A Soft computing Optimization based Two Ware-House Inventory Model for Deteriorating Items with shortages using Genetic Algorithm

Ajay Singh Yadav, PhD
SRM University
NCR Campus, GZB.

Kusum Gupta, PhD
Banasthali University
Jaipur, Rajasthan

Ankur Garg
MIET College
Meerut, U.P.

Anupam Swami
Govt. P. G. College
Sambhal, U.P.

ABSTRACT
In this paper a two warehouse inventory model for deteriorating items is considered under assumption that the Inventory cost (including holding cost and deterioration cost) in RW (Rented Warehouse) is higher than those in OW (Owned Warehouse) due to better preservation facilities in RW. The demand and holding cost, both are taken variable. Shortages are allowed in the OW and a fraction of shortages backlogged at the next replenishment cycle .This paper mainly dealt with deteriorating items with time dependent demand and variable holding cost which is constant for a definite time period and after that it increases according to length of ordering cycle in RW and remains constant in OW. Transportation cost is taken to be negligible and goods are transported on the basis of bulk release pattern. A genetic algorithm with varying population size is used to solve the model. In this GA a subset of better children is included with the parent population for next generation and size of this subset is a percentage of the size of its parent set. A numerical example is presented to illustrate the model and sensitivity is performed for a parameter keeping rest unchanged.

Keywords
Two warehouses, Instantaneous deterioration, Time-dependent Demand, Variable holding cost, shortages and Genetic Algorithm

1. INTRODUCTION
The term soft computing was also introduced by Prof. Zadeh in 1992. It is a collection of some biologically inspired methodologies such as Fuzzy Logic (FL), Neural Network (NN), Genetic Algorithm (GA), and other and their different combined forms Namely GA-FL, GA-NN, NNFL, GA-FN-NN, In Which Precision is traded for tractability robustness ease of implementation and a low cost solution.

1.1 Genetic Algorithm
Genetic algorithms are very different from most of the traditional optimization methods. Genetic algorithms need design space to be converted into genetic space. So, genetic algorithms work with a coding of variables. The advantage of working with a coding of variable space is that coding discretizes the search space even though the function may be continuous. A more striking difference between genetic algorithms and most of the traditional optimization methods is that GA uses a population of points at one time in contrast to the single point approach by traditional optimization methods. This means that GA processes a number of designs at the same time. As we have seen earlier to improve the search direction in traditional optimization methods transition rules are used and they are deterministic in nature but GA uses randomized operators. Random perators improve the search space in an adaptive manner.

Compute $t_n(u)$ for given u by Genetic Algorithm

For each given state u in order compute the optimal cost with respect to it, we must solve the following optimization problem

$$t_n(u) = \min_{v \geq u} T_n(u) - c_n \, . u,$$

$$u \in \Pi_n, v \in \Xi_n$$

Now, we give a genetic algorithm procedure for solving the above optimization problem

Genetic Algorithm Procedure for optimal cost:

Step1: Initialize pop_size chromosomes randomly.

Step2: Update the chromosomes by crossover and mutation operations.

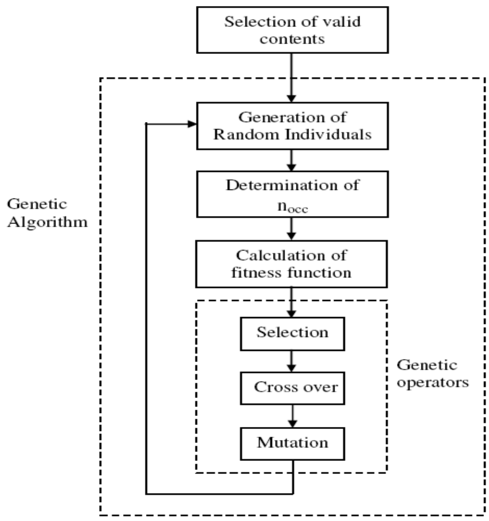
Step3: Calculate the objective values for all chromosomes.

Step4: Compute the fitness of each chromosome according to the objective values.

Step5: Select the chromosomes by spinning the roulette wheel.

Step6: Repeat the second to fifth steps for a given number of cycles.

Step7: Report the best chromosomes as the optimal cost for the given state.



The classical inventories models are basically developed with the single warehouse system .In the past, researchers have established a lot of research in the field of Inventory management and Inventory control system. Inventory management and control system basically deals with demand and supply chain problems and for this, production units (Producer of finished goods), vender's, suppliers and retailers need to store the raw materials, finished goods for future demand and supply in the market and to the customers. In the traditional models it is assumed that the demand and holding cost are constant and goods are supplied instantly under infinite replenishment policy, when demanded but as time passed away many researchers considered that demand may vary with time, due to price and on the basis of other factors and holding cost also may vary with time and depending on other factors. Many models have been developed considering various time dependent demand with shortages and without shortage. All those models that consider demand variation in response to inventory level, assume that the holding cost is constant for the entire inventory cycle. In studies of inventory models, unlimited warehouse capacity is often assumed. However, in busy marketplaces, such as super markets, corporation markets etc. the storage area for items may be limited. Another case, of inadequate storage area, can occur when a procurement of a large amount of items is decided. That could be due to, an attractive price discount for bulk purchase which is available or, when the cost of procuring goods is higher than the other inventory related costs or, when demand for items is very high or, when the item under consideration is a seasonal product such as the yield of a harvest or, when there are some problems in frequent procurement. In this case these items cannot be accommodated in the existing store house (the own warehouse, abbreviated as OW). Hence, in order to store the excess items, an additional warehouse (the rented warehouse, abbreviated as RW),which may be located at a short distance from the OW or a little away from it, due to non-availability of warehouse nearby, is hired on a rental basis.

2. RELATED WORKS

Hartely (1976) discussed an inventory model with two storage facilities. Ghare and Schrader (1963) initially worked in this field and they extended Harris (1915) EOQ model with deterioration and shortages. Goyal and Giri (2001) gave a survey on recent trends in the inventory modelling of deteriorating items. Lee and Wu (2004) developed a note on EOQ model for items with mixtures of exponential distribution deterioration, shortages and time varying demand. It is generally assumed, that the holding cost in the RW is higher than that in the OW, due to the additional cost of maintenance, material handling, etc. To reduce the inventory costs, it will be cost-effective to consume the goods of the RW at the earliest.

In the many literature deterioration phenomenon was not taken into account. Since many items are deteriorate with time, some instantly and some after a fixed life time of its own. Assuming the deterioration in both warehouses, Sarma (1987), extended his earlier model to the case of infinite replenishment rate with shortages. Pakkala and Achary (1992) extended the two-warehouse inventory model for deteriorating items with finite replenishment rate and shortages, taking time as discrete and continuous variable, respectively. In these models mentioned above the demand rate was assumed to be constant. Subsequently, the ideas of time varying demand and stock dependent demand considered by some authors, such as Goswami and Chaudhary (1998), Bhunia and Maiti(1998), Bankerouf (1997), Kar et al. (2001) and others. Goel and Giri (2001) suggested a review of deteriorating inventory literature in which all the inventory models for deteriorating items assume that the deterioration occurs as soon as the retailer receives the commodities.

Last so many years, some researchers have given attention to the situation where holding cost is taken constant per item per unit of time but due to the time value of money it is not assumed that holding cost will remain always constant and thus holding cost may vary with time. Chang (2004) developed an inventory models with stock dependent demand and nonlinear holding costs for deteriorating items. Ajanta Roy (2008) developed an inventory model for deteriorating items with time varying holding cost and price dependent demand. C. Sugapriya and K. Jeyaraman (2008) developed an EPQ model for deteriorating items in which holding cost varies with time. Maya Gyan and A. K. Pal (2009) developed a two-warehouse inventory model for deteriorating items with stock dependent demand rate and holding cost. Bindu Vaish and Garima Garg (2011) consider variable holding cost for development of Optimal Ordering and Transfer Policy for an Inventory System. Mukesh Kumar et. al. (2012) developed a Deterministic Inventory Model for Deteriorating Items with Price Dependent Demand and Time Varying Holding Cost under Trade credits .Yadav A.S. and Swami A. (2013) developed a two- warehouse inventory model for deteriorating items with exponential demand and variable holding cost. K. D. Rathor and P.H Bhathawala (2013) constructed a model with variable holding cost and inventory level dependent demand. R. P. Tripathi (2013) developed an Inventory model for varying demand and variable holding cost.Vinod Kumar Mishra et.al. (2013) developed an inventory model with variable holding cost and salvage value. Vipin Kumar et. al. (2013) developed an inventory model with selling price dependent demand and variable holding cost. Meghna Tyagi and S. R. Singh (2013) developed two-ware-house inventory model with time dependent demandand variable holding cost.

Nia. et.al. (2015) suggested a hybrid genetic and imperialist competitive algorithm for green vendor managed inventory of multi-item multi-constraint EOQ model under shortage. Ren Qing-dao-er-ji, et.al. (2013) developed Inventory based two-objective job shop scheduling model and its hybrid genetic algorithm. Seyed Hamid Reza Pasandideh, et.al. (2011) extended A genetic algorithm for vendor managed inventory control system of multi-product multi-constraint economic order quantity model. Ata Allah Taleizadeh, et.al. (2013) suggested A hybrid method of fuzzy simulation and genetic algorithm to optimize constrained inventory control systems with stochastic replenishments and fuzzy demand Ilkay Saracoglu, et.al. (2014) developed A genetic algorithm approach for multi-product multi-period continuous review inventory models. R.K. Gupta, et.al. (2009) extended an application of Genetic Algorithm in solving an inventory model with advance payment and interval valued inventory costs and an application of genetic algorithm in a marketing oriented inventory model with interval valued inventory costs and three-component demand rate dependent on displayed stock level. M.J. Li, et.al. (2010) extended Optimizing emission inventory for chemical transport models by using genetic algorithm. Seyed Hamid Reza Pasandideh, et.al. (2011) developed a parameter-tuned genetic algorithm to optimize two-echelon continuous review inventory systems. Javad Sadeghi andSeyed Taghi Akhavan Niaki et. al. (2015) extended two parameter tuned multi-objective evolutionary algorithms for a bi-objective vendor managed inventory model with trapezoidal fuzzy demand. Masao Yokoyama (2002) extended integrated optimization of inventory-distribution systems by random local search and a genetic algorithm. Tamás Varga, et.al. (2013) suggested 19 - Improvement of PSO Algorithm by Memory-Based Gradient Search—Application in Inventory Management. Sasan Khalifehzadeh, et.al. (2015) developed A four-echelon supply chain network design with shortage: Mathematical modeling and solution methods. A.K. Bhunia and Ali Akbar Shaikh (2015) extended An application of PSO in a two-warehouse inventory model for deteriorating item under permissible delay in payment with different inventory policies. Yoshiaki Shimizu and Takatobu Miura (2012) extended Effect of Topology on Parallel Computing for Optimizing Large Scale Logistics through Binary PSO. Salah Alden Ghasimi, et.al. (2014) developed a genetic algorithm for optimizing defective goods supply chain costs using JIT logistics and each-cycle lengths. Ying-Hua Chang (2010) gave Adopting co-evolution and constraint-satisfaction concept on genetic algorithms to solve supply chain network design problems. Bongju Jeong, et.al. (2002) suggested a computerized causal forecasting system using genetic algorithms in supply chain management. Fulya Altiparmak, et.al. (2006) developed a genetic algorithm approach for multi-objective optimization of supply chain networks. Antonio Costa, et.al. (2010) suggested a new efficient encoding/decoding procedure for the design of a supply chain network with genetic algorithms. Fulya Altiparmak, et.al. (2009) extended a steady-state genetic algorithm for multi-product supply chain network design. Miguel Zamarripa, et.al. (2012) gave Supply Chain Planning under Uncertainty using Genetic Algorithms. Reza Zanjirani Farahani, Mahsa Elahipana (2008) gave a genetic algorithm to optimize the total cost and service level for just-in-time distribution in a supply chain. David Naso, et.al. (2007) extended Genetic algorithms for supply-chain scheduling: A case study in the distribution of ready-mixed concrete. S.H. Zegordi, et.al. (2010) developed a novel genetic algorithm for solving production and transportation scheduling in a two-stage supply chain. R.J. Kuo, Y.S. Han (2011) suggested a hybrid of genetic algorithm and particle swarm optimization for solving bi-level linear programming problem – A case study on supply chain model

In this paper a deterministic Inventory model for deteriorating items with two level of storage system and time dependent demand with partial backlogged shortages is developed. Stock is transferred RW to OW under bulk release pattern and the transportation cost is taken to be negligible. The deterioration rates in both the warehouses are constant but different due to the different preservation procedures. Holding cost is considered to be constant up to a definite time and is increases. A genetic algorithm with varying population size is used to solve the model. In this GA a subset of better children is included with the parent population for next generation and size of this subset is a percentage of the size of its parent set. The numerical example is presented to demonstrate the development of mode land to validate it. Sensitivity analysis is performed separately for each parameter.

3. ASSUMPTION AND NOTATIONS

The mathematical model of two warehouse inventory model for deteriorating items is based on the following notation and assumptions.

3.1 Notations:

CA : Ordering cost per Order.

W1 : Capacity of OW.

W2 : Capacity of RW.

T : The length of replenishment cycle.

Qmax : Maximum Inventory level per cycle to be ordered.

t_1 : The time up to which inventory vanishes in RW.

t_2 : The time at which inventory level reaches to zero in OW and shortages begins.

k : Definite time up to which holding cost is constant.

h_m : The holding cost per unit time in OW .

h_r : The holding cost per unit time in RW.

s_c : The shortages cost per unit per unit time.

L_c : The opportunity cost per unit per time

$I^r(t)$: The level of inventory in RW.

$I^i(t)$: The level of inventory in OW where i=1,2.

$I^s(t)$: Determine the inventory level at time t in which the product has shortages.

α : Deterioration rate in RW Such that $0<\alpha<1$;

β : Deterioration rate in OW such that $0<\beta<1$;

Cp : Purchase cost per unit of items.

IB : Maximum amount of inventory backlogged.

IL : Amount of inventory lost

CP : Cost of purchase

CS : The present worth cost of shortages

CL : The present worth cost of lost sale

H_C : The present worth cost of holding inventory

$T^C(t_1, T)$: The total relevant inventory cost per unit time of inventory system.

3.2 Assumption

1. Replenishment rate is infinite and lead time is negligible i.e. zero.
2. The time horizon of the inventory system is infinite.
3. Goods of OW are consumed only after the consumption of goods kept in RW due to the more holding cost in RW than in OW.
4. The OW has the limited capacity of storage and RW has unlimited capacity.
5. Demand vary with time and is linear function of time and given by $D(t)=\begin{cases} a & if \quad t=0 \\ a+bt & if \quad t>0 \end{cases}$; wherea>0 and b>0 ;
6. For deteriorating items a fraction of on hand inventory deteriorates per unit time in both the warehouse with different rate of deterioration.
7. Shortages are allowed and demand is partially backlogged at the beginning of next replenishment.
8. The unit inventory cost (Holding cost) in RW>OW.
9. We assume that the holding cost will be fixed till a definite time in RW and the will increased according to a fraction of ordering cycle length. So for holding cost (h_r) ,we have k a time moment before which holding cost is constant.

$$h_r=\begin{cases} h_r & if \quad t \le k \\ h_r t & if \quad t > k \end{cases}$$

4. MATHEMATICAL FORMULATION OF MODEL AND ANALYSIS

In the beginning of the cycle at t=0 a lot size of Q_{max} units of inventory enters into the system in which backlogged(Q_{max}-IB) units are cleared and the remaining units M is kept into two storage as W_1 units in OW and W_2 units in RW.(See Figure-1)

During the time interval [0 t_1] the inventory in RW decrease due to the demand and deterioration and is governed by the following differential equation:

$$\frac{dI^r(t)}{dt} = -(a + bt) - (\alpha I^r(t)); \qquad 0\le t\le t_1 \qquad (1)$$

In the time interval [0 t_2] the inventory level decreases in OW decreases due to deterioration only and is governed by differential equation

$$\frac{dI^{1w}(t)}{dt} = -\beta I^{1w}(t) \quad ; \qquad 0\le t\le t_1 \qquad (2)$$

During time interval [$t_1 t_2$] the inventory level in OW is decreases due to demand and deterioration both and is governed by the following differential equation

$$\frac{dI^{2w}(t)}{dt} = -(a + bt) - (\beta I^r(t)); \qquad t_1\le t\le t_2 \qquad (3)$$

Now at t=t_2 the inventory level vanishes and the shortages occur in the time interval [t_2T] a fraction f of the total shortages is backlogged and the shortages quantity supplied to the customers at the beginning of the next replenishment cycle. The shortages is governed by the differential equation

$$\frac{dI^s(t)}{dt} = -f(a + bt) \quad ; \qquad t_3\le t\le T \qquad (4)$$

At the time t=T replenishment cycle restarts. The objective of the model is to minimize the total inventory cost by the relevant cost as low as possible.

Now inventory level at different time intervals is given by solving the above differential equations (1) to (4) under boundary conditions

$I^r(t_1)=0; I^{1w}(0)=W_1; I^{2w}(t_2)=0; I^S(t_2)=0$; therefore Differential eq. (1) gives

$$I^r(t)=\{\frac{a}{\alpha}+\frac{b}{\alpha^2}\{(\alpha t_1-1)\}e^{\alpha(t_1-t)} - \{\frac{a}{\alpha}+\frac{b}{\alpha^2}(\alpha t-1)\}; \qquad (5)$$

$$I^{1w}(t)=W_1 e^{-\beta t_1} \quad ; \qquad (6)$$

$$I^{2w}(t)=\{\frac{a}{\beta}+\frac{b}{\beta^2}\{(\beta t_2-1)\}e^{\beta(t_2-t)} - \{\frac{a}{\beta}+\frac{b}{\beta^2}(\beta t-1)\}; \qquad (7)$$

$$I^S(t)= f\{a(t_2-t)+\frac{b}{2}(t_2^2-t^2\}; \qquad (8)$$

Now at $t=0$, $I^r(0)= W_2$ therefore equation (5) yield

$$W_2=\{(\frac{b}{\alpha^2}-\frac{a}{\alpha})+\{\frac{a}{\alpha}+\frac{b}{\alpha^2}(\alpha t_1-1)e^{-\alpha t_1}\}; \qquad (9)$$

Maximum amount of inventory backlogged during shortages period (at t=T) is given by

$$IB= -I^S(T) = f\{a(T- t)+\frac{b}{2}(T^2-t^2)\}; \qquad (10)$$

Amount of inventory lost during shortages period

$$LI= (1-IB)= (1- \quad f\{a(T- t)+\frac{b}{2}(T^2-t^2)\}) \qquad (11)$$

The maximum Inventory to be ordered is given as

$$Q_{max}= W_1+I^r(0)+IB= W_1 + \{(\frac{b}{\alpha^2}-\frac{a}{\alpha}) + \{\frac{a}{\alpha}+\frac{b}{\alpha^2}(\alpha t_1-1)e^{\alpha t_1}\} + f\{a(T-t_2)+\frac{b}{2}(T-t_2^2)\}; \qquad (12)$$

Now continuity at $t=t_1$shows that $I^{1w}(t_1)=I^{2w}(t_1)$ therefore from eq. (6) & (7) we have

$$b\beta^2 t_2^2 - a\beta^2 t_2 - (\beta^2(W_1+Z)+b-a\beta)=0 \qquad (13)$$

Where $Z=\{\frac{a}{\beta}+\frac{b}{\beta^2}(\beta t_1-1)\}e^{-\beta t_1}$

which is quadratic in t_2 and further can be solved for t_2 in terms of t_1 i.e.

$$t_2=\varphi(t_1) \qquad (14)$$

where $\varphi(t_1)=\frac{-a^2\beta^4\pm\sqrt{D}}{2b\beta^2}$ and $D=a^2\beta^4 + 4b\beta^2(b-a\beta + \beta^2(W_1+\{\frac{a}{\beta}+\frac{b}{\beta^2}(\beta t_1-1)\}e^{-\beta t_1})$

Next the total relevant inventory cost per cycle includes following parameters:

1. Ordering costper cycle=C_A

2. Purchase cost per cycle $=P^*Q_{max}$

3.The present worth holding cost=H_C

Case-1 When k<T and $0 \le k < t_1$ in RW

$Hc = \int_0^k h_r I^r(t)dt + \int_k^{t_1} h_r\, t\, I^r(t)dt + \int_0^{t_1} h_w I^{1w}(t)dt + \int_{t_1}^{t_2} h_w\, I^{2w}(t)dt$

Case-2:When k > T

$\int_0^{t_1} h_r I^r(t)dt + + \int_0^{t_1} h_w I^{1w}(t)dt + \int_{t_1}^{t_2} h_w\, I^{2w}(t)dt$

Holding cost for Case -1

$$H_C = h_r\left(at_1k + bt_1^2k - \frac{bk^2}{2\alpha} - \frac{bt_1^2}{3\alpha} + at_1^3 + bt_1^4 - \frac{a\,k}{\alpha} - bt_1k^2 - at_1k^2 - bt_1^2k^2 + \frac{b\,t_1k^2}{\alpha} + \frac{a\,k^2}{\alpha} + \frac{b\,k^3}{3\alpha}\right) + h_w\left(W_1t_1 + \frac{bt_2^2}{\beta} - \frac{bt_1t_2}{\beta} + \frac{bt_1^2}{2\beta} - \frac{bt_2^2}{2\beta}\right) \quad (15)$$

Holding cost for Case -2

$$H_C = h_r\left(at_1^2 + bt_1^3 + \frac{b\,t_1^2}{\alpha} - \frac{b\,t_1^2}{2\alpha}\right) + h_w\left(W_1t_1 + \frac{bt_2^2}{\beta} - \frac{bt_1t_2}{\beta} + \frac{bt_1^2}{2\beta} - \frac{bt_2^2}{2\beta}\right) \quad (16)$$

The present worth of shortages cost

$$CS = S_c f\left(\frac{aT^2}{2} - \frac{at_2^2}{2} + \frac{bT^3}{6} - \frac{bt_2^3}{6} - at_1T + at_2^2 - \frac{bt_2^2\,T}{2} + \frac{bt_2^3}{2}\right) \quad (17)$$

The present worth opportunity cost/Lost sale cost

$$CL = L_c\left(1 - \left(\frac{aT^2}{2} - \frac{at_2^2}{2} + \frac{bT^3}{6} - \frac{bt_2^3}{6} - at_1T + at_2^2 - \frac{bt_2^2\,T}{2} + \frac{bt_2^3}{2}\right)\right) \quad (18)$$

Present worth purchase cost

$$CP = P_c\left(W_1 + \{(\frac{b}{\alpha^2} - \frac{a}{\alpha}) + \{\frac{a}{\alpha} + \frac{b}{\alpha^2}(at_1 - 1)e^{-\alpha t_1}\} + f\{a(T - t_2) + \frac{b}{2}(T^2 - t_2^2)\}\}\right) \quad (19)$$

Therefore Total relevant inventory cost per unit per unit of time is denoted and given by Case-1

T^C (t_1, T) $= \frac{1}{T}[C_A + CP + CS + CL + H_C]$

$$= \frac{1}{T}[C_A + P_c\left(W_1 + \{(\frac{b}{\alpha^2} - \frac{a}{\alpha}) + \{\frac{a}{\alpha} + \frac{b}{\alpha^2}(at_1 - 1)e^{-\alpha t_1}\} + f\{a(T - t_2) + \frac{b}{2}(T^2 - t_2^2)\}\}\right)$$
$$+ S_C f\left(\frac{aT^2}{2} - \frac{at_2^2}{2} + \frac{bT^3}{6} - \frac{bt_2^3}{6} - at_1T + at_2^2 - \frac{bt_2^2\,T}{2} + \frac{bt_2^3}{2}\right) + L_C\left(1 - \left(\frac{aT^2}{2} - \frac{at_2^2}{2} + \frac{bT^3}{6} - \frac{bt_2^3}{6} - at_1T + at_2^2 - \frac{bt_2^2\,T}{2} + \frac{bt_2^3}{2}\right)\right) + h_r\left(at_1k + bt_1^2k - \frac{bk^2}{2\alpha} - \frac{bt_1^2}{3\alpha} + at_1^3 + bt_1^4 - \frac{a\,k}{\alpha} - bt_1k^2 - at_1k^2 - bt_1^2k^2 + \frac{b\,t_1k^2}{\alpha} + \frac{a\,k^2}{\alpha} + \frac{b\,k^3}{3\alpha}\right) + h_w\left(W_1t_1 + \frac{bt_2^2}{\beta} - \frac{bt_1t_2}{\beta} + \frac{bt_1^2}{2\beta} - \frac{bt_2^2}{2\beta}\right)] \quad (20)$$

Case-2

T^C (t_1, T) $= \frac{1}{T}[C_A + CP + CS + CL + H_C]$

$$= \frac{1}{T}[C_A + P_C\left(W_1 + \{(\frac{b}{\alpha^2} - \frac{a}{\alpha}) + \{\frac{a}{\alpha} + \frac{b}{\alpha^2}(at_1 - 1)e^{-\alpha t_1}\} + f\{a(T - t_2) + \frac{b}{2}(T^2 - t_2^2)\}\}\right) + S_c f\left(\frac{aT^2}{2} - \frac{at_2^2}{2} + \frac{bT^3}{6} - \frac{bt_2^3}{6} - at_1T + at_2^2 - \frac{bt_2^2\,T}{2} + \frac{bt_2^3}{2}\right) + L_c\left(1 - \left(\frac{aT^2}{2} - \frac{at_2^2}{2} + \frac{bT^3}{6} - \frac{bt_2^3}{6} - at_1T + at_2^2 - \frac{bt_2^2\,T}{2} + \frac{bt_2^3}{2}\right)\right) + h_r\left(at_1^2 + bt_1^3 + \frac{b\,t_1^2}{\alpha} - \frac{b\,t_1^2}{2\alpha}\right) + h_w\left(W_1t_1 + \frac{bt_2^2}{\beta} - \frac{bt_1t_2}{\beta} + \frac{bt_1^2}{2\beta} - \frac{bt_2^2}{2\beta}\right) \quad (21)$$

The total relevant inventory cost is minimum if

$\frac{\partial T^C}{\partial t_1} = 0$ and $\frac{\partial T^C}{\partial T} = 0$

and subject to the satisfaction of following:-

$\left(\frac{\partial^2 T^C}{\partial t_1^2}\right)\left(\frac{\partial^2 T^C}{\partial T^2}\right) - \frac{\partial^2 T^C}{\partial t_1 \partial T} > 0$

5. NUMERICAL ANALYSIS

The following randomly chosen data in appropriate units has been used to find the optimal solution and validate the model of the three players the producer, the distributor and the retailer. The data are given as a=500, C=1500, W=2000, b=0.50, h_w=60, h_r=75, P_C=1500, α=0.013, β=0.014, S_C=250, k=1.61, f=0.06 and Lc=100.The values of decision variables are computed for the model for two cases separately. The computational optimal solutions of the models are shown in Table-1.

The actual values are to be tuned to the specific GA through experience and trial-and-error. However some standard settings are reported in literature. One of the widely acclaimed standards was proposed by Dejong and Spears (1990) as given below.

Population Size = 150
Number of generations = 3000
Crossover type = two Point
Crossover rate = 1.8
Mutation types = Bit flip
Mutation rate = 0.003 per bit

If single cut-point crossover instead of two cut-point crossover is employed the crossover rate can be lowered to a maximum of 1.50

5.1 Numerical comparison between two cases of the Model

Using the same value of parameters as given in numerical example we obtain the total relevant inventory costs of the model for two cases as given in table-1.From the table we observe that model with instantaneous deterioration and partially backlogged in case-1. Where holding cost vary in RW after a certain time is most expensive state of affairs. So for the case-2 where the holding cost does not vary with the cycle length, the model is the most flexible model and it corresponds to the least expensive circumstances.

Table-1:

Case	Cost function	t_1^*	t_2^*	T^*	Total relevant cost	Genetic Algorithm
1	$T^c(t_1^*, T)$	2.47477	62.7053	74.2487	135249	3.5249
2	$T^c(t_1^*, T)$	7.31247	35.7460	37.9896	61042.2	4.6528

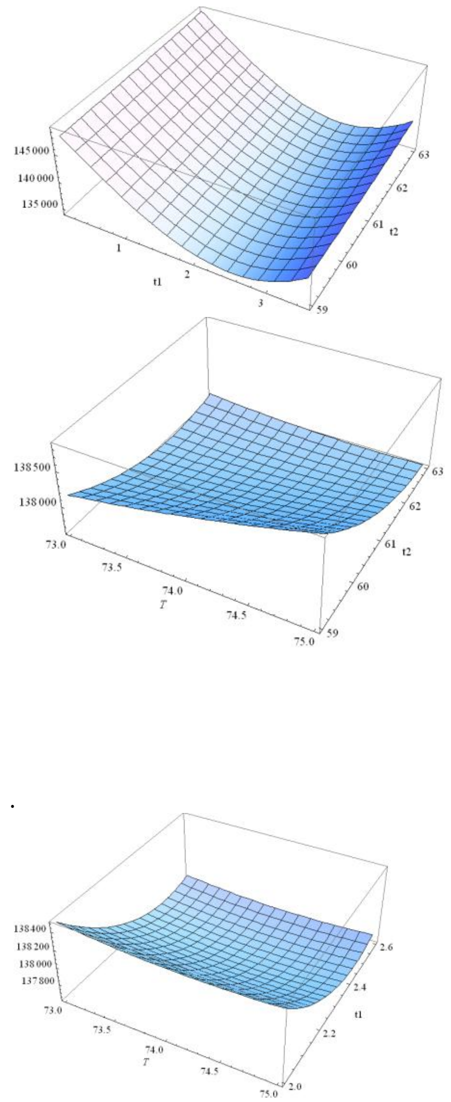


Figure-2 Convexity of the Model for the case-1 is represented graphically when the optimal values of t_1^* , t_2^* and T^* are taken respectively.

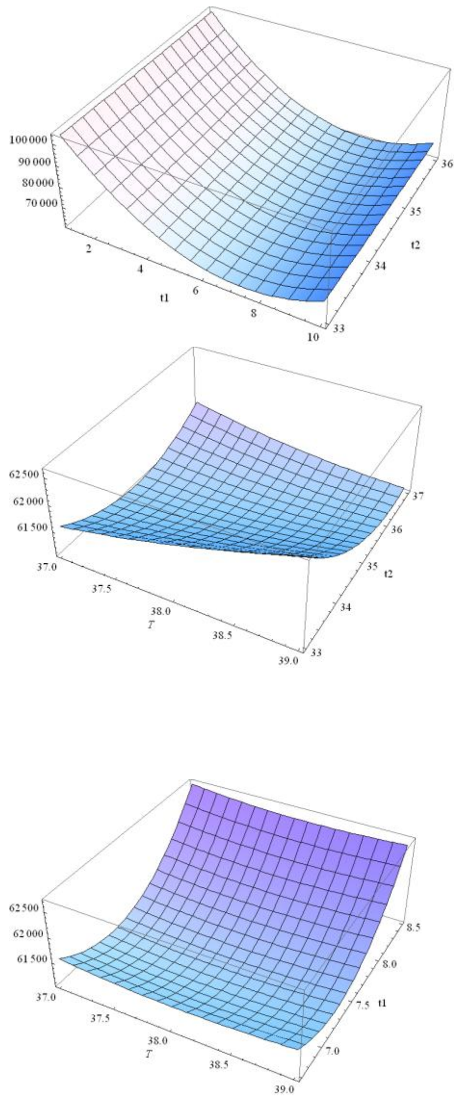


Figure-3 Convexity of the Model for the case -2 is represented graphically when the optimal values of t_1^* , t_2^* and T^* are taken respectively.

6. SENSITIVITY ANALYSIS

Sensitivity analysis performed on every parameter of the model. The analysis is carried out by changing the value of only one parameter at a time by increasing and decreasing by 10% , 20% and 50%,keepingthe rest parameters at their initial values. The change in the values of decision variables t_2^*,t_3^* ,$T*$ and the percentage change in the value of $\boldsymbol{T^c(t_1^*,}$ **T*)** is taken as a measure of sensitivity with respect to the changes in the value of the parameter and the result is shown in tables.

The following observation is made from the tables given below.

(1) The total average cost $\boldsymbol{T^c(t_1^*,}$ **T)** directly proportional to the values of a, b, k, C_p, h_w, and h_r and highly sensitive to these parameters however the value of $\boldsymbol{T^c(t_1^*,}$ **T*)** changes slightly with respect to change in the values of ordering cost, shortages cost and opportunity cost.

(2) The increase in the value of deterioration rate in RW does not affect the value of total relevant inventory cost.

(3) The total relevant inventory cost is indirectly proportional to the deterioration rate in the both ware-houses and moderately sensitive to these parameters.

If parameters k, $C_{p\ ,}h_{r\ ,}\beta$increases, the length of order cycle is increases and hence total inventory cost is also increases with respect to increase in the value of k, $C_{p\ ,}h_r$ and decreases with respect to change in the value of β.

Table-2: Sensitivity analysis with respect to constant demand rate

A	t_1^*	t_2^*	T	$T^c(t_1^*,$ T)	% change in $T^c(t_1^*,$ T*)
550	2.49618	64.1910	74.6198	140800	4.10
600	2.51352	65.5790	75.0507	143858	6.37
750	2.55002	69.3147	76.5811	151940	12.34
450	2.44771	61.0971	73.4559	134125	-0.83
400	2.41251	59.3314	73.7677	130524	-3.49
250	2.1942	52.4114	74.3116	116654	-13.75

Table-3: Sensitivity analysis with respect to variable demand rate

B	t_1^*	t_2^*	T	$T^c(t_1^*,$ T)	% change in $T^c(t_1^*,$ T*)
0.55	2.46488	59.4764	71.6827	143436	6.05
0.60	2.45425	56.6482	69.4710	148950	10.13
0.75	2.41861	49.8863	64.3341	163695	21.03
0.45	2.48385	66.4413	77.2679	131372	-2.87
0.40	2.49201	70.8351	80.8821	124634	-7.85
0.25	2.50921	90.9246	98.1062	100468	-25.72

Table-4: Sensitivity analysis with respect to ordering cost

C_A	t_1^*	t_2^*	T	$T^c(t_1^*,$ T)	% change in $T^c(t_1^*,$, T*))
1650	2.47477	62.7062	74.2498	137624	1.76
1800	2.47478	62.7071	74.2709	137626	1.76
2250	2.47479	62.7098	74.2542	137632	1.76
1350	2.47477	62.7045	74.2475	137620	1.76
1200	2.47475	62.7045	74.2475	137620	1.76
750	2.47475	62.7009	74.2431	137612	1.76

Table-5: Sensitivity analysis with respect to shortages cost

C_s	t_1^*	t_2^*	T	$T^c(t_1^*,$ T)	% change in $T^c(t_1^*,$, T*)
275	2.47605	63.0333	73.6067	138282	2.24
300	2.47715	63.3126	73.0670	138844	2.66
375	2.47963	63.9473	71.8635	140123	3.60
225	2.47324	62.3148	75.0249	136835	3.60
200	2.47138	61.8417	75.9826	135.883	0.47
125	2.46235	59.5406	80.9334	131257	-2.95

Table-6: Sensitivity analysis with respect to opportunity cost

L_c	t_1^*	t_2^*	T	$T^c(t_1^*,$ T)	% change in $T^c(t_1^*,$, T*)
110	2.47468	62.6829	74.2602	137572	1.72
120	2.47459	62.6604	74.2717	137522	1.68
150	2.47433	62.5924	74.3056	137371	1.57
90	2.47486	62.7278	74.2371	137671	1.79
80	2.47494	62.7501	74.2254	137721	1.83
50	2.47521	62.8169	74.1899	137868	1.94

Table-7: Sensitivity analysis with respect to time up to which holding cost remain constant

K	t_1^*	t_2^*	T	$T^c(t_1^*, T)$	% change in $T^c(t_1^*, T^*)$
1.771	2.52073	68.6183	81.7013	151730	12.19
1.932	2.56961	74.4808	89.3418	166340	22.99
2.415	2.73079	92.8567	112.8710	212288	56.96
1.449	2.43217	57.1093	67.0702	124162	-8.20
1.288	2.39351	51.8280	60.2908	111562	-17.51
0.805	2.31021	39.5857	44.5368	82565.8	-38.95

Table-8: Sensitivity analysis with respect to purchase cost

C_p	t_1^*	t_2^*	T	$T^c(t_1^*, T)$	% change in $T^c(t_1^*, T^*)$
1650	2.59282	63.9583	75.2051	140218	3.67
1800	2.70547	65.1431	76.0761	142667	5.48
2250	3.01070	68.3322	78.2298	149574	10.59
1350	2.35049	61.3789	73.1999	134868	-2.82
1200	2.2189	59.9723	72.0506	131945	-2.44
750	1.76310	55.1799	67.8856	121981	-9.81

Table-9: Sensitivity analysis with respect to holding cost in OW

h_w	t_1^*	t_2^*	T	$T^c(t_1^*, T)$	% change in $T^c(t_1^*, T^*)$
66	2.46324	59.6488	71.9610	143077	5.79
72	2.45096	56.9546	69.9022	148202	9.58
90	2.41038	50.4533	65.2274	161955	19.75
54	2.48546	66.2148	76.9845	131789	-2.57
48	2.49519	70.3033	80.2418	125519	-7.19
30	2.51653	88.4028	95.3834	103172	-23.72

Table-10: Sensitivity analysis with respect to holding cost in RW

h_r	t_1^*	t_2^*	T	$T^c(t_1^*, T)$	% change in $T^c(t_1^*, T^*)$
82.5	2.37132	64.3002	76.3952	142302	5.21
90.0	2.28116	65.8104	78.4338	146798	8.54
112.5	2.06749	69.9336	84.0350	159432	17.88
67.5	2.5951	61.0109	71.9746	132718	-1.87
60.0	2.73743	59.1963	69.5458	127538	-5.70
37.5	3.39647	52.6439	60.8134	109302	-19.18

Table-11: Sensitivity analysis with respect to Deterioration rate in RW

A	t_1^*	t_2^*	T	$T^c(t_1^*, T)$	% change in $T^c(t_1^*, T^*)$
0.0143	2.48562	61.0270	72.1171	133742	-1.11
0.0156	2.49444	59.5881	70.2887	130418	-3.57
0.0195	2.51304	56.2776	66.0783	122777	-9.22
0.0117	2.46111	64.6911	76.7690	142216	5.15
0.0104	2.44345	67.0818	79.8010	147752	9.24
0.0065	2.33759	78.6239	94.4048	174561	29.07

Table-12: Sensitivity analysis with respect to deterioration rate in OW

β	t_1^*	t_2^*	T	$T^c(t_1^*, T)$	% change in $T^c(t_1^*, T^*)$
0.0154	2.46512	66.0232	76.9065	132683	-1.90
0.0168	2.45643	69.1559	79.4509	128287	-5.15
0.0210	2.43479	77.6484	86.5026	117490	-13.13
0.0126	2.48558	59.1742	71.4646	143201	5.88
0.0112	2.49781	55.3941	68.5391	149575	10.60
0.007	2.54720	42.0103	58.7006	175946	30.09

7. CONCLUSIONS

In this paper, we proposed a deterministic two-warehouse inventory model for deteriorating items with linear time-dependent demand and varying holding cost with respect to ordering cycle length with the objective of minimizing the total inventory cost. Shortages are allowed and partially backlogged. Two different cases has been discussed one with variable holding cost during the cycle period and other with constant holding cost during total cycle length and it is observed that during variable holding cost the total inventory cost is much more than the other case. Furthermore the proposed model is very useful for the items which are highly deteriorating, since as the deterioration rate increases in both ware-houses the total inventory cost decreases. This model can be further extended by incorporation with other deterioration rate, probabilistic demand pattern and other realistic combinations.

8. REFERENCES

[1] A.K. Bhunia, Ali Akbar Shaikh (2015). An application of PSO in a two-warehouse inventory model for deteriorating item under permissible delay in payment with different inventory policies Applied Mathematics and Computation, Volume 256, Pages 831-850.

[2] A.K. Bhunia, M. Maiti (1998). A two-warehouse inventory model for deteriorating items with a linear trend indemand and shortages, Journal of the Operational Research Society 49, 287-292.

[3] Ajay Singh Yadav, Anupam Swami (2013). A Two-Warehouse Inventory Model for Decaying Items with Exponential Demand and Variable Holding Cost, International Journal of Inventive Engineering and Sciences (IJIES) ISSN: 2319–9598, Volume-1, Issue-5.

[4] Ali Roozbeh Ren Qing-dao-er-ji, Yuping Wang, Xiaoli Wang (2013). Inventory based two-objective job shop scheduling model and its hybrid genetic algorithm Applied Soft Computing, Volume 13, Issue 3, Pages 1400-1406.

[5] Antonio Costa, Giovanni Celano, Sergio Fichera, Enrico Trovato (2010). A new efficient encoding/decoding procedure for the design of a supply chain network with genetic algorithms Computers & Industrial Engineering, Volume 59, Issue 4, Pages 986-999.

[6] Ata Allah Taleizadeh, Seyed Taghi Akhavan Niaki, Mir-Bahador Aryanezhad, Nima Shafii (2013). A hybrid method of fuzzy simulation and genetic algorithm to optimize constrained inventory control systems with stochastic replenishments and fuzzy demand Information Sciences, Volume 220, Pages 425-441.

[7] Bongju Jeong, Ho-Sang Jung, Nam-Kyu Park (2002). A computerized causal forecasting system using genetic algorithms in supply chain management Journal of Systems and Software, Volume 60, Issue 3, Pages 223-237.

[8] David Naso, Michele Surico, Biagio Turchiano, Uzay Kaymak (2007). Genetic algorithms for supply-chain scheduling: A case study in the distribution of ready-mixed concrete European Journal of Operational Research, Volume 177, Issue 3, Pages 2069-2099.

[9] Fulya Altiparmak, Mitsuo Gen, Lin Lin, Ismail Karaoglan (2009). A steady-state genetic algorithm for multi-product supply chain network design Computers & Industrial Engineering, Volume 56, Issue 2, Pages 521-537.

[10] Fulya Altiparmak, Mitsuo Gen, Lin Lin, Turan Paksoy (2006). A genetic algorithm approach for multi-objective optimization of supply chain networks Computers & Industrial Engineering, Volume 51, Issue 1, Pages 196-215.

[11] Hertely V. Ronald. (1976). On the EOQ model two levels of storage. Opsearch, 13, 190-196.

[12] Ilkay Saracoglu, Seyda Topaloglu, Timur Keskinturk (2014). A genetic algorithm approach for multi-product multi-period continuous review inventory models Expert Systems with Applications, Volume 41, Issue 18, Pages 8189-8202.

[13] Javad Sadeghi, Seyed Taghi Akhavan Niaki (2015) Two parameter tuned multi-objective evolutionary algorithms for a bi-objective vendor managed inventory model with trapezoidal fuzzy demand Applied Soft Computing, Volume 30, Pages 567-576.

[14] K. D. Rathod and P. H. Bhathawala (2013). Inventory model with inventory-level dependent demand rate, variable holding cost and shortages, International Journal of Scientific & Engineering Research, Volume 4, Issue 8,

[15] K.V.S. Sarma (1987) A deterministic order level inventory model for deteriorating items with two storage facilities. European Journal of Operational Research 29,70-73.

[16] K.V.S. Sarrna (1983). A deterministic inventory model with two level of storage and an optimum release rule, Opsearch 20, 175-180.

[17] M.J. Li, D.S. Chen, S.Y. Cheng, F. Wang, Y. Li, Y. Zhou, J.L. Lang (2010). Optimizing emission inventory for chemical transport models by using genetic algorithm Atmospheric Environment, Volume 44, Issue 32, Pages 3926-3934.

[18] Masao Yokoyama (2002). Integrated optimization of inventory-distribution systems by random local search and a genetic algorithm Computers & Industrial Engineering, Volume 42, Issues 2–4, Pages 175-188

[19] Maya Gyan, A.K. Pal (2009). A two warehouse Inventory model for deteriorating items with stock dependent demand rate and holding cost,Oper. Res. Int. J.vol(9),153-165.

[20] Miguel Zamarripa, Javier Silvente, Antonio Espuña (2012). Supply Chain Planning under Uncertainty using Genetic Algorithms Computer Aided Chemical Engineering, Volume 30, Pages 457-461.

[21] Nia, Mohammad Hemmati Far, Seyed Taghi Akhavan Niaki (2015). A hybrid genetic and imperialist competitive algorithm for green vendor managed inventory of multi-item multi-constraint EOQ model under shortage Applied Soft Computing, Volume 30, Pages 353-364.

[22] R.J. Kuo, Y.S. Han (2011). A hybrid of genetic algorithm and particle swarm optimization for solving bi-level linear programming problem – A case study on supply chain model Applied Mathematical Modelling, Volume 35, Issue 8, Pages 3905-3917.

[23] R.K. Gupta, A.K. Bhunia, S.K. Goya (2007). An application of genetic algorithm in a marketing oriented inventory model with interval valued inventory costs and three-component demand rate dependent on displayed stock level Applied Mathematics and Computation, Volume 192, Issue 2, Pages 466-478l.

[24] R.K. Gupta, A.K. Bhunia, S.K. Goyal (2009). An application of Genetic Algorithm in solving an inventory model with advance payment and interval valuedinventory costs Mathematical and Computer Modelling, Volume 49, Issues 5–6, Pages 893-905.

[25] Reza Zanjirani Farahani, Mahsa Elahipana (2008). A genetic algorithm to optimize the total cost and service level for just-in-time distribution in a supply chain International Journal of Production Economics, Volume 111, Issue 2, Pages 229-243

[26] S. Kar. A.K. Bhunia, M. Maiti (2001). Deterministic inventory model with two levels of storage, a linear trendin demand and a fixed time horizon. Computers & Operations Research 28 , 1315-1331.

[27] S.H. Zegordi, I.N. Kamal Abadi, M.A. Beheshti Nia (2010). A novel genetic algorithm for solving production and transportation scheduling in a two-stage supply chain Computers & Industrial Engineering, Volume 58, Issue 3, Pages 373-381

[28] Salah Alden Ghasimi, Rizauddin Ramli, Nizaroyani Saibani (2014). A genetic algorithm for optimizing defective goods supply chain costs using JIT logistics and each-cycle lengths Applied Mathematical Modelling, Volume 38, Issue 4, Pages 1534-1547

[29] Sasan Khalifehzadeh, Mehdi Seifbarghy, Bahman Naderi (2015). A four-echelon supply chain network design with shortage: Mathematical modeling and solution methods Journal of Manufacturing Systems, Volume 35, Pages 164-175.

[30] Seyed Hamid Reza Pasandideh, Seyed Taghi Akhavan Niaki, Ali Roozbeh Nia (2011). A genetic algorithm for vendor managed inventory control system of multi-product multi-constraint economic order quantity model Expert Systems with Applications, Volume 38, Issue 3, Pages 2708-2716.

[31] Seyed Hamid Reza Pasandideh, Seyed Taghi Akhavan Niaki, Nafiseh Tokhmehchi (2011). A parameter-tuned genetic algorithm to optimize two-echelon continuous review inventory systems Expert Systems with Applications, Volume 38, Issue 9, Pages 11708-11714

[32] T.A. Murdeshwar, Y.S. Sathe (1985). Some aspects of lot size model with two levels of storage, Opsearch 22, 255-262.

[33] T.P.M. Pakkala, K.K. Achary (1992). A deterministic inventory model for deteriorating items with twowarehousesand finite replenishment rate, European Journal of Operational Research 57, 157- 167.

[34] Tamás Varga, András Király, János Abonyi (2013). 19 - Improvement of PSO Algorithm by Memory-Based Gradient Search—Application in Inventory Management Swarm Intelligence and Bio-Inspired Computation, Pages 403-422.