

Implementation of Rational Function Model for Rad-Orthokit Generation on Cartosat-1 Data

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ABSTRACT

Sensor models are required to establish the relationship between 3D object space and 2D image space. Traditionally this is done using the physical sensor model where the complete parameters of physical imaging system are known. The replacement sensor models are required to establish this relation without the knowledge of the physical sensor model. The rational function model (RFM) is one of the replacement model used in remote sensing with 78 rational polynomial coefficients (RPCs). RFM is a complete mathematical model, which approximately describes the physical imaging process in photogrammetry and remote sensing. In the absence of interior and exterior orientation such as camera model, position and orientation information of specific sensor, large number of ground control points (GCPs) are needed to solve all the unknown coefficients of the RFM and to achieve higher accuracies in the photogrammetric processing. The rational function model (RFM) can be used either as a replacement for physical sensor model (terrain dependent) or to express the physical model in the form of RPCs (terrain independent) for further processing.

In this paper the implementation aspects of terrain dependent RFM model for Cartosat-1 data for the Chitrapur, Simla, Himachal Pradesh state, India and the accuracies achieved and the stability of the model are discussed. The direct least square solutions to the RFM are implemented using row reduction. The validation of RFM is done at check points and achieved planimetric accuracy 1.5m,3.38m with respect to CE90 in X and Y directions respectively.

General Terms

Check Points ,Ground Control Points, RMSE.

Keywords

Feature editing, RFM , row reduction, Sensor Model,CE90.

1. INTRODUCTION

A sensor model describes the geometric relationship of 3D object coordinates to 2D image coordinates. There are various models to provide the relation and each model has its own merits and demerits. There are two broad categories of sensor models, which are generalized and physical. A physical sensor model needs the physical parameters of the sensor such as the position and the orientation of the sensor with respect to an object space co-ordinate system and these parameters are not correlated and yield accurate results. But in the recent past the

satellite data providers are not ready to part with the sensor model. As an alternative remote sensing data providers started RPC models where sensor model is provided in the form of RPC coefficients. But when RPCs are not provided by the data suppliers generalized terrain dependent methods are to be used to carry out ortho-rectification of the data. The generalized models represent the transformation of the co-ordinates from 3D to 2D by rational functions or polynomials. This paper describes implementation of RFM to establish relationship between object space and image space for Cartosat-1 data.

2. THE RATIONAL FUNCTION MODEL

RFM is a mathematical transformation between object space and image space co-ordinates. The RFM is defined as ratio of two cubic polynomials separately for row and column coordinates of image and hence it is given in[3] as

$$r = \frac{N_r(X, Y, Z)}{D_r(X, Y, Z)}$$

$$c = \frac{N_c(X, Y, Z)}{D_c(X, Y, Z)}$$

1

Where r ,c are normalized row and column coordinates of image space and X, Y, Z are normalized coordinates of object space. The constant term in the denominator of r and c is taken as 1 to avoid singularity in equation(1). Also $N_r(X, Y, Z), D_r(X, Y, Z), N_c(X, Y, Z), D_c(X, Y, Z)$ defined as in[3]

$$N_r(X, Y, Z) = a_1 + a_2X + a_3Y + a_4Z + a_5XY + a_6XZ + a_7YZ + a_8X^2 + a_9Y^2 + a_{10}Z^2 + a_{11}XYZ + a_{12}X^3 + a_{13}XY^2 + a_{14}XZ^2 + a_{15}X^2Y + a_{16}Y^3 + a_{17}YZ^2 + a_{18}ZX^2 + a_{19}ZY^2 + a_{20}Z^3$$

$$D_r(X, Y, Z) = b_1 + b_2X + b_3Y + b_4Z + b_5XY + b_6XZ + b_7YZ + b_8X^2 + b_9Y^2 + b_{10}Z^2 + b_{11}XYZ + b_{12}X^3 + b_{13}XY^2 + b_{14}XZ^2 + b_{15}X^2Y + b_{16}Y^3 + b_{17}YZ^2 + b_{18}ZX^2 + b_{19}ZY^2 + b_{20}Z^3$$

$$b_1 = 1$$

$$Nc(X, Y, Z) = c_1 + c_2X + c_3Y + c_4Z + c_5XY + c_6XZ + c_7YZ + c_8X^2 + c_9Y^2 + c_{10}Z^2 + c_{11}XYZ + c_{12}X^3 + c_{13}XY^2 + c_{14}XZ^2 + c_{15}X^2Y + c_{16}Y^3 + c_{17}YZ^2 + c_{18}ZX^2 + c_{19}ZY^2 + c_{20}Z^3$$

$$Dc(X, Y, Z) = d_1 + d_2X + d_3Y + d_4Z + d_5XY + d_6XZ + d_7YZ + d_8X^2 + d_9Y^2 + d_{10}Z^2 + d_{11}XYZ + d_{12}X^3 + d_{13}XY^2 + d_{14}XZ^2 + d_{15}X^2Y + d_{16}Y^3 + d_{17}YZ^2 + d_{18}ZX^2 + d_{19}ZY^2 + d_{20}Z^3$$

$$d_1 = 1$$

Where a_i 's, b_i 's, c_i 's, d_i 's are polynomial coefficients.

In RFM model, the distortions caused by optical projection can be expressed as 1st-order polynomial coefficients, and the error caused by the earth curvature, atmospheric refraction and lens distortion can be corrected by 2nd-order polynomial coefficients, and that caused by other unknown distortions can be simulated by 3rd-order polynomial coefficients[2].

2.1 RFC Generation Methodology

In order to avoid time consuming process and to improve the numerical stability of the equations in 1, both image and object space co-ordinates are normalized to the range of -1.0 to 1.0 [1].The normalization of the coordinates is computed as follows[1]:

$$r_n = \frac{r - r_0}{r_s}, \quad c_n = \frac{c - c_0}{c_s}$$

$$X_n = \frac{X - X_0}{X_s}, \quad Y_n = \frac{Y - Y_0}{Y_s}, \quad Z_n = \frac{Z - Z_0}{Z_s}$$

Where, r_0, c_0, X_0, Y_0, Z_0 are themean values for scan line number, pixel number, latitude, longitude and height respectively, and r_s, c_s, X_s, Y_s, Z_s are the scale values for scan line number, pixel number, latitude, longitude and height respectively. So from the expression of scan line and pixel line functions, we have [1]

$$r = \frac{(1 \ X \ Y \ Z \ \dots \ Y^3 Z^3) \cdot (a_1 \ a_2 \ \dots \ a_{20})^T}{(1 \ X \ Y \ Z \ \dots \ Y^3 Z^3) \cdot (1 \ b_2 \ \dots \ b_{20})^T}$$

$$c = \frac{(1 \ X \ Y \ Z \ \dots \ Y^3 Z^3) \cdot (c_1 \ c_2 \ \dots \ c_{20})^T}{(1 \ X \ Y \ Z \ \dots \ Y^3 Z^3) \cdot (1 \ d_2 \ \dots \ d_{20})^T}$$

Where a_i 's, b_i 's, c_i 's, d_i 's are polynomial coefficients.

$$\text{Let } M = \begin{bmatrix} 1 & X_1 & \dots & -r_1 Z_1^3 \\ 1 & X_2 & \dots & -r_n Z_2^3 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & X_3 & \dots & -r_n Z_n^3 \end{bmatrix}$$

$R=(X_1, X_2, \dots, X_n)$ is set of n number of ground control points. $J = (a_0 a_1 \dots a_{19} b_1 b_2 \dots b_{19})^T$ is the result matrix containing the RFCs for row of image coordinates.

We get, from equation number(1) as in [1]

$$V = MJ - R \rightarrow (2)$$

Where V is the error matrix. Since the error to be minimum, first take the values of V to be zero for direct least square solution[1]. It is noticed that solving equation(2) normally is critical because the matrix size is large and hence to solve the above equation multiply both the sides of the equation by the inverse of M matrix in order to get square matrices which can be worked easily with in terms of taking inverse and hence the equation becomes

$$0 = (M^T M)J - M^T R$$

Simplifying it further gives

$$J = (M^T M)^{-1} M^T R$$

Inverse of $M^T M$ is found with row reduction method by C-code and then computed the rational polynomial coefficients for a particular scene of Cartosat -1 from a set of ground control points.

2.2 Dataset used and Implementation

Cartosat-1 data acquired during the period 10th March, 2009 over Chitrapur, Simla, Himachal Pradesh state, India where the terrain undulations vary between 330 to 850m and ground control points which have an accuracy of 1m are used for generation of RPC's. The 600 GCPs are identified using Leica photogrammetric software. After adjustment with the control points and scales, offsets in image space and object space derived for 600 points and only 387 points are used in RFM model to generate RPC's. A text file is generated for 387 points containing the line, column pixel, latitude, longitude and height respectively. This file is used as an interface to the developed s/w to the RFM model.

The Input data is normalized using line offsets and scales. The variable matrix inverse is described by row reduction method using elementary operation by augmenting the identity matrix with the same dimension of the variable matrix. The same is done for column. 39 RPCS for row and 39 RPCS for column were generated.

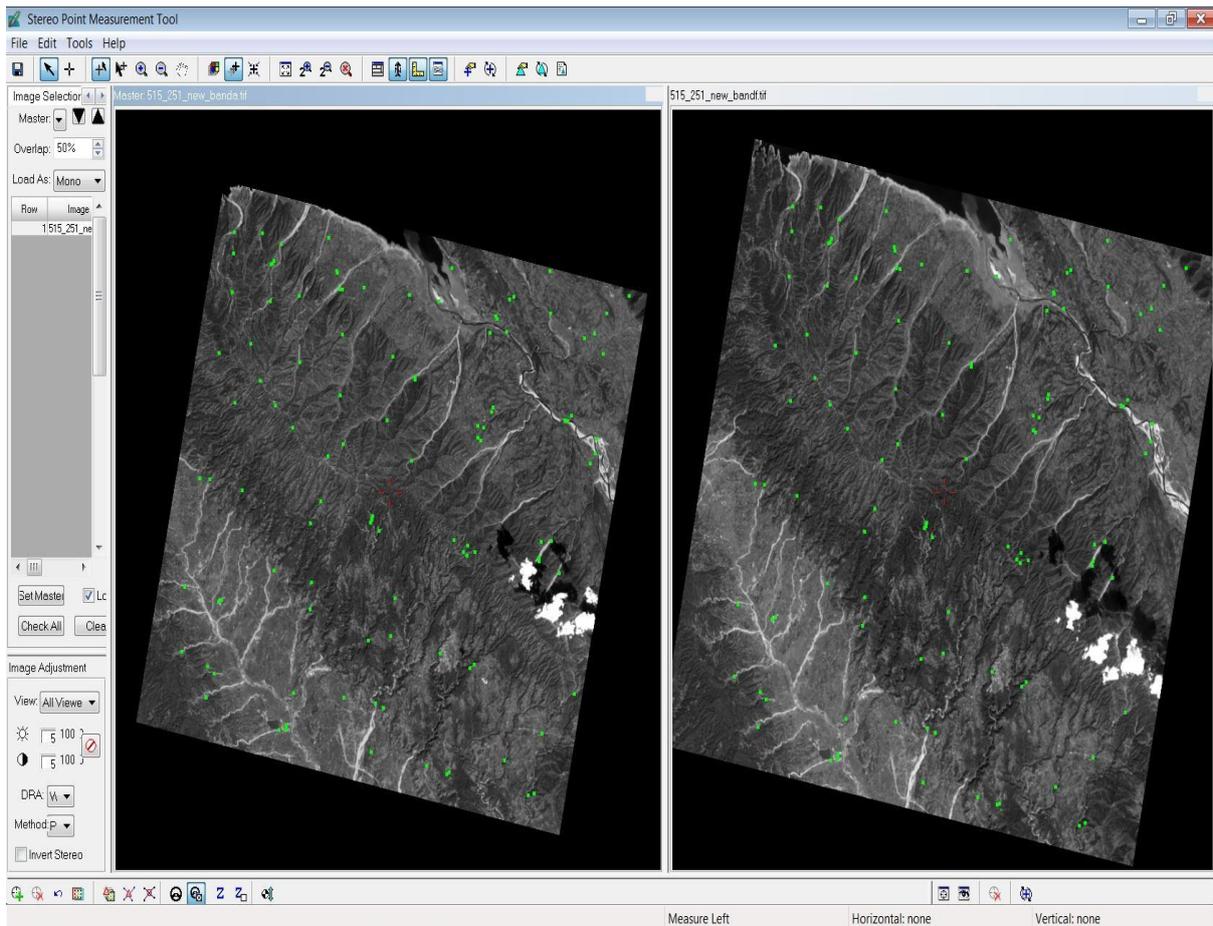


Figure 1: Marking of tie points on stereo pair

3. VALIDATION OF RFM

To validate Rational Function Model, the rational polynomial coefficients and the values of latitude, longitude and height of check points are substituted in the equations (1) to get the

values of line and pixels back. The errors between the check points image coordinates (line, pixel) and the computed image coordinates of same points using obtained RPC's through RFM model are shown Table1. For total image obtained RMSE=0.1137580pixels and calculated CE90(circular error/planimetric accuracy) at control points, check points with CE90=1.5175*RMSE and vertical accuracyLE90=1.6449*RMSE are shown in Table2.

Table 1. Differences between actual and calculated image coordinates of check points

Check Points Line No's	Check Points Pixel No's		Obtained Line No's	Obtained Pixel No's	Difference of Lines	Difference of Pixels
387.594196	477.448157	1	387.6063	477.5092	-0.01209	-0.06102
755.371459	405.801379	2	755.4197	405.8771	-0.04819	-0.07572
1122.864876	332.88501	3	1122.849	332.9274	0.016291	-0.04238
1490.90964	262.335203	4	1490.871	262.3728	0.039004	-0.0376
1858.735615	190.837214	5	1858.732	190.8752	0.003484	-0.03797
2226.667735	119.927684	6	2226.658	119.9675	0.009549	-0.03982
115.179527	932.798355	7	115.1485	932.8159	0.03107	-0.01753
483.036669	861.241314	8	483.0476	861.2563	-0.01092	-0.01504
850.690214	788.79148	9	850.5924	788.8009	0.097846	-0.00943
1218.607123	717.383069	10	1218.55	717.3751	0.057041	0.008004
1586.415846	645.433877	11	1586.418	645.4467	-0.00217	-0.01281
1954.325819	573.962853	12	1954.332	573.9712	-0.00653	-0.00833
2322.467739	503.844709	13	2322.461	503.8578	0.006574	-0.01312
2690.445071	433.007933	14	2690.437	433.0215	0.007707	-0.01358
3058.374649	361.986327	15	3058.362	362.0003	0.012528	-0.01395
3426.301548	291.164262	16	3426.284	291.1792	0.017566	-0.01493
3794.161441	219.932388	17	3794.178	219.9391	-0.0162	-0.00668
4162.006206	148.726358	18	4162.012	148.7333	-0.00625	-0.00692
4529.794365	77.013266	19	4529.798	77.01804	-0.00334	-0.00477
210.422471	1316.029239	20	210.3445	1316.051	0.078017	-0.0219
578.463045	1244.88674	21	581.6181	1244.908	-3.15502	-0.02101
946.052818	1171.847475	22	946.0258	1171.861	0.026981	-0.01365

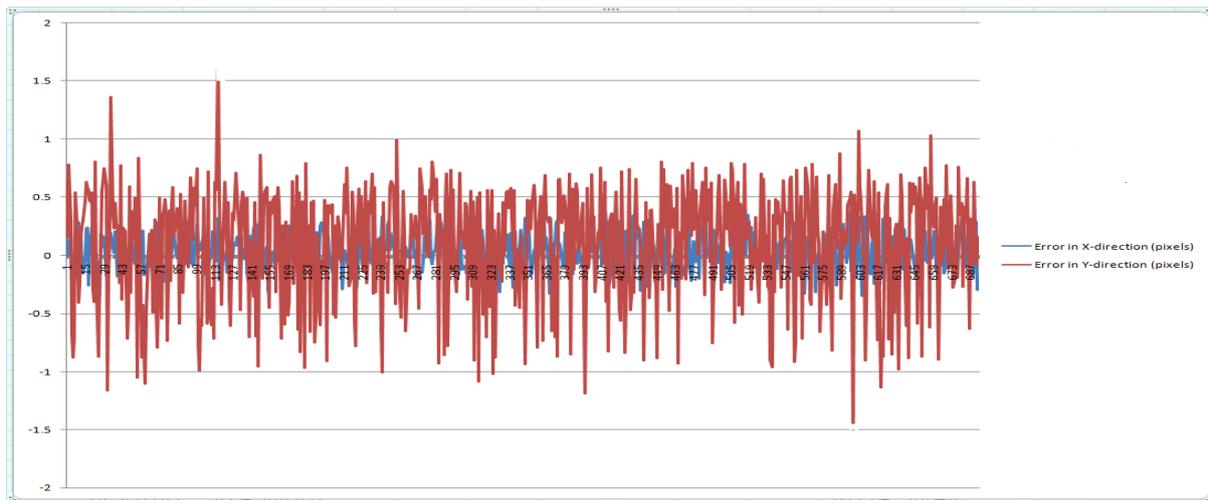


Figure 2: RPC accuracy in pixels.

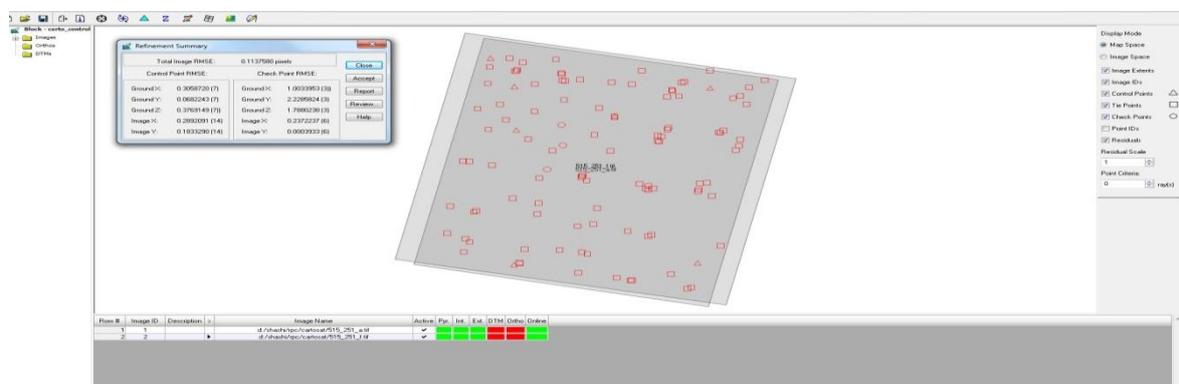


Figure 3: RMSE of total image, control points, check points.

Table 2. CE90 and LE90 values.

	Control Points RMSE	Check Points RMSE	Control Points		Check points	
			CE90	LE90	CE90	LE90
Ground X	0.30587206(7)	1.0033953(3)	0.464160851	-	1.522652368	-
Ground Y	0.0682243(7)	2.2285824(3)	0.103530375	-	3.381873792	-
Ground Z	0.3769149(7)	1.7880238(3)	-	0.619987	-	2.941120

3.1 Conclusions

Approaches for Cartosat-1 Orthokit product generation are presented including the test results for the Datasets used. From the tests performed on the given datasets, it can be inferred that RPCs can be provided to the users for photogrammetric processing at their end without compromising on the accuracy when sufficient GCP'S are available. The Rational Function model is able to model all the distortions in the image and achieved planimetric accuracy 1.5m, 3.38m in X and Y directions with respect to CE90 respectively and vertical accuracy is up to 3m with respect to LE90 at check points.

3.2 Acknowledgments

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4. REFERENCES

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