$f(u_4) = 4$$

and for $1 \leq i \leq \frac{n-4}{6}$,

$$f(u_{6i-1}) = 6i+1$$
$$f(u_{6i}) = 6i-1$$
$$f(u_{6i+1}) = 6i$$
$$f(u_{6i+2}) = 6i+2$$
$$f(u_{6i+3}) = 6i+3$$
$$f(u_{6i+4}) = 6i+4.$$

Then $e_f(-1) = e_f(0) = e_f(1) = \frac{n-1}{3}$.

Case(v): $n \equiv 5(mod\,6)$
Define

$$f(u_1) = 2$$
$$f(u_2) = 3$$
$$f(u_3) = 1$$
$$f(u_4) = 4,$$

for $1 \leq i \leq \frac{n-5}{6}$,

$$f(u_{6i-1}) = 6i+1$$
$$f(u_{6i}) = 6i-1$$
$$f(u_{6i+1}) = 6i$$
$$f(u_{6i+2}) = 6i+2$$
$$f(u_{6i+3}) = 6i+3$$
$$f(u_{6i+4}) = 6i+4$$

and

$$f(u_n) = n.$$

Then $e_f(-1) = e_f(0) = \frac{n-2}{3}$ and $e_f(1) = \frac{n+1}{3}$.

Case(vi): $n \equiv 0(mod\,6)$
Define

$$f(u_1) = 2$$
$$f(u_2) = 3$$
$$f(u_3) = 1$$
$$f(u_4) = 4,$$

for $1 \leq i \leq \frac{n-6}{6}$,

$$f(u_{6i-1}) = 6i+1$$
$$f(u_{6i}) = 6i-1$$
$$f(u_{6i+1}) = 6i$$
$$f(u_{6i+2}) = 6i+2$$
$$f(u_{6i+3}) = 6i+3$$
$$f(u_{6i+4}) = 6i+4$$

and

$$f(u_{n-1}) = n-1$$
$$f(u_n) = n.$$

Then $e_f(-1) = e_f(1) = \frac{n}{3}$ and $e_f(0) = \frac{n-3}{3}$.
Thus in all cases, $|e_f\,(i) - e_f\,(j)\,| \leq 1$ for all $-1 \leq i, j \leq 1$ and therefore P_n is a square difference 3-equitable graph. □

EXAMPLE 2. *The square difference 3-equitable labeling of P_{10} is shown below.*

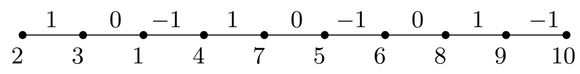

Fig 2. Square difference 3-equitable labeling of P_{10}

THEOREM 2. *The cycle C_n admits square difference 3-equitable labeling.*

PROOF. Let $C_n : u_1 u_2 ... u_n u_1$ be the cycle.

The square difference 3-equitable labeling of C_3 is given as follows.

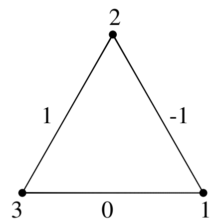

Fig 3. Square difference 3-equitable labeling of C_3

If $n \geq 4$, we consider the following cases.
Case(i): $n \equiv 1(mod\,6)$
Define

$$f(u_1) = 2$$
$$f(u_2) = 3$$
$$f(u_3) = 1$$
$$f(u_4) = 4,$$

for $1 \leq i \leq \frac{n-7}{6}$,

$$f(u_{6i-1}) = 6i+1$$
$$f(u_{6i}) = 6i-1$$
$$f(u_{6i+1}) = 6i$$
$$f(u_{6i+2}) = 6i+2$$
$$f(u_{6i+3}) = 6i+3$$
$$f(u_{6i+4}) = 6i+4$$

and

$$f(u_{n-2}) = n$$
$$f(u_{n-1}) = n-2$$
$$f(u_n) = n-1.$$

Then $e_f(-1) = e_f(1) = \frac{n-1}{3}$ and $e_f(0) = \frac{n+2}{3}$.

Case(ii): $n \equiv 2(mod\,6)$
Define

$$f(u_1) = 2$$
$$f(u_2) = 1$$
$$f(u_3) = 4$$
$$f(u_4) = 5$$
$$f(u_5) = 3,$$

for $1 \leq i \leq \frac{n-8}{6}$,

$$f(u_{6i}) = 6i+2$$
$$f(u_{6i+1}) = 6i$$
$$f(u_{6i+2}) = 6i+1$$
$$f(u_{6i+3}) = 6i+3$$
$$f(u_{6i+4}) = 6i+4$$
$$f(u_{6i+5}) = 6i+5$$

and

$$f(u_{n-2}) = n$$
$$f(u_{n-1}) = n-2$$
$$f(u_n) = n-1.$$

Then $e_f(-1) = e_f(1) = \frac{n+1}{3}$ and $e_f(0) = \frac{n-2}{3}$.

Case(iii): $n \equiv 3(mod\,6)$
Define

$$f(u_1) = 2$$
$$f(u_2) = 3$$
$$f(u_3) = 5$$
$$f(u_4) = 4$$
$$f(u_5) = 1$$
$$f(u_6) = 6,$$

for $1 \leq i \leq \frac{n-9}{6}$,

$$f(u_{6i+1}) = 6i+3$$
$$f(u_{6i+2}) = 6i+1$$
$$f(u_{6i+3}) = 6i+2$$
$$f(u_{6i+4}) = 6i+4$$
$$f(u_{6i+5}) = 6i+5$$
$$f(u_{6i+6}) = 6i+6$$

and

$$f(u_{n-2}) = n$$
$$f(u_{n-1}) = n-2$$
$$f(u_n) = n-1.$$

Then $e_f(-1) = e_f(0) = e_f(1) = \frac{n}{3}$.

Case(iv): $n \equiv 4(mod\,6)$
Define

$$f(u_1) = 2$$
$$f(u_2) = 3$$
$$f(u_3) = 1$$
$$f(u_4) = 4$$

and for $1 \leq i \leq \frac{n-4}{6}$,

$$f(u_{6i-1}) = 6i+1$$
$$f(u_{6i}) = 6i-1$$
$$f(u_{6i+1}) = 6i$$
$$f(u_{6i+2}) = 6i+2$$
$$f(u_{6i+3}) = 6i+3$$
$$f(u_{6i+4}) = 6i+4.$$

Then $e_f(-1) = e_f(1) = \frac{n-1}{3}$ and $e_f(0) = \frac{n+2}{3}$.

Case(v): $n \equiv 5(mod\,6)$
Define

$$f(u_1) = 2$$
$$f(u_2) = 1$$
$$f(u_3) = 4$$
$$f(u_4) = 5$$
$$f(u_5) = 3$$

and for $1 \leq i \leq \frac{n-5}{6}$,

$$f(u_{6i}) = 6i+2$$
$$f(u_{6i+1}) = 6i$$
$$f(u_{6i+2}) = 6i+1$$
$$f(u_{6i+3}) = 6i+3$$
$$f(u_{6i+4}) = 6i+4$$
$$f(u_{6i+5}) = 6i+5.$$

Then $e_f(-1) = e_f(1) = \frac{n+1}{3}$ and $e_f(0) = \frac{n-2}{3}$.

Case(vi): $n \equiv 0(mod\, 6)$
Define

$$f(u_1) = 2$$
$$f(u_2) = 3$$
$$f(u_3) = 5$$
$$f(u_4) = 4$$
$$f(u_5) = 1$$
$$f(u_6) = 6$$

and for $1 \leq i \leq \frac{n-6}{6}$,

$$f(u_{6i+1}) = 6i + 3$$
$$f(u_{6i+2}) = 6i + 1$$
$$f(u_{6i+3}) = 6i + 2$$
$$f(u_{6i+4}) = 6i + 4$$
$$f(u_{6i+5}) = 6i + 5$$
$$f(u_{6i+6}) = 6i + 6.$$

Then $e_f(-1) = e_f(0) = e_f(1) = \frac{n}{3}$.
Thus in all cases, $|e_f\,(i) - e_f\,(j)\,| \leq 1$ for all $-1 \leq i, j \leq 1$ and therefore C_n is a square difference 3-equitable graph. □

EXAMPLE 3. *The square difference 3-equitable labeling of C_9 is shown below.*

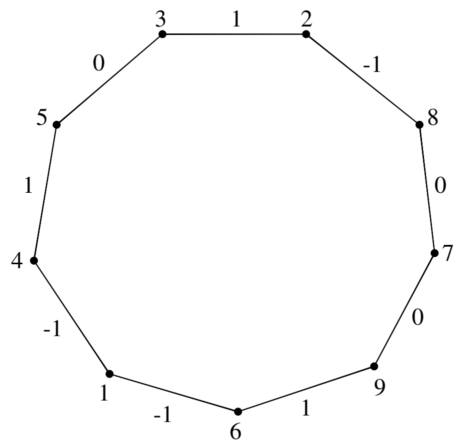

Fig 4. Square difference 3-equitable labeling of C_9

3. CONCLUSION

We have discussed here a new labeling called square difference 3-equitable labeling and we have investigated for paths and cycles only. The results reported here are new and expected to add new dimension to the theory of 3- equitable graphs. It is possible to investigate similar results for other graph families.

4. REFERENCES

[1] F. Harary, *Graph Theory*, Addition-Wesley, Reading, Mass, 1972.

[2] J. A. Gallian, *A dynamic survey of graph labeling*, Electronic Journal of Combinatorics, 17 (2010), DS6.

[3] J.Shiama, *Square sum labeling for some middle and total graphs*, International Journal of Computer Applications (0975-8887) Volume 37- No.4 January 2012.

[4] J.Shiama, *Square difference labeling for some graphs*, International Journal of Computer Applications (0975-8887) Volume 44- No.4, April 2012.

[5] J.Shiama, *Some Special types of Square difference graphs*, International Journal of Mathematical archives- 3(6), 2012, 2369-2374 ISSN 2229-5046.

[6] J.Shiama, *Square difference labeling for some path, fan and gear graphs*, International Journal of Scientific and Engineering Research volume 4, issues 3, March -2013, ISSN 2229-5518.