Reliability Measures of a Computer System with Priority for Repair and Hardware Redundancy

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ABSTRACT
This paper concentrates on the evaluation of reliability measures of a computer system by introducing the concept of priority of hardware repair over software up-gradation. The system operates with one more hardware in cold standby. The failure times of hardware and software are independent random variables which follow negative exponential distribution. The repair facility (called server) attends the faults immediately which occur during operation of the system. Repair of the hardware is done at failure while software undergoes for up-gradation. The distributions of hardware repair and software up-gradation times are taken as arbitrary with different probability density functions. The system model has been analyzed using semi-Markov process and regenerative point technique. The trends of some important measures of system effectiveness have been observed for arbitrary values of the parameters. The profit of the present model has also been compared with that of the system model in which no priority is given to hardware repair.

Keywords
Computer System, Hardware Redundancy, Priority to Repair, Software Up-gradation, Hardware Repair and Reliability Measures

1. INTRODUCTION
The rigorous reliability requirements of computer systems have forced the hardware and software engineers to probe the techniques that can be used in improving their performance. The technique of redundancy in different standby modes has been proved as one of the effective way to meet out this requirement. Therefore, over the few years, a lot of research work on stochastic modeling of repairable and non repairable systems has been appeared in the literature of reliability with the technique of redundancy. Welke et al. (1995) have discussed reliability modeling of a hardware/software system. The technique of unit wise redundancy in cold standby mode has also been used in computer systems. Malik and Anand (2010), Kumar et al. (2013) and Malik (2013) analyzed different computer system models with unit wise cold standby redundancy and different repair policies. But, it is also proved that component wise redundancy is better than unit wise redundancy so far as reliability is concerned. Malik and Munday (2014) developed a stochastic model for a computer system with hardware component in cold standby redundancy. Further, reliability of operating systems may be improved by giving priority in repair disciplines of one unit over the other. Anand and Malik (2012) studied a cold standby computer system by giving priority to hardware repair activities over software replacement.

Thus, the main aim of the present study is to evaluate reliability measures of a computer system by introducing the concept of priority of hardware repair over software up-gradation. The system operates with one more hardware in cold standby. The failure times of hardware and software are independent random variables which follow negative exponential distribution. The repair facility (called server) attends the faults immediately which occur during operation of the system. Repair of the hardware is done at failure while software undergoes for up-gradation. The hardware repair and software up-gradation done by the server are perfect. The distributions of hardware repair and software up-gradation times are taken as arbitrary with different probability density functions. The system model has been analyzed using semi-Markov process and regenerative point technique. The trends of some important measures of system effectiveness including mean time to system failure (MTSF), availability and profit function have been observed for arbitrary values of the parameters. The profit of the present model has also been compared with that of the model Malik and Munday (2014).

2. NOTATIONS

E	:	Set of regenerative states
$\bar{E}$	:	Set of non-regenerative states
O	:	Computer system is operative
Hcs	:	Hardware is in cold standby
a/b	:	Probability that the system has hardware / software failure
λ_1/ λ_2	:	Hardware/Software failure rate
HFUr /HFWr	:	The hardware is failed and under repair/waiting for repair
SFUg/SFWUg	:	The software is failed and under/waiting up-gradation
HFUR/HFWR	:	The hardware is failed and continuously under repair / waiting for repair from previous state
SFUG/SFWUG	:	The software is failed and continuously under up-gradation /waiting for up- gradation from previous state
g(t)/G(t)	:	pdf/cdf of hardware repair time
f(t)/F(t)	:	pdf/cdf of software up-gradation time
$q_{ij}(t)$/ $Q_{ij}(t)$	:	pdf / cdf of first passage time from regenerative state Si to

a regenerative state Sj or to a failed state Sj without visiting any other regenerative state in (0, t]

$q_{ij.k}(t)/Q_{ij.k}(t)$: pdf/cdf of direct transition time from regenerative state Si to a regenerative state Sj or to a failed state Sj visiting state S_k once in (0, t]

$M_i(t)$: Probability that the system up initially in state $S_i \in E$ is up at time t without visiting to any regenerative state

$W_i(t)$: Probability that the server is busy in the state S_i up to time 't' without making any transition to any other regenerative state or returning to the same state via one or more non-regenerative states.

μ_i : The mean sojourn time in state S_i which is given by

$$\mu_i = E(T)$$

$$= \int_0^\infty P(T > t)\, dt = \sum_j m_{ij},$$

where T denotes the time to system failure.

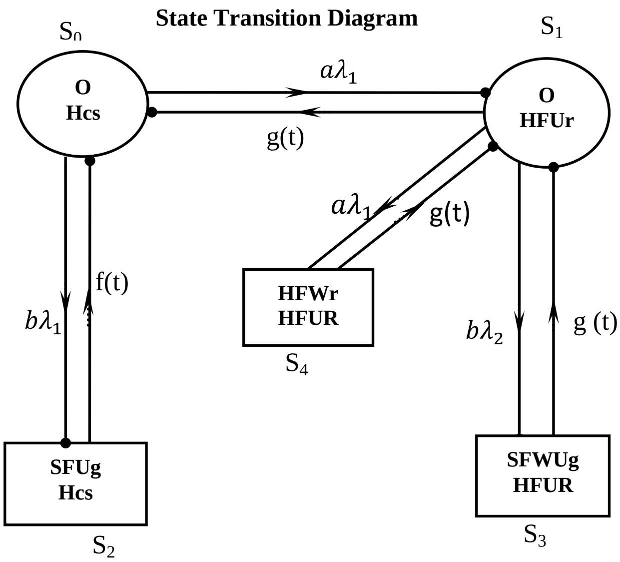
m_{ij} : Contribution to mean sojourn time (μ_i) in state S_i when system transits directly to state S_j so that

$$\mu_i = \sum_j m_{ij} \text{ and}$$

$$m_{ij} = \int_0^\infty t dQ_{ij}(t) = -q_{ij}^{*\,\prime}(0)$$

& / © : Symbol for Laplace-Stieltjes convolution/Laplace convolution

*/** : Symbol for Laplace Transformation (LT)/Laplace Stieltjes Transformation (LST)



○ **Up-State** ☐ **Failed State** • **Regenerative Point**

Figure 1

3. TRANSITION PROBABILITIES AND MEAN SOJOURN TIMES

Simple probabilistic considerations yield the following expressions for the non-zero elements.

$$p_{ij} = Q_{ij}(\infty) = \int_0^\infty q_{ij}(t)dt$$

$$p_{01} = \frac{a\lambda_1}{a\lambda_1 + b\lambda_2}, \qquad p_{02} = \frac{b\lambda_2}{a\lambda_1 + b\lambda_2},$$

$$p_{10} = g^*(a\lambda_1 + b\lambda_2),\ p_{13} = \frac{b\lambda_2}{a\lambda_1 + b\lambda_2}\{1 - g^*(a\lambda_1 + b\lambda_2)\}$$

$$p_{14} = \frac{a\lambda_1}{a\lambda_1 + b\lambda_2}\{1 - g^*(a\lambda_1 + b\lambda_2)\},$$

$$p_{20} = f^*(0),\quad p_{31} = p_{41} = g^*(0)$$

For $g(t) = \alpha e^{-\alpha t}$ and $f(t) = \theta e^{-\theta t}$

But, $f^*(0) = g^*(0) = 1$ and $a + b = 1$

we have

$$p_{11.3} = \frac{b\lambda_2}{a\lambda_1 + b\lambda_2 + \alpha}, \qquad p_{11.4} = \frac{a\lambda_1}{a\lambda_1 + b\lambda_2 + \alpha}$$

It can be easily verified that

$p_{01} + p_{02} = p_{10} + p_{13} + p_{14} = p_{20} = p_{31}$

$= p_{41} = p_{10} + p_{11.3} + p_{11.4} = 1$

The mean sojourn times (μ_i) is the state S_i are

$\mu_0 = \frac{1}{a\lambda_1 + b\lambda_2}$ $\quad \mu_1 = \frac{1}{a\lambda_1 + b\lambda_2 + \alpha}$ $\quad \mu_2 = \frac{1}{\theta}$

$\mu_3 = \mu_4 = \frac{1}{\alpha}$ $\quad \mu_1' = \frac{1}{\alpha}$

Also

$\mu_0 = m_{01} + m_{02}$,

$\mu_1 = m_{10} + m_{13} + m_{14}$,

$\mu_2 = m_{20}$ $\quad \mu_3 = m_{31}$ $\quad \mu_4 = m_{41}$

And $\mu_1' = m_{10} + m_{11.3} + m_{11.4}$

4. RELIABILITY AND MEAN TIME TO SYSTEM FAILURE (MTSF)

Let $\phi_i(t)$ be the cdf of first passage time from regenerative state $S_i$to a failed state. Regarding the failed state as absorbing state, we have the following recursive relations for $\phi_i(t)$,

$\phi_0(t) = Q_{01}(t)$ & $\phi_1(t) + Q_{02}(t)$

$\phi_1(t) = Q_{10}(t)$ & $\phi_0(t) + Q_{13}(t) + Q_{14}(t)$ (1)

Taking LST of above relations (1) and solving for $\phi_0^{**}(s)$

We have

$R^*(s) = \frac{1 - \phi_0^{**}(s)}{s}$

The reliability of the system model can be obtained by taking Laplace inverse transform of the above equation. The mean time to system failure (MTSF) is given by

$MTSF = \lim_{s \to 0} \frac{1 - \phi_0^{**}(s)}{s} = \frac{N_1}{D_1}$ (2)

where $N_1 = p_{01}\mu_1 + \mu_0$ and $D_1 = 1 - p_{01}p_{10}$ (3)

5. STEADY STATE AVAILABILITY

Let $A_i(t)$ be the probability that the system is in up-state at an instant't' given that the system entered regenerative state S_i at $t = 0$. The recursive relations for $A_i(t)$ are given as:

$A_0(t) = M_0(t) + q_{01}(t)©A_1(t) + q_{02}(t)©\, A_2(t)$

$A_1(t) = M_1(t) + q_{10}(t)©A_0(t)$

$\quad +\{q_{11.3}(t) + q_{11.4}(t)\}©\, A_1(t)$

$A_2(t) = q_{20}(t)©A_0(t)$ (4)

where

$M_0(t) = e^{-(a\lambda_1 + b\lambda_2)t}$ and $\mathrm{M_1(t)} = e^{-(a\lambda_1 + b\lambda_2)t}\, \overline{G(t)}$

Taking LT of relations (4) and solving for $A_0^*(s)$, the steady state availability is given by

$A_0(\infty) = \lim_{s \to 0} s\, A_0^*(s) = \frac{N_2}{D_2}$ (5)

where $N_2 = p_{10}\mu_0 + p_{01}\mu_1$ and

$D_2 = p_{10}\mu_0 + p_{01}\mu_1'\left(+\mu_3' p_{13}\right) + p_{10}p_{02}\mu_2$ (6)

6. BUSY PERIOD OF THE SERVER

6.1 Due to Hardware Repair

Let $B_i^H(t)$ be the probability that the server is busy in repairing the unit due to hardware failure at an instant't' given that the system entered state S_i at $t = 0$. The recursive relations for $B_i^H(t)$ are as follows:

$B_0^H(t) = q_{01}(t)©B_1^H(t) + q_{02}(t)©B_2^H(t)$

$B_1^H(t) = W_1^H(t) + q_{10}(t)©B_0^H(t)$

$\quad +\{q_{11.3}(t) + q_{11.4}(t)\}©B_1^H(t)$

$B_2^H(t) = q_{20}(t)©B_0^H(t)$ (7)

where

$W_1^H(t) = e^{-(a\lambda_1 + b\lambda_2)t}\, \overline{G(t)} + \left(a\lambda_1 e^{-(a\lambda_1 + b\lambda_2)t}©1\right)\overline{G(t)}$

$\quad +\left(b\lambda_2 e^{-(a\lambda_1 + b\lambda_2)t}©1\right)\overline{G(t)}$

6.2 Due to Software Up-gradation

Let $B_i^S(t)$ be the probability that the server is busy in upgrading the unit due to software failure at an instant't' given that the system entered state S_i at $t = 0$. The recursive relations for $B_i^S(t)$ are as follows:

$B_0^S(t) = q_{01}(t)©B_1^S(t) + q_{02}(t)©B_2^S(t)$

$B_1^S(t) = q_{10}(t)©B_0^S(t) + \{q_{11.3}(t) + q_{11.4}(t)\}©B_1^S(t)$

$B_2^S(t) = W_2^S(t) + q_{20}(t)©B_0^S(t)$ (8)

where $W_2^S(t) = \overline{F(t)}$

Taking LT of relations (7) & (8), solving for $B_0^{H^*}(t)$ and $B_0^{S^*}(t)$. The time for which server is busy due to repairs and up-gradations respectively are given by

$B_0^H(t) = \lim_{s \to 0} s\, B_0^{H^*}(t) = \frac{N_3^H}{D_2}$ (9)

$B_0^S(t) = \lim_{s \to 0} s\, B_0^{S^*}(t) = \frac{N_3^S}{D_2}$ (10)

where

$N_3^H = p_{01}W_1^{H^*}(0)$, $N_3^S = p_{10}p_{02}W_2^{S^*}$

$N_3^{Rp} = p_{01}p_{13}W_3^{Rp^*}(0)$ and D_2 is $already$ $mentioned.$

(11)

7 EXPECTED NUMBER OF HARDWARE REPAIRS

Let $NHR_i(t)$ be the expected number of hardware repairs by the server in (0, t] given that the system entered the regenerative state S_i at $t = 0$. The recursive relations for $NHR_i(t)$ are given as:

$NHR_0(t) = Q_{01}(t)$ & $(1 + NHR_1(t))$

$\quad +Q_{02}(t)$ & $NHR_2(t)$

$NHR_1(t) = Q_{10}(t)$ & $NHR_0(t)$

$\quad +(Q_{11.4}(t) + Q_{11.49}(t))$ & $NHR_1(t)$

$NHR_2(t) = Q_{20}(t)$ & $NHR_0(t)$ (12)

Taking LST of relations (12) and solving for $NHR_0^{**}(s)$. The expected number of hardware repair is given by

$NHR_0 = \lim_{s \to 0} sNHR_0^{**}(s) = \frac{N_4}{D_2}$ (13)

Where, $N_4 = p_{01}p_{10}$ *and* D_2 *is already mentioned.* (14)

8 EXPECTED NUMBER OF SOFTWARE UP-GRADATIONS

Let $NSU_i(t)$ be the expected number of software up-gradations in (0, t] given that the system entered the regenerative state S_i at $t = 0$. The recursive relations for $NSU_i(t)$ are given as follows:

$$NSU_0(t) = Q_{01}(t) \,\&\, NSU_1(t) + Q_{02}(t) \,\&\, (1 + NSU_2(t))$$

$$NSU_1(t) = Q_{10}(t) \,\&\, NSU_0(t) + \left(Q_{11.3}(t) + Q_{11.4}(t)\right) \,\&\, NSU_1(t)$$

$$NSU_2(t) = Q_{20}(t) \,\&\, NSU_0(t) \quad (15)$$

Taking LST of relations (15) and solving for $NSU_0^{**}(s)$. The expected numbers of software up-gradation are given by

$$NSU_0(\infty) = \lim_{s\to 0} sNSU_0^{**}(s) = \frac{N_5}{D_2} \quad (16)$$

Where

$N_5 = p_{10}p_{02}$ *and* D_2 *is already mentioned.* (17)

9 COST-BENEFIT ANALYSIS

The profit incurred to the system model in steady state can be obtained as:

$$P = K_0A_0 - K_1B_0^H - K_2B_0^S - K_3NHR_0 - K_4NSU_0 \quad (18)$$

where

$K_0 =$ Revenue per unit up-time of the system

$K_1 =$ Cost per unit time for which server is busy due to hardware repair

$K_2 =$ Cost per unit time for which server is busy due to software up-gradation

$K_3 =$ Cost per unit repair of the failed hardware

$K_4 =$ Cost per unit up-gradation of the failed software

and $A_0, B_0^H, B_0^S, NHR_0, NSU_0$ are already defined.

10 PARTICULAR CASES

Suppose $g(t) = \alpha e^{-\alpha t}$ *and* $f(t) = \theta e^{-\theta t}$

We can obtain the following results:

$$MTSF\ (T_0) = \frac{N_1}{D_1}$$

$$Availability\ (A_0) = \frac{N_2}{D_2}$$

Busy period due to hardware failure $(B_0^H) = \dfrac{N_3^H}{D_2}$

Busy period due to software failure $(B_0^S) = \dfrac{N_3^S}{D_2}$

Expected number of hardware repairs $(NHR_0) = \dfrac{N_4}{D_2}$

Expected number of software upgradations $(NSU_0) = \dfrac{N_5}{D_2}$

Where

$$N_1 = \frac{2a\lambda_1 + b\lambda_2 + \alpha}{(a\lambda_1 + b\lambda_2)(a\lambda_1 + b\lambda_2 + \alpha)}$$

$$D_1 = \frac{(a\lambda_1 + b\lambda_2)(a\lambda_1 + b\lambda_2 + \alpha) - \alpha a\lambda_1}{(a\lambda_1 + b\lambda_2)(a\lambda_1 + b\lambda_2 + \alpha)}$$

$$N_2 = \frac{\alpha + a\lambda_1}{(a\lambda_1 + b\lambda_2)(a\lambda_1 + b\lambda_2 + \alpha)}$$

$$D_2 = \frac{\alpha^2(b\lambda_2 + \theta) + \theta a\lambda_1(a\lambda_1 + b\lambda_2 + \alpha)}{\alpha\theta(a\lambda_1 + b\lambda_2)(a\lambda_1 + b\lambda_2 + \alpha)}$$

$$N_3^H = \frac{a\lambda_1}{(a\lambda_1 + b\lambda_2)(a\lambda_1 + b\lambda_2 + \alpha)}$$

$$N_3^S = \frac{\alpha b\lambda_2}{\theta(a\lambda_1 + b\lambda_2)(a\lambda_1 + b\lambda_2 + \alpha)}$$

$$N_4 = \frac{\alpha a\lambda_1}{(a\lambda_1 + b\lambda_2)(a\lambda_1 + b\lambda_2 + \alpha)}$$

$$N_5 = \frac{\alpha b\lambda_2}{(a\lambda_1 + b\lambda_2)(a\lambda_1 + b\lambda_2 + \alpha)}$$

11 CONCLUSION

The results for a particular case are obtained to depict the behaviour of some important reliability measures such as mean time to system failure (MTSF), availability and profit function as shown respectively in figures 2, 3, and 4. It is analyzed that these measures go on decreasing with the increase of failure rates (λ_1 and λ_2). However, they keep on increasing with the increase of hardware repair rate (α) and software up-gradation rate (θ) provided system has more chances of hardware failure than that of software failure (a>b). It is interesting to note that system becomes more profitable by interchanging the values of a and b (a<b). Hence, study reveals that a computer system with hardware redundancy in cold standby can be made more profitable by giving priority to hardware repair over software up-gradation. The comparison of profit of the system model is also shown in figure 5.

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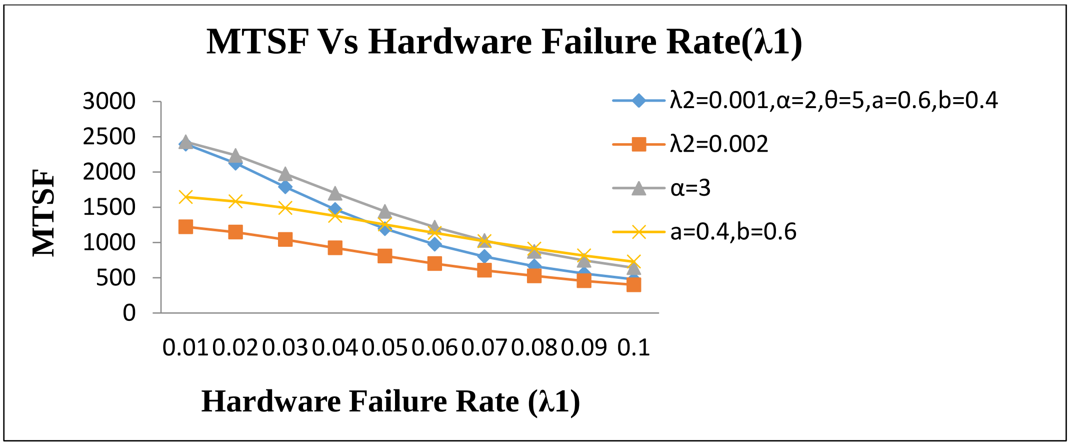

Fig:2

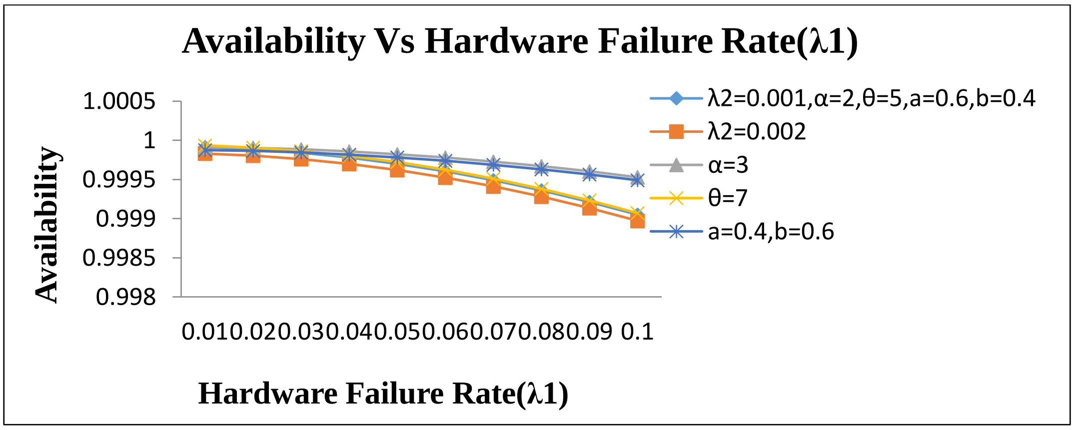

Fig:3

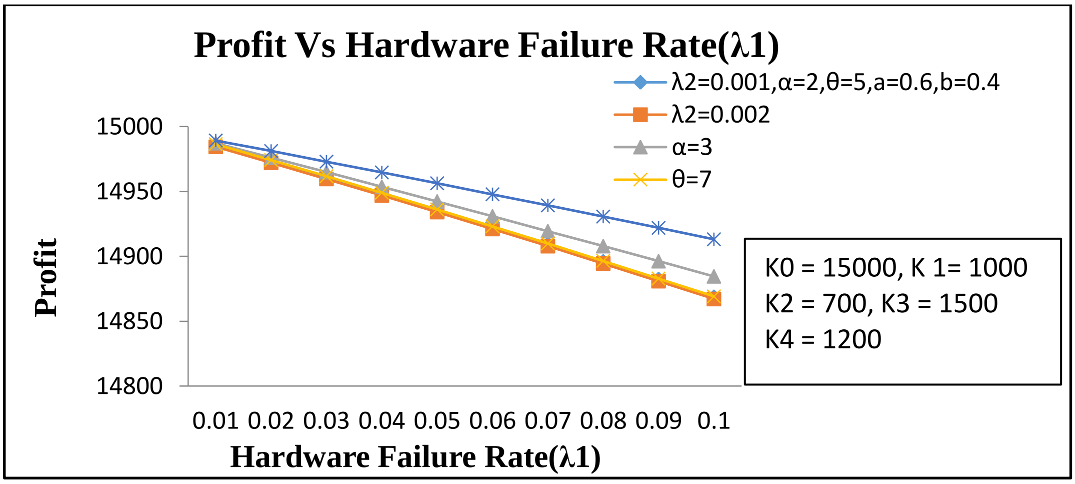

Fig:4

P – Profit of the present model and P1 – Profit of the model Malik and Munday (2014)
Graph of Profit Difference (P – P1)

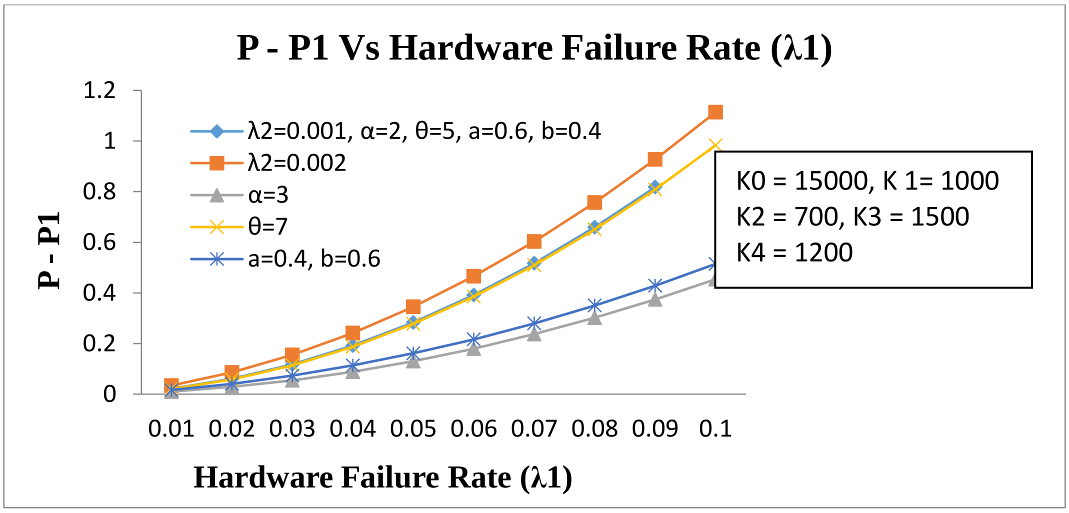

Fig. 5