A Study on the Performance Analysis of a Batch Arrival Queue with Two Stages of Service, Bernoulli Schedule Vacation, Extended Vacation and Service Interruption

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ABSTRACT
In this paper a $M^{[X]}/G\ /1$ Queueing model with two stages of service is studied. Service interruption is considered as a major phenomenon. On completion of a service, the server will go for a vacation .An additional aspect of Optional extended vacation is considered in this model . In this model, repair process start immediately. Service time, Vacation time & Repair time follows general distribution. Steady state solution & Performance measures are derived.

Mathematics Subject Classification: 60K25, 60K30

Keywords
Random breakdown, Repair process, extended vacation, Steady state, Queue size distribution.

1. INTRODUCTION
We study a single server batch arrival queue with poisson arrivals and general service times. The service is given in two stages. On completion of two stages of service, the server has the option of taking vacation. Here the vacation follows a Bernoulli schedule vacation. An additional phenomenon of extended vacation is considered here. During service, breakdown arrives at random. So repair process follows immediately.. Moreover the server has a option of taking a extended vacation with probability r or continue service with probability 1-r. We assume that the service times, vacation times, breakdown times and extended vacation times each have a general distribution. Many researchers have made their efforts in queueing theory by considering various aspects like multi stages of service, Bernoulli vacation, Break down, repair process, general vacation and extended vacation. Madan [13, 14] studied a single server with two types of service with deterministic server vacation. Gautam Choudury et a;.[6] discussed a batch arrival with two phases of service with delay time in starting the repair process. Thangaraj and Vanitha [15] have obtained transient solution of two stage heterogeneous service with compulsory vacation and random breakdowns .Madan[2] discussed a queueing model with random failures and delayed repairs. Takagi[4] investigated time dependent analysis of M/G/1 vacation models with exhaustive service. This paper is organized as follows. The model assumption is given in section 2. In section 3 all the equations governing the mathematical system in the steady state are formulated. The supplementary variable technique is used in this section to obtain the closed form of the probability generating function of the queue length, average queue size and the average waiting time are given in section 4

2. MODEL ASSUMPTIONS
a) Customers arrive at the system in batches of variable size in a compound Poisson process and they are provided one by one service on a ‘first come’-first served basis. Let $\lambda c_i(i = 1,2,3 \ldots.)$ be the first order probability that a batch of i customers arrives at the system during a short interval of time $(t, t+dt)$, where $0 \le c_i \le 1$ and $\sum_{i=1}^{\infty} c_i = 1$ and $\lambda > 0$ is the mean arrival rate of batches

b) Each customer undergoes two stages of service provided by a single server on a first come first served basis. The service time follows general(arbitrary)distribution with distribution function G(s) and density function g(s). Let $\mu(x)dx$ be the conditional probability density of service completion during the interval $(x, x+dx)$,given that the elapsed time is x,so that

$$\mu\,(x) = \frac{g(x)}{1-G(x)} \tag{1}$$

and therefore

$$g(s) = \mu(s)e^{-\int_0^s \mu(x)dx} \quad i = 1,2 \tag{2}$$

c) As soon as a service is completed, the server may go for a vacation. Also the server may go for a extended vacation with probability r or return back to the system to continue the service for the next customer with probability 1-r. Let $\beta(x)dx$ be the conditional probability of a completion of a vacation during the interval $(x, x+dx)$,so that

d)The server’s vacation time follows a general(arbitrary)

$$\beta(x) = \frac{m(x)}{1-M(x)} \tag{3}$$

And, therefore

$m(s) = \beta(s)e^{-\int_0^s \beta(x)dx}$ with distribution function $M(s)$ and density function $m(s)$.

The server’s Extended vacation time follows a general(arbitrary) distribution with distribution function $H(s)$ and density function $h(s)$. Let $\theta(x)dx$ be the conditional probability of a completion of a Extended vacation during the interval $(x, x+dx)$,so that

$$\theta(x) = \frac{h(x)}{1-H(x)} \tag{3a}$$

And, therefore

$$h(s) = \theta(s)e^{-\int_0^s \theta(x)dx} \tag{4}$$

e) The system may breakdown at random and breakdowns are assumed to occur according to a Poisson stream with mean breakdown rate $\alpha > 0$.)

f)The server's repair time follows a general(arbitrary) distribution with distribution function $F(x)$ and density function $f(x)$. Let $\gamma(x)dx$ be the conditional probability of a completion of a repair during the interval $(x, x+dx)$, so that

Repair time $\gamma(x) = \frac{f(x)}{1-F(x)}$ and $f(s) = \gamma(s)e^{-\int_0^s \gamma(x)dx}$
(5)

g) Various stochastic process involved in the system are assumed to be independent of each other.

3. EQUATIONS GOVERNING THE SYSTEM

The equations governing the system are as follows:

$$\frac{\partial}{\partial x}P_n^{(1)}(x,t) + \frac{\partial}{\partial x}P_n^{(1)}(x,t) + \frac{\partial}{\partial x}(\lambda + \mu_1(x) + \alpha)P_n^{(1)}(x,t) = \lambda \sum_{i=1}^{n-1} C_i\ P_{n-i}^{(1)}(x,t)\ \ n \geq 1$$
(6)

$$\frac{\partial}{\partial x}P_0^{(1)}(x,t) + \frac{\partial}{\partial x}P_0^{(1)}(x,t) + \frac{\partial}{\partial x}(\lambda + \mu_1(x) + \alpha)P_0^{(1)}(x,t) = 0$$
(7)

$$\frac{\partial}{\partial x}P_n^{(2)}(x,t) + \frac{\partial}{\partial x}P_n^{(1)}(x,t) + \frac{\partial}{\partial x}(\lambda + \mu_2(x) + \alpha)P_n^{(2)}(x,t) = \lambda \sum_{i=1}^{n-1} C_i\ P_{n-i}^{(2)}(x,t)$$
(8)

$$\frac{\partial}{\partial x}P_0^{(2)}(x,t) + \frac{\partial}{\partial x}P_0^{(2)}(x,t) + \frac{\partial}{\partial x}(\lambda + \mu_2(x) + \alpha)P_0^{(2)}(x,t) = 0$$
(9)

$$\frac{\partial}{\partial x}V_n(x,t) + \frac{\partial}{\partial x}V_n(x,t) + \frac{\partial}{\partial x}(\lambda + \beta(x) + \delta)V_n(x,t) = \lambda \sum_{i=1}^{n-1} C_i\ V_{n-i}(x,t) + \delta V_{n+1}(x) \quad n \geq 1$$
(10)

$$\frac{\partial}{\partial x}V_o(x,t) + \frac{\partial}{\partial x}V_o(x,t) + \big(\lambda + \beta(x)\big)V_o(x,t) = \delta V_1(x)$$
(11)

$$\frac{\partial}{\partial x}E_n(x,t) + \frac{\partial}{\partial x}E_n(x,t) + \frac{\partial}{\partial x}(\lambda + \theta(x))E_n(x,t) = \lambda \sum_{i=1}^{n-1} C_i\ E_{n-i}(x,t) \quad n \geq 1 \quad (12)$$

$$\frac{\partial}{\partial x}E_0(x,t) + \frac{\partial}{\partial x}E_0(x,t) + \frac{\partial}{\partial x}(\lambda + \theta(x))E_0(x,t) = 0$$
(13)

$$\frac{\partial}{\partial x}R_n(x,t) + \frac{\partial}{\partial x}R_n(x,t) + (\lambda + \gamma(x))R_n(x,t) = \lambda \sum_{i=1}^{n-1} C_i\ R_{n-i}(x,t)$$
(14)

$$\frac{\partial}{\partial x}R_0(x,t) + \frac{\partial}{\partial x}R_0(x,t) + (\lambda + \gamma x)R_0(x,t) = 0$$
(15)

As $t \to \infty$, the above equation becomes

$$\lambda Q = \int_0^\infty R_0(x)\gamma(x)dx + (1-p)\int_0^\infty P_0^{(2)}(x)\mu_2(x)dx + (1-r)\int_0^\infty V_0(x)\beta(x)dx + \int_0^\infty E_0(x)\theta(x)dx$$
(16)

The following boundary Conditions are used to solve the above equations:

$$P_n^{(1)}(0) = (1-p)\int_0^\infty P_{n+1}^{(2)}(x)\mu_2(x)dx + (1-r)\int_0^\infty V_{n+1}(x)\beta(x)dx + \int_0^\infty E_{n+1}(x)\theta(x)dx \int_0^\infty R_{n+1}(x)\gamma(x)dx + \lambda C_{n+1}Q$$
(17)

$$P_n^{(2)}(0) = \int_0^\infty P_n^{(1)}(x)\mu_1(x)dx$$
(18)

$$V_n(0) = p\int_0^\infty P_n^{(2)}(x)\,\mu_2(x)dx$$
(19)

$$E_n(0) = r\int_0^\infty V_n(x)\,\beta(x)dx$$
(20)

$$R_n(0) = \alpha\left[\int_0^\infty P_n^{(1)}(x)\,dx + \int_0^\infty P_n^{(2)}(x)\,dx\right]$$
(21)

$$R_n^{(1)}(0) = \int_0^\infty D_n(x)\delta(x)dx$$
(22)

$$R_0(0) = 0$$
(23)

Multiplying Equation (6) by Z^n sum over n from 1 $to\ \infty$ and adding to (7), using Probability generating function, we obtain

$$\frac{\partial}{\partial x}P_q^{(1)}(x,z) + (\lambda - \lambda(z) + \mu_1(x) + \alpha)P_q^{(2)}(x,z) = 0$$
(24)

$$\frac{\partial}{\partial x}P_q^{(2)}(x,z) + (\lambda - \lambda(z) + \mu_2(x) + \alpha)P_q^{(2)}(x,z) = 0$$
(25)

$$\frac{\partial}{\partial x}V_q(x,z) + \big(\lambda - \lambda(z) + \beta(x)\big)V_q(x,z) = 0$$
(26)

$$\frac{\partial}{\partial x}R_q(x,z) + \big(\lambda - \lambda(z) + \gamma(x)\big)R_q(x,z) = 0$$
(27)

$$\frac{\partial}{\partial x}E_q(x,z) + \big(\lambda - \lambda(z) + \theta(x)\big)E_q(x,z) = 0$$
(28)

Multiplying Equation (17) by z^{n+1}, sum over n from $0\ to\ \infty$,

$$zP_q^{(1)}(0,z) = (1-p)\int_0^\infty P_q^{(2)}(x,z)\mu_2(x)dx + (1-r)\int_0^\infty V_q(x,z)\beta(x)dx + \int_0^\infty R_q(x,z)\gamma(x)dx + \int_0^\infty E_q(x,z)\theta(x)dx - \lambda Q\,(C(z)-1)$$
(29)

$$P_q^{(2)}(0,z) = \int_0^\infty P_q^{(1)}(x,z)\mu_1(x)dx$$
(30)

$$P_q^{(3)}(0,z) = \int_0^\infty P_q^{(2)}(x,z)\mu_2(x)dx$$
(30a)

$$V_q(0,z) = p\int_0^\infty P_q^{(2)}(x,z)\mu_2(x)dx$$
(31)

$$E_q(0,z) = r\int_0^\infty V_q(x,z)\beta(x)dx$$
(32)

$R_q(0,z) = \alpha z\left[\int_0^\infty P_q^{(1)}(x,z)dx + \int_0^\infty P_q^{(2)}(x,z)dx\right] = \alpha z\left[P_q^{(1)}(z) + P_q^{(2)}(z)\right]$

(33)

Integrating Equation (24) from 0 to ∞

$P_q^{(1)}(x,z) = P_q^{(1)}(0,z)e^{-(\lambda-\lambda C(z)+\alpha)x-\int_0^\infty \mu_1(t)dt}$
(34)

Let $\lambda - \lambda C(z) + \alpha = m$

Again Integrating the above equation, by parts with respect to x,

$P_q^{(1)}(z) = P_q^{(1)}(0,z)\left(\frac{1-\overline{G_1}(m)}{m}\right)$
(35)

Where $\overline{G_1}(m) = \int_0^\infty e^{-(\lambda-\lambda C(z)+\alpha)x}\, dG_1(x)$ is Laplace Stieltjes Transform of the service time $G_1(x)$.

Multiply both sides of Equation (34) by $\mu(x)$ & integrating over x, we get

$\int_0^\infty P_q^{(1)}(x,z)\mu_1(x)dx = P_q^{(1)}(0,z)\overline{G_1}(m)$
(36)

Similarly from equations (26), (27),(28) & (29) applying the same process we have,

$P_q^{(2)}(z) = \frac{P_q^{(2)}(0,z)(1-\overline{G_2}(m))}{m}$
(36a)

$$\int_0^\infty P_q^{(2)}(x,z)\mu_2(x)dx = P_q^{(2)}(0,z)\overline{G_2}(m)$$
$$= \left[\int_0^\infty P_q^{(1)}(x,z)\mu_1(x)dx\right]\overline{G_2}(m)$$
$$= P_q^{(1)}(0,z)\overline{G_1}(m)\overline{G_2}(m)$$
(37)

From (27), using (32) & (33), we have

$V_q(o,z) = p\, P_q^{(1)}(0,z)\overline{G_1}(m)\overline{G_2}(m)$
(38)

Integrating equation (22) from $0\ to\ x$, we have

$V_q(x,z) = V_q(o,z)e^{-(\lambda-\lambda C(z)+\alpha)x-\int_0^x \beta(t)dt}$

$= p\, P_q^{(1)}(0,z)\overline{G_1}(m)\overline{G_2}(m)e^{-\left(\lambda-\lambda C(z)+\alpha+\delta-\frac{\delta}{z}\right)x-\int_0^x \beta(t)dt}$
(39)

Let $s = \lambda - \lambda C(z) + \delta - \frac{\delta}{z}$

Integrating equation (35) by parts with respect to x, we have

$V_q(z) = \frac{p\, P_q^{(1)}(0,z)\overline{G_1}(m)\overline{G_2}(m)(1-\overline{K}(s))}{s}$
(40)

Where $\overline{K}(s) = \int_0^\infty e^{-\left(\lambda-\lambda C(z)+\alpha+\delta-\frac{\delta}{z}\right)x}\, dK(x)$ is the Laplace Stieltjes transform of the vacation time $K(x)$.

Multiplying equation (35) both $\beta(x)$ & integrating over x, we get

$\int_0^\infty V_q(x,z)\beta(x)dx = p\, P_q^{(1)}(0,z)\overline{G_1}(m)\overline{G_2}(m)\overline{K}(s)$
(41)

Applying the same process for equation (23) & (24)

$E_q(z) = \frac{rp\, P_q^{(1)}(0,z)\overline{G_1}(m)\overline{G_2}(m)\overline{K}(s)(1-\overline{H}(w))}{w}$
(42)

And therefore

$$\left[\begin{array}{c}\int_0^\infty E_q(x,z)\theta(x)dx = rp\, P_q^{(1)}(0,z)\overline{G_1}(m)\overline{G_2}(m)\overline{K}(s)\overline{H}(w) \\ \text{Where } \overline{H}(w) = \int_0^\infty e^{-(\lambda-\lambda C(z))x}\, dH(x) \text{is the Laplace} \\ \text{Stieltjes transform of the Extended vacation } H(x)\end{array}\right]$$

(43)

Also we have, integrating (24) from $0\ to\ x$

$R_q(x,z) = R_q(0,z)\left[e^{-\left(\lambda-\lambda C(z)+\delta-\frac{\delta}{z}\right)x-\int_0^x \gamma(t)dt}\right]$

$= \alpha z\, P_q^{(1)}(0,z)\left[\frac{1-\overline{G_1}(m)}{m} + \frac{\overline{G_1}(m)(1-\overline{G_2}(m))}{m}\right]\left[e^{-\left(\lambda-\lambda C(z)+\delta-\frac{\delta}{z}\right)x-\int_0^x \gamma(t)dt}\right]$

(44)

Integrating (40) by parts with respect to x, we obtain

$R_q(z) = \frac{\alpha z\, P_q^{(1)}(0,z)(1-\overline{G_1}(m)\overline{G_2}(m))(1-\bar{F}(s))}{ms}$
(45)

Multiply Equation (40) by $\gamma(x)$ on both sides and integrating with respect to x, we get

$\int_0^\infty R_q(x,z)\,\gamma(x)dx = \frac{\alpha z\, P_q^{(1)}(0,z)(1-\overline{G_1}(m)\overline{G_2}(m)\bar{F}(s))}{ms}$
(46)

Using Equation (33),(37), (39) and (42) Equation (25) becomes,

$$z\, P_q^{(1)}(0,z) = (1-p)\, P_q^{(1)}(0,z)\overline{G_1}(m)\overline{G_2}(m) + p\, P_q^{(1)}(0,z)\overline{G_1}(m)\overline{G_2}(m)\overline{K}(s)$$
$$+rp\, P_q^{(1)}(0,z)\overline{G_1}(m)\overline{G_2}(m)\overline{K}(s)\overline{H}(w)$$
$$+\frac{\alpha z\, P_q^{(1)}(0,z)(1-\overline{G_1}(m)\overline{G_2}(m)\bar{F}(s))}{m} + \lambda Q(C(z)-1)$$

$$P_q^{(1)}(0,z)\left(mz - m\left[\begin{array}{c}(1-p)\overline{G_1}(m)\overline{G_2}(m) - p\overline{G_1}(m)\overline{G_2}(m)\overline{K}(s) \\ -rp\overline{G_1}(m)\overline{G_2}(m)\overline{K}(s)\overline{H}(w)\end{array}\right] - \alpha z + \alpha z\overline{G_1}(m)\overline{G_2}(m)\bar{F}(s)\right)$$
$= m\lambda Q(C(z)-1)$

$P_q^{(1)}(0,z) = \frac{m\lambda Q(C(z)-1)}{Dr} = \frac{Nr}{Dr}$
(47)

$Nr = m\lambda Q(C(z)-1) = \overline{m}Qw$
(48)

$Dr = m\left(z - \overline{G_1}(m)\overline{G_2}(m)(1-p-p\overline{K}(s)-rp\overline{K}(s)\overline{H}(w))\right) - \alpha z + \alpha z\overline{G_1}(m)\overline{G_2}(m)\bar{F}(s)$ (49)

Therefore

$$P_q^{(1)}(z) = \frac{\bar{m}Qw}{m\left(z-\overline{G_1}(m)\overline{G_2}(m)\left(1-p-p\bar{K}(s)-rp\bar{K}(s)\bar{H}(w)\right)\right) -\alpha z+\alpha z\overline{G_1}(m)\overline{G_2}(m)\bar{F}(s)} \frac{1-\overline{G_1}(m)}{m}$$

(50)

$$P_q^{(2)}(z) = \frac{\bar{m}Qw}{m\left(z-\overline{G_1}(m)\overline{G_2}(m)\left(1-p-p\bar{K}(s)-rp\bar{K}(s)\bar{H}(w)\right)\right) -\alpha z+\alpha z\overline{G_1}(m)\overline{G_2}(m)\bar{F}(s)} \frac{1-\overline{G_1}(m)\overline{G_2}(m)}{m}$$

(51)

$$V_q(z) = \frac{\bar{m}Qw}{m\left(z-\overline{G_1}(m)\overline{G_2}(m)\left(1-p-p\bar{K}(s)-rp\bar{K}(s)\bar{H}(w)\right)\right) -\alpha z+\alpha z\overline{G_1}(m)\overline{G_2}(m)\bar{F}(s)} \frac{p\overline{G_1}(m)\overline{G_2}(m)\left(1-\bar{K}(s)\right)}{s}$$

(52)

$$E_q(z) = \frac{\bar{m}Qw}{m\left(z-\overline{G_1}(m)\overline{G_2}(m)\left(1-p-p\bar{K}(s)-rp\bar{K}(s)\bar{H}(w)\right)\right) -\alpha z+\alpha z\overline{G_1}(m)\overline{G_2}(m)\bar{F}(s)} \frac{rp\overline{G_1}(m)\overline{G_2}(m)\bar{K}(s)\left(1-\bar{H}(w)\right)}{w}$$

(53)

$$R_q(z) = \frac{\bar{m}Qw}{m\left(z-\overline{G_1}(m)\overline{G_2}(m)\left(1-p-p\bar{K}(s)-rp\bar{K}(s)\bar{H}(w)\right)\right) -\alpha z+\alpha z\overline{G_1}(m)\overline{G_2}(m)\bar{F}(s)} \frac{\alpha z\left(1-\overline{G_1}(m)\overline{G_2}(m)\right)\left(1-\bar{F}(s)\right)}{ms}$$

(54)

Let $M_q(z)$ denotes the probability generating function of the queue size

$$M_q(z) = P_q^{(1)}(z) + P_q^{(2)}(z) + V_q(z) + E_q(z) + R_q(z)$$

(55)

In order to obtain Q, We use the normalization condition

$$M_q(1) + Q = 1$$

(56)

4. THE AVERAGE QUEUE SIZE AND THE AVERAGE WAITING TIME

Let L_q denote the mean number of customers in the queue under the steady state. Then

$$L_q = \frac{d}{dz} M_q(z)\Big|_{z=1}$$

Since $S_q(z) = 0/0$ at $z = 1$, we use the result

$$L_q = \lim_{z\to 1} \frac{d}{dz} M_q(z)$$

$$= P_q(1) = \lim_{z\to 1} \frac{D'(z)N''(z) - N'(z)D''(z)}{2\left(D'(z)\right)^2}$$

$$= \frac{D'(1)N''(1) - N'(1)D''(1)}{2\left(D'(1)\right)^2}$$

(56)

$$N'(1) = \lambda Q E(I)[1 - \overline{G_1}(\alpha) + \overline{G_1}(\alpha)\overline{G_2}(\alpha)]$$

(57)

$$N''(1) = 2\lambda Q E(I)\left[\lambda E(I)\overline{G'}_1(\alpha) + \left(-\lambda E(I)\right)[\overline{G'}_1(\alpha)\overline{G_2}(\alpha) + \overline{G_1}(\alpha)\overline{G'}_2(\alpha)] - p\alpha\overline{G_1}(\alpha)\overline{G_2}(\alpha)E(K) + rp\alpha\overline{G_1}(\alpha)\overline{G_2}(\alpha)E(K) + \alpha\left(1 - \overline{G_1}(\alpha)\overline{G_2}(\alpha)\right)\left(-E(F)\right)\right] + \lambda Q E(I(I-1))[1 - \overline{G_1}(\alpha) + \overline{G_1}(\alpha)\overline{G_2}(\alpha)]$$

(58)

$$D'(1) = -\lambda E(I)[1 - \overline{G_1}(\alpha)\overline{G_2}(\alpha)(1 - p - p - rp)] + \alpha\left[1 - \left\{\left(-\lambda E(I)\left(\overline{G'}_1(\alpha)\overline{G_2}(\alpha) + \overline{G_1}(\alpha)\overline{G'}_2(\alpha)\right)\right)(1 - p - p - rp) + \overline{G_1}(\alpha)\overline{G_2}(\alpha)\left(-pE(K)(-\lambda E(I) + \delta)\right) - rp[E(k)(-\lambda E(I) + \delta) + E(H)(-\lambda E(I))]\right\}\right]$$

(59)

$$D''(1) = -\lambda E\left(I(I-1)\right)[1 - \overline{G_1}(\alpha)\overline{G_2}(\alpha)(1 - 2p - rp)] + 2\left(-\lambda E(I)\right)\left[1 - \left\{-\lambda E(I)\left(\overline{G'}_1(\alpha)\overline{G_2}(\alpha) + \overline{G_1}(\alpha)\overline{G'}_2(\alpha)\right)(1 - 2p - rp) + \overline{G_1}(\alpha)\overline{G_2}(\alpha)\left(-pE(k)(-\lambda E(I) + \delta)\right) - rp\left[E(K)(-\lambda E(I) + \delta) + E(H)\left(-\lambda E(I)\right)\right]\right\}\right] + \alpha\left\{\left[-\lambda E\left(I(I-1)\right)\left(\overline{G'}_1(\alpha)\overline{G_2}(\alpha) + \overline{G_1}(\alpha)\overline{G'}_2(\alpha)\right)\right](1 - 2p - rp) + \left(\lambda E(I)\right)^2[\overline{G''}_1(\alpha)\overline{G_2}(\alpha) + 2\overline{G'}_1(\alpha)\overline{G'}_2(\alpha) + \overline{G_1}(\alpha)\overline{G''}_2(\alpha)](1 - 2p - rp) - \lambda E(I)[\overline{G'}_1(\alpha)\overline{G_2}(\alpha) + \overline{G_1}(\alpha)\overline{G'}_2(\alpha)]\left[-\left(-\lambda E(I)E(H)\right)\right] + \left(-\lambda E(I)\right)\left[\left(\overline{G'}_1(\alpha)\overline{G_2}(\alpha) + \overline{G_1}(\alpha)\overline{G'}_2(\alpha)\right)\right][-pE(K)(-\lambda E(I) + \delta)E(K)] - rp[(-\lambda E(I) + \delta)E(K) - \lambda E(I)E(H)] + \left(-\lambda E(I)\right)\left(\overline{G'}_1(\alpha)\overline{G_2}(\alpha) + \overline{G_1}(\alpha)\overline{G'}_2(\alpha)\right)\left[-pE(K)(-\lambda E(I) + \delta) - rp[(-\lambda E(I) + \delta)E(K) + (-\lambda E(I)E(H))]\right] + \overline{G_1}(\alpha)\overline{G_2}(\alpha)(-p[(-\lambda E(I) + \delta)^2 E(K^2) + E(K)(-\lambda E(I)(I-1) - 2\delta)]) - rp\left\{\left((-\lambda E(I) + \delta)^2 E(K^2) + E(K)(-\lambda E(I(I-1) - 2\delta) + E(K)(-\lambda E(I) + \delta)\left(-\lambda E(I)E(H)\right) + (-\lambda E(I(I-1)E(H)) + \left(-\lambda E(I)\right)^2 E(H^2) - \lambda E(I)E(H)(-\lambda E(I) + \delta)E(K)\right)\right\}\right\}$$

(60)

Where primes and double primes in (56) denote first and second derivatives at $z = 1$, respectively.Carrying out the derivatives at $z = 1$ and if we substitute the values of $N'(1), N''(1), D'(1)$ and $D''(1)$ into (56) we obtain L_q in closed form. Further, the mean waiting time of a customer could be found using $W_q = L_q/\lambda$.

5. CONCLUSIONS

In this paper, we study a batch arrival queue with two stages of service, vacation, extended vacation and service interruptions. The average number of customers in the queue is found using the Supplementary variable method. This paper clearly analyses the steady state results and some performance measures of the model. As a future work in this model, various aspects like multi stages of service, optional repair, deterministic service time and delay time can be discussed.

6. REFERENCES

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