Image Enhancement based on Nonsubsampled Contourlet Transform using Matrix Factorization Techniques

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ABSTRACT
A unique method for image enhancement using the nonsubsampled Contourlet transform (NSCT) is presented here. Existing methods for image enhancement cannot capture the geometric information of images and tend to amplify noises when they are applied to noisy images since they cannot distinguish noises from weak edges. In contrast, the nonsubsampled Contourlet transform extracts the geometric information of images, which can be used to distinguish noises from weak edges. In this paper, we take the low pass subband of the \image obtained after nonsubsampled Contourlet decomposition. QR decomposition is applied on the lowest frequency subband. SVD decomposition technique is applied on the QR decomposed coefficients to obtain singular values. Therefore, changing the singular values will directly affect the illumination of the image; hence, the other information in the image will not be changed. Experimental results show the pro-posed method achieves better enhancement results than a wavelet-based image enhancement method.

Keywords
Discrete Wavelet Transform, Non Subsampled Contourlet Transform, Singular Value Decomposition, QR Decomposition, Image Equalization.

1. INTRODUCTION
Image enhancement is widely used in medical and biological imaging to improve the image quality. Traditional image enhancement methods such as unsharp masking split an image into different frequency subbands and amplify the high pass subbands. More recent methods are based on the discrete wavelet transform in a multiscale framework and achieve better results [1]. However, all existing methods decompose images in a separable fashion, and thus cannot use the geometric information in the trans-form domain to distinguish weak edges from noises. There-fore, they either amplify noises or introduce visible artifacts, when they are applied to noisy images.

Recently do and Vetterli proposed an efficient directional multiresolution image representation called the con-tourlet transform [2]. The Contourlet transform employs Laplacian pyramids to achieve multiresolution decomposition. Owing to the geometric information, the con-tourlet transform achieves better results than discrete wavelet Transform in image analysis applications such as denoising And texture retrieval [3]. Due to down sampling and upsam-pling, the Contourlet transform is shift-variant. However, shift-invariance is desirable in image analysis applications such as edge detection, contour characterization, and image enhancement [4]. Discrete wavelet transform is one of the suitable tool for contrast enhancement [5], but cannot capture information at edges and contours.

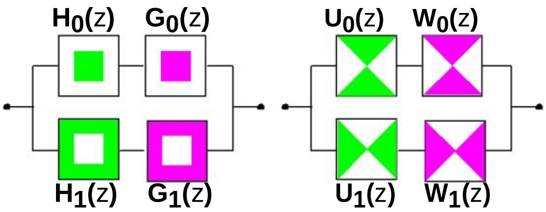
In this paper, we present the nonsubsampled Contourlet transform (NSCT), which is a *shift-invariant* version of the Contourlet transform. The NSCT is built upon iterated non-subsampled filter banks to obtain a shift-invariant directional multiresolution image representation. Based on the NSCT, we propose a new method for image enhancement using matrix factorization. The input image *is* first processed by using General Histogram Equalization (GHE) [6] to generate equalized image. Then, the equalized image and original image are transformed by NSCT into different subband images. Then QR Decomposition is applied to both the subband images. The QR decomposed coefficients are further factorized by SVD technique. The correction coefficients for the singular value matrix of both the images are calculated and the new LL subband is obtained. Then the new LL subband image is combined with other subband images by applying inverse transformation to generate equalized image.

2. CONSTRUCTION
We briefly introduce the construction of the nonsubsam-pled Contourlet transform. For the filter design, we refer readers to [7]. The Contourlet transform employs Laplacian pyramids for multiscale decomposition, and directional filter banks (DFB) for directional decomposition. To achieve the shift-invariance, the *nonsubsampled* Contourlet transform is built upon nonsubsampled pyramids and nonsubsampled DFB.

2.1 Nonsubsampled Pyramids
The nonsubsampled pyramid is completely different from the counterpart of the Contourlet transform, the Laplacian pyramid. The building block of the nonsubsampled pyra-mid is a two-channel nonsubsampled filter bank as shown in Fig. 1(a). A nonsubsampled filter bank has no down-sampling or up sampling, and hence it is shift-invariant. The Perfect reconstruction condition is given as

$$\mathbf{H_0(z)G_0(z) + H_1(z)G_1(z) = 1}$$

This condition is much easier to satisfy than the perfect reconstruction condition for critically sampled filter banks, and thus allows better filte-rs to be designed.



(a) (b)

Fig 1: Ideal frequency response of the building block of: (a) Nonsubsampled pyramid; (b) nonsubsampled DFB.

The ideal frequency response of the building block of the nonsubsampled pyramid is given in Fig. 1(a). To achieve the multiscale decomposition, we construct nonsubsampled pyramids by iterated nonsubsampled filter banks. For the next level, we up sample all filters by **2** in both dimensions.

Therefore, they also satisfy the perfect reconstruction condition. Note that filtering with the upsampled filter **H** (z^M **)** has the same complexity as filtering with **H (**z**)** using the `a trous' algorithm. The cascading of the analysis part is shown in Fig. 2. The equivalent filters of a **k**-th level cascading nonsubsampled pyramid are given by

$$H_n^{eq}(z) = \begin{cases} H_1(z^{2^{n-1}}) \prod_{j=0}^{n-2} H_0(z^{2^j}) & 1 \leq n < 2^k \\ \prod_{j=0}^{n-1} H_0(z^{2^j}) & n = 2^k \end{cases}$$

Where z^j stands for **[**z_1^j **,** z_2^j **]**. These filters achieve multiresolution analysis as shown in Fig. 3(a).

2.2 Nonsubsampled Directional Filter Banks

The nonsubsampled DFB is a shift-invariant version of the critically sampled DFB in the Contourlet transform. The building block of a nonsubsampled DFB is also a two-channel nonsubsampled filter bank. However, the ideal frequency response for a nonsubsampled DFB is different, as shown Fig 3.b.

To obtain finer directional decomposition, we iterate non-subsampled DFB's. For the next level, we up sample all filters by a quincunx matrix given by

$$Q = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

The frequency responses of two upsampled filters are given in Fig. 4 and the cascading of the analysis part is shown in Fig.

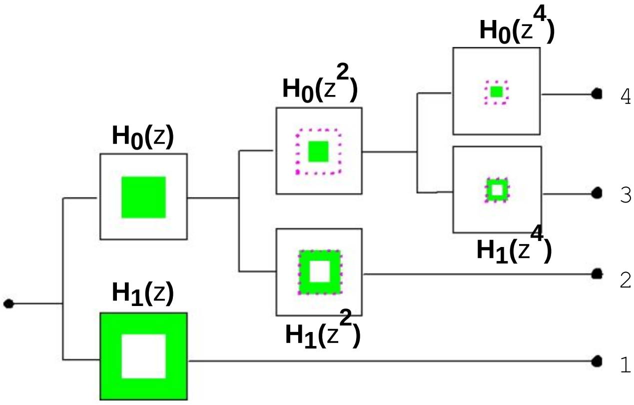
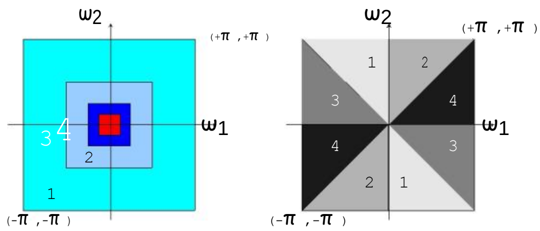


Fig 2. Iteration of two-channel nonsubsampled filter banks in the analysis part of a nonsubsampled pyramid. For upsampled filters, only effective passbands within dotted boxes are shown.



(a) (b)

Fig. 3. Frequency divisions of: (a) a nonsubsampled pyramid given in Fig. 2. (b) a nonsubsampled DFB given in Fig. 5.

2.3 Nonsubsampled Contourlet Transform

The nonsubsampled Contourlet transform combines nonsub-sampled pyramids and nonsubsampled DFB's as shown in Fig. 6. Nonsubsampled pyramids provide multiscale de-composition and nonsubsampled DFB's provide directional

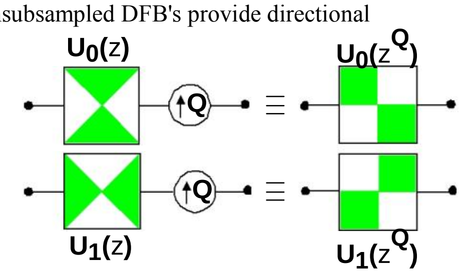


Fig. 4. Upsampling filters by a quincunx matrix Q.

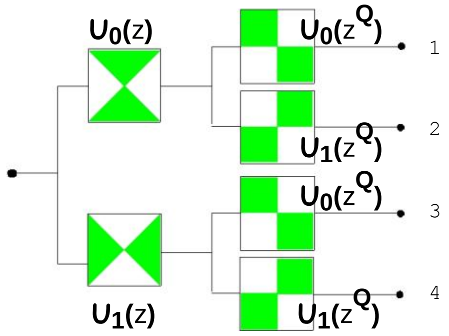
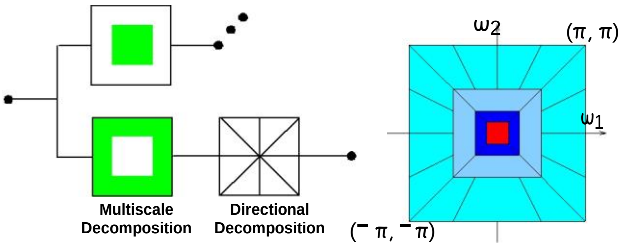


Fig. 5. The analysis part of an iterated nonsubsampled directional filter bank decomposition. This scheme can be iterated repeatedly on the lowpass subband outputs of nonsubsampled pyramids.



(a) (b)

Fig. 6. The nonsubsampled contourlet transform: (a) Block diagram. First, a nonsubsampled pyramid split the input into a lowpass subband and a highpass subband. Then a nonsubsampled DFB decomposes the highpass subband into several directional subbands. The scheme is iterated re-peatedly on the lowpass subband. (b) Resulting frequency division, where the number of directions is increased with frequency.

3. MATRIX FACTORIZATION TECHNIQUES

Matrix Factorization is a technique to reduce the matrix into some canonical form that can be easily accessed. The matrix decomposition is realized by the factorization of matrix. The factorization can be based on solving linear equation system or eigenvalues. There are several matrix decomposition methods such as SVD, QR etc. For solving linear equation system, singular value decomposition (SVD) technique is a good approach. For eigenvalue decomposition, schur factorization technique is appropriate. QR decomposition is a popular technique for matrix factorization in linear algebra for square and rectangular matrix.

3.1 QR DECOMPOSITION

It factors a matrix into an orthogonal and triangular component. In QR decomposition, a matrix can be represented as:

$$H = Q\,R. \qquad (1)$$

Where Q is an orthogonal matrix of size PxP and R is an upper triangular matrix of size PxQ. It decomposes an image matrix H of size PxQ.

3.2 SINGULAR VALUE DECOMPOSITION

The singular-value-based image equalization (SVE) technique is based on equalizing the singular value matrix obtained by singular value decomposition (SVD) [8]. SVD of an image, which can be interpreted as a matrix, is written as follows:

$$A = U_A \Sigma_A V_A^T \qquad (1)$$

where U_A and V_A are orthogonal square matrices known as hanger and aligner, respectively, and the Σ_A matrix contains the sorted singular values on its main diagonal. The idea of using SVD for image equalization comes from this fact that Σ_A contains the intensity information of a given image . The method uses the ratio of the largest singular value of the generated normalized matrix, with mean zero and variance of one, over a normalized image which can be calculated according to

$$\xi = \frac{\max \Sigma_{N(\mu=0,\mathrm{var}=1)}}{\max(\Sigma_A)} \qquad (2)$$

where $\Sigma_{(\mu=0,\mathrm{var}=1)}$ is the singular value matrix of the synthetic intensity matrix. This coefficient can be used to regenerate an equalized image using

$$E_{\mathrm{equalized}A} = U_A(\xi\Sigma_A)V_A^T \qquad (3)$$

which eliminates the illumination problem.

4. IMAGE ENHANCEMENT ALGORITHM

Existing image enhancement methods amplify noises when they amplify weak edges since they cannot distinguish noises from weak edges. In the frequency domain, both weak edges and noises lead to low-value coefficients. The non-subsampled Contourlet transform provides not only multiresolution analysis, but also geometric and directional representation. Since weak edges are geometric structures, while noises are not, we can use this geometric representation to distinguish them. The NSCT is shift-invariant such that each pixel of the transform subbands corresponds to that of the original image in the same location. Therefore, we gather the geometric information pixel by pixel from the NSCT coefficients.

The general procedure is as follows. The input image is processed by histogram equalization and both the equalized image and input image undergoes Non Subsampled Contourlet Transform to produce Low pass subband image and Band pass sub band image. QR Decomposition is applied on the lowest subband of both the images. SVD decomposition technique is applied on the QR decomposed coefficients to obtain singular values. The correction coefficient for the singular value matrix is calculated by using the following equation:

$$\partial = \frac{\max \sum LL_{\bar{A}}}{\max \sum LL_A} \qquad (1)$$

Where $\Sigma_{LL\bar{A}}$ the Low pass singular value matrix of the input image and Σ_{LLA} is the low pass singular value matrix of the output of the GHE. The new low pass subband image is

$$\Sigma LLA = \partial\, \Sigma LLA$$

$$LLA = ULLA\ \Sigma LL\bar{A}\, \Sigma LLA \qquad (2)$$

Now, the *LLA*, *LHA*, *HLA*, and *HHA* subband images of the original image are recombined by applying INSCT to generate the resultant equalized image. In the following section, the experimental results and the comparison of NSCT, CT and DWT are discussed.

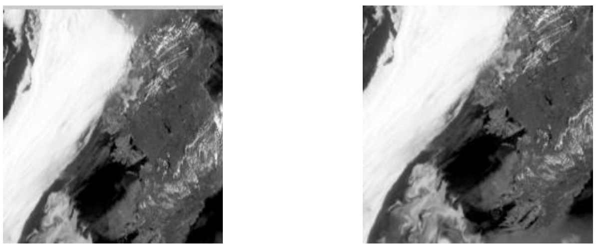
5. EXPERIMENTAL RESULTS

The results analysis has been done for image contrast enhancement with DWT,CT and NSCT. The Low contrast satellite image is given as the input and an equalized image is obtained. The Transforms are individually applied both on the equalized image as well as directly on the Low contrast satellite image. Then QR factorization is done. Then SVD is calculated for both the decomposed image and the correction coefficient is also calculated. A new subband image is obtained from which the original image is reconstructed.

In an attempt to estimate the quantitative performance, analysis is done using estimated Gaussian distribution of the enhanced images. In probability theory, Gaussian distribution, is a continuous probability distribution that is often used as a first approximation to describe real-valued random variables that tend to cluster around a single mean value. The graph of the associated probability density function is "bell"-shaped, and is known as the Gaussian function or bell curve. The width of the bell shaped curve illustrates illumination of the image. The proposed algorithm is implemented using Matlab. Experiments have been performed on over 50 randomly selected images from various sources which confirmed the qualitative results.

(a) (b)

Fig 7 (a) Original Image (b) Enhanced Image using DWT



(a) (b)

Fig 8 (a) Enhanced Image using CT (b) Enhanced Image using NSCT

In order to support the qualitative conclusions on the superiority of the proposed method, a quantitative analysis is done using Gaussian Distribution. The enhanced images are modelled by using the calculated mean (μ) and standard deviation (σ) of the output images. Any pixel of an image can be considered as a random variable with a distribution function. According to the central limit theorem, the sum of a sequence of random variables tends to have a Gaussian distribution [9]. It is clear from these distributions that in the estimated Gaussian functions the (σ) (measure of the width of the distribution) values of Contourlet Transform are higher than DWT. However, the estimated Gaussian distribution of the Contourlet Transform covers a wider gray level range which is verified by calculating the standard deviation (σ). PSNR values are also calculated .A higher PSNR would normally indicate that the reconstruction is of higher quality. Even a 0.5db changes in PSNR would improve image quality visible to the eye. The Table I shows the PSNR and σ of the Fig 7(b) , 8(a) and 8(b)

TABLE 1. PSNR and σ of different transforms

Parameters	Enhanced Image by DWT	Enhanced Image by CT	Enhanced Image by NSCT
σ	75.68	76.28	76.31
PSNR	24.29	24.29	26.93

6. CONCLUSION

We present the nonsubsampled contourlet transform con-structed by iterated nonsubsampled filter banks. This trans - form provides shift-invariant directional multiresolution im-age representation. We propose a new algorithm for im-age enhancement using the nonsubsampled contourlet trans-form. Experimental results show that the proposed algo-rithm achieves better enhancement results than the undeci-mated wavelet transform and contourlet transform

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