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Robust System Architecture for DOA Estimation based on Total Forward Backward Matrix Pencil Algorithm

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ABSTRACT
Radio Direction Finding (DF) is a technique that identifies the bearing angle or the coordinates of an incoming radio signal(s). The primary function of DF is to get the Direction of Arrival (DOA) information. In this paper, we propose the robust system architecture for DOA estimation using the Total Forward - Backward Matrix Pencil Method (TFBMP). This method works directly on signal samples of incoming signals received by an M-element Uniform Linear Antenna (ULA) array. The simulation results for DOA estimation of RF signals using the proposed system will be shown and analyzed to verify its performance.

Keywords
Direction of arrival (DOA); Wideband signal; Total Forward-Backward Matrix Pencil (TFBMP), Uniform Linear Antenna Array (ULA).

1. INTRODUCTION
The most important information that estimated by a Radio Direction Finding (DF) system is the Direction of Arrival (DOA). The DF systems have many applications in Radio Navigation, Emergency Aid and intelligent operations, etc... The efficience of DF system could be improved by many ways such as developing of system architecture and research on DOA estimation algorithms or the hybrid of the two.

Thank to technology development such as Software Defined Radio technique, all kind of DF system can be designed as all digital radio receiver which is a flexible open-architecture platform with dynamic selection of parameters for individual modules. Recently, a lot of digital radio receiver architecture has been investigated [1][2]. In all digital receiver architecture (System 1), the RF signal is directly digitalized by an Analog – to – Digital (ADC) which is ideally connected to the antenna or as close to the antenna as possible. The output of ADC will be processed in the digital domain by a digital signal processor (DSP) where all tasks such as frequency conversion, filtering and so on are performed. One of the biggest obstacles of this architecture is the ADC sampling rate. The ADCs usually set the receiver's bandwidth equal to a half of ADC's sampling which is not easily satisfied when signal is in millimeter band. This problem has been solved in [2] for the DOA purpose (System 2).

In our research, we develop an innovative architecture in comparison to the system 1. With this architecture, the RF signal is still directly sampled by ADCs with a lower sampling rate than signal frequency. However, the structure will be reconstructed with simpler platform and changed in the processing data manner in order to calculate the DOA information. Moreover, in this paper, we develop an approach based on an extension of the Matrix Pencil Method [3][4][5][6] called Total Forward - Backward Matrix Pencil Method (TFBMP) [7] to solve the DOA estimation problem. One of the most remarkable advantages of this technique is that it can extract the DOA information with very smaller number of snapshot than other technique such as MUSIC [8]. Although TFBMP deals with a larger database, it is more efficient than the original method, especially for a high correlative environment. The performance of the proposed system with TFBMP will be assessed in many cases that depend on the characteristics of incoming signals as well as number of snapshots of data.

The paper is organized as follows. Section 2 describes the proposed architecture and model of the incoming signals. Section 3 presents in details signal processing procedure for DOA estimation. The simulation results are shown in the section 4. The conclusion is given in the section 5.

2. PROPOSED SYSTEM ARCHITECTURE

2.1 System Architecture

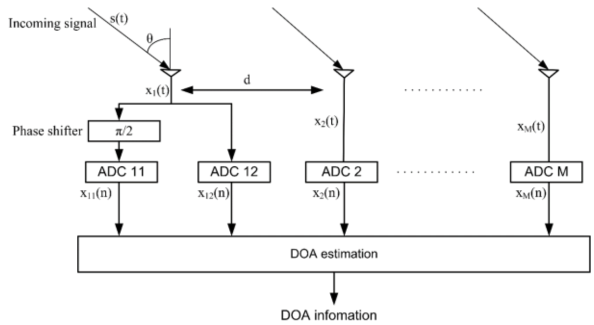

Fig1: Proposed system architecture

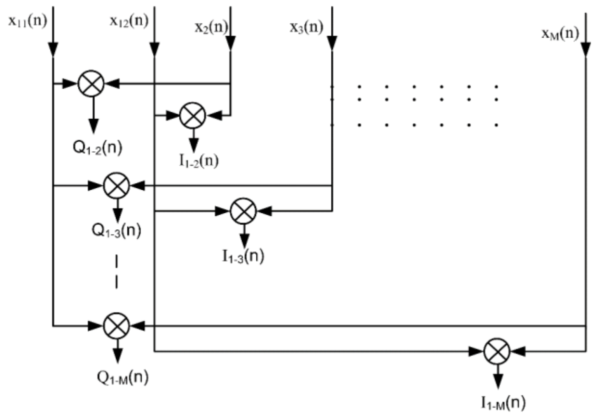

Fig2: DOA estimation processing block

The proposed architecture for Radio Direction Finding system is depicted in Fig.1. The system includes an M – element Uniform Linear Antenna (ULA) array. The received signal at each antenna element will be digitalized by the ADCs connecting to them. However, the improvement compared with the System 2 is that only for the first element of the array, the received signal is divided into two branches, one is directly connected to an ADC and the other passed through a 90° phase shifter before entering another ADC. The output of ADCs will be used to estimate the complex envelops which are processed to determine the DOA information in the DOA estimation block shown in Fig.2. All these tasks are carried out by digital signal processing in the digital domain.

2.2 Signal Model

In our research, an M-element Uniform Linear Antenna Array (ULA) model is used. For the theoretical analysis, let us model the mathematic signal received at the ULA. This is one of the most convenient mathematical models for array processing due to its simplicity and regularity. The ULA model can be described as a set of M isotropic antenna elements spaced at a uniform interval d along some line in space as in Fig.3. However, this work only estimates the DOA of interest in azimuth(θ). The reception of plane wave with ULA is depicted in Fig.4, where θ is measured down from the perpendicular of the ULA and $|\theta| < \frac{\pi}{2}$.

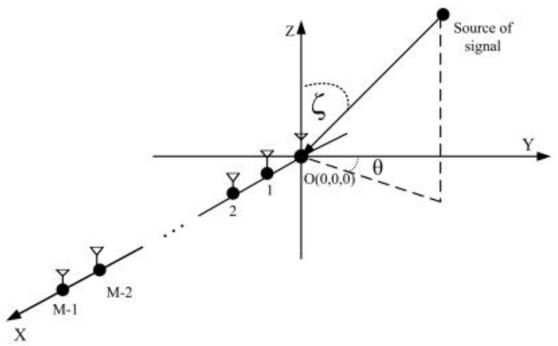

Fig. 3: ULA Model

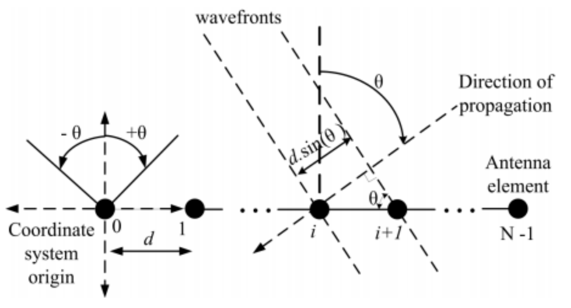

Fig. 4: The reception of plane wave with ULA

The phase at the m^{th} antenna relative to the reference point is given as:

$$\phi_m = \frac{2\pi}{\lambda} md\sin(\theta) \quad (m = 0 \div M-1) \tag{1}$$

where λ is the wavelength of signal of interested. The phase response at the m^{th} antenna relative to the reference point is

$$a_m = g_m e^{j\phi_m} \tag{2}$$

where g_m is the gain of the m^{th}antenna element. The baseband output signal at the m^{th} antenna is:

$$x_m(t) = s(t)a_m = A(t)e^{j\frac{2\pi}{\lambda}md\sin(\theta)} \tag{3}$$

where s(t) is the incoming signal and A(t) = s(t).g_m.

In case of K signals approaching the array from some azimuth directions $\theta_1, \theta_2, \ldots, \theta_K$, the received signal at the m^{th} antenna is:

$$x_m(t) = \sum_{i=1}^{K} A_i(t)e^{j\beta md\sin(\theta_i)} = \sum_{i=1}^{K} A_i(t)\alpha_i^m \tag{4}$$

with

$$\alpha_i = e^{j\beta md\sin(\theta_i)} \tag{5}$$

where $\beta = \frac{2\pi}{\lambda}$ is the propagation factor.

Equation 3 represents the received signal in the absence of noise at each antenna element as a continuous time function. After passing the ADCs block, this signal will be sampled with period T_s at discrete time instants nT_s as

$$x_m(nT_s) = A(nT_s).e^{j\frac{2\pi}{\lambda}md\sin(\theta)} \tag{6}$$

In order to simplify this equation, the discrete time sampled version of signal $x_m(nT_s)$ is represented as $x_m(n)$. This mean that $x_m(n)$ is a discrete version of $x_m(t)$ sampled with the period T_s.

In practice, when K signals impinging on the array simultaneously, based on Eq.4 and Eq.6, the discrete sampled form of the output signal at each antenna element can be written as:

$$x_m(n) = \sum_{i=1}^{K} A_i(n)e^{j\beta md\sin(\theta_i)} \tag{7}$$

Each sample of the output signal is defined as one snapshot of the data which will be processed to produce the DOA information. It can be simply expressed as:

$$x_m = \sum_{i=1}^{K} A_i e^{j\beta md\sin(\theta_i)} \tag{8}$$

3. SIGNAL PROCESSING FOR DOA ESTIMATION

3.1 Data contruction

For simplification, let us consider one incoming signal with the DOA at θ in azimuth. Equation (3) is the baseband complex envelope of the received signal at each antenna element. However, in practical, the continuous wave impinging on the array is described as

$$x_m(t) = E.Cos(2\pi ft + \phi_m) = Re\{E_m e^{j(2\pi ft+\phi_m)}\} \tag{9}$$

The signal at the first and the second antenna elements can be expressed as:

$$x_1(t) = Re\{E_1 e^{j2\pi ft}\} \tag{10}$$

$$x_2(t) = Re\{E_2 e^{j2\pi ft+\Delta\varphi}\} \tag{11}$$

where E_1 and E_2 are their amplitude; $\Delta\varphi$ is the phase difference of the received signal at two elements:

$$\Delta\varphi = \beta d sin\theta \tag{12}$$

In order to determine the DOA information, the complex envelope form of the signal will be established. In conventional analog receiver, the complex envelope could be extracted from I/Q demodulation which is complicated and depends on the ADCs speed. Based on proposed system, the DOA information can be easily achieved by signal processing in the digital domain as follow.

For the first antenna element, the continuous time function of the signal in the left branch after the 90^o phase shifter and in the right Zranch can be expressed as:

$$x_{11}(t) = Re\left\{\frac{E_1}{\sqrt{2}}e^{j\left(2\pi ft+\frac{\pi}{2}\right)}\right\} \tag{13}$$

$$x_{12}(t) = Re\left\{\frac{E_1}{\sqrt{2}}e^{j2\pi ft}\right\} \tag{14}$$

In a simple manner, the real and imaginary parts of the correlation product between $x_1(t)$ and $x_2(t)$ obtained in the discrete time domain as follows:

$$\begin{aligned} I_{1-2}(n) &= Re\{x_{12}(n).x_2^*(n)\} \\ &= Re\left\{\frac{E_1E_2}{\sqrt{2}}.e^{j2\pi fn}.e^{-j(2\pi fn+\Delta\varphi)}\right\} \\ &= \frac{E_1E_2}{\sqrt{2}}\cos(\Delta\varphi) \end{aligned} \tag{15}$$

$$\begin{aligned} Q_{1-2}(n) &= Re\{x_{11}(n).x_2^*(n)\} \\ &= Re\left\{\frac{E_1E_2}{\sqrt{2}}.e^{j(2\pi fn+\frac{\pi}{2})}.e^{-j(2\pi fn+\Delta\varphi)}\right\} \\ &= \frac{E_1E_2}{\sqrt{2}}.\sin(\Delta\varphi) \end{aligned} \tag{16}$$

The complex envelop is then obtained by:

$$z_1(n) = I_{1-2}(n) + jQ_{1-2}(n) = \frac{E_1E_2}{\sqrt{2}}e^{j\Delta\varphi} \tag{17}$$

By the same way, the complex envelope of the correlation product between the first element and the others are determined and the output vector of the array is:

$$\begin{aligned} x(n) &= [z_1(n), z_2(n) \dots\dots z_{M-1}(n)]^T \\ &= \left[\frac{E_1E_2}{\sqrt{2}}e^{j\Delta\varphi}, \frac{E_1E_3}{\sqrt{2}}e^{j2\Delta\varphi} \dots . \frac{E_1E_M}{\sqrt{2}}e^{j(M-1)\Delta\varphi}\right]^T \end{aligned} \tag{18}$$

Based on the above analysis, it is easy to see that the complex envelop can be calculated with only one snapshot of the signal. When the signal is periodic as CW signal, one snapshot can be taken on any periods. Because of this reason, the sampling frequency can be much smaller than signal frequency.

3.2 DOA estimation

From the measured data, the DOA of incident signals will be estimated based on TFBMP. First of all, the number of signals (K) must be calculated. In order to do that the corresponding data covariance matrix in Equation (18) is established as:

$$R_{xx} = E\{x(n).x(n)^H\} \tag{19}$$

By eigenvalue analysis of R_{xx} and based on the noise energy, the number of incoming signals is found [8] and then the DOA information θ_i could be extracted by using TFBMP as follow.

Based on [7], the "all data" matrix is given by

$$\boldsymbol{Y}_{fb_{2(M-L)\times(L+1)}} = \begin{bmatrix} Z_0 & Z_1 & \cdots & Z_{L-1} & Z_L \\ Z_L^* & Z_{L-1}^* & \cdots & Z_1^* & Z_0^* \end{bmatrix} \tag{20}$$

where '*' denotes complex conjugate, L is chosen as pencil parameter with the condition:

$$\begin{aligned} &K \le L \le M-K \text{ if } M \text{ is even,} \\ &K \le L \le M-K+1 \text{ if } M \text{ is odd} \end{aligned} \tag{21}$$

and z_j $(j = 0, \dots, L)$ is defined as

$$z_j^T = [x_j \; x_{j+1} \; \dots \; x_{M-L+j-1}] \; ; \; j = 0, \dots, L \tag{22}$$

The SVD of Y_{fb} is:

$$\begin{aligned} &\boldsymbol{Y}_{fb_{2(M-L)\times(L+1)}} \\ &= \boldsymbol{U}_{2(M-L)\times 2(M-L)}\,\boldsymbol{\Sigma}_{2(M-L)\times(L+1)}\,\boldsymbol{V}^H_{(L+1)\times(L+1)} \end{aligned} \tag{23}$$

where H denotes complex conjugate transpose of a matrix, $\boldsymbol{U}, \boldsymbol{\Sigma}$, and $\boldsymbol{V}$ are given by

$$\boldsymbol{\Sigma} = diag\{\sigma_1, \sigma_2 \dots \sigma_p\}, \tag{24}$$

where $p = min\{2(M-L), L+1\}$ and $\sigma_1 \ge \sigma_2 \ge \cdots \ge \sigma_p \ge 0$,

$$\boldsymbol{U} = [u_1, u_2, \dots, u_{2(M'-L)}], \tag{25}$$

$$\begin{aligned} &\boldsymbol{Y}_{fb}^H u_k = \sigma_k v_k, \\ &(k = 1, \dots, p) \end{aligned} \tag{26}$$

$$\boldsymbol{V} = [v_1, v_2, \dots, v_{(L+1)}], \tag{27}$$

$$\begin{aligned} &\boldsymbol{Y}_{fb}^H v_k = \sigma_k u_k, \\ &(k = 1, \dots, p) \end{aligned} \tag{28}$$

$$\boldsymbol{U}^H\boldsymbol{U} = \boldsymbol{I}, \boldsymbol{V}^H\boldsymbol{V} = \boldsymbol{I}, \tag{29}$$

where σ_k are the singular values of $\boldsymbol{Y}_{fb}$ and the vector u_k and v_k are the k^{th} left and right singular vector, respectively.

The problem can be computationally improved by applying the singular value filtering to obtain the K largest singular values of $\boldsymbol{Y}_{fb}$.

$$\overline{\boldsymbol{Y}}_{fb_{2(M-L)\times(L+1)}} = \overline{\boldsymbol{U}}_{2(M-L)\times K}\overline{\boldsymbol{\Sigma}}_{K\times K}\overline{\boldsymbol{V}}^H_{K\times(L+1)} \tag{30}$$

where

$$\overline{\boldsymbol{\Sigma}} = diag\{\sigma_1, \sigma_2 \dots \sigma_K\} \tag{31}$$

has the K largest singular values of $\boldsymbol{\Sigma}$, and the matrices $\overline{\boldsymbol{U}}$ and $\overline{\boldsymbol{V}}$are formed by extracting the singular vectors corresponding to K singular values. Extract $\overline{\boldsymbol{V}_0}$ and $\overline{\boldsymbol{V}_1}$ from $\overline{\boldsymbol{V}}$ as follows:

$$\overline{\boldsymbol{V}} = [\overline{\boldsymbol{V}}_{\boldsymbol{0}}, v_{L+1}], \overline{\boldsymbol{V}} = [v_1, \overline{\boldsymbol{V}}_1]. \tag{32}$$

Same as above, $\overline{\boldsymbol{Y}}_{0fb}$ and $\overline{\boldsymbol{Y}}_{1fb}$ are established as

$$\overline{\boldsymbol{Y}}_{0fb} = \overline{\boldsymbol{U}}\overline{\boldsymbol{\Sigma}}\overline{\boldsymbol{V}}_{\boldsymbol{0}}^H \tag{33}$$

$$\overline{\boldsymbol{Y}}_{1fb} = \overline{\boldsymbol{U}}\overline{\boldsymbol{\Sigma}}\overline{\boldsymbol{V}}_1^H \tag{34}$$

Now, consider the matrix pencil

$$\bar{\boldsymbol{Y}}_{1fb} - z\bar{\boldsymbol{Y}}_{0fb} \quad (35)$$

and left multiplying Eq.35 by $\bar{\boldsymbol{Y}}^{+}_{0fb}$ yields

$$q^H(\bar{\boldsymbol{Y}}_{1fb}\bar{\boldsymbol{Y}}^{+}_{0fb} - z\boldsymbol{I}) = 0^H \quad (36)$$

where $\bar{\boldsymbol{Y}}^{+}_{0fb}$ is the Moore – Penrose pseudo – inverse of $\boldsymbol{Y}_{0fb}$,

$$\bar{\boldsymbol{Y}}^{+}_{0fb} = (\bar{\boldsymbol{V}}_0^H)^{+}\bar{\boldsymbol{\Sigma}}^{-1}\bar{\boldsymbol{U}}^{+} \quad (37)$$

Substituting Eq.34 and Eq.37 into Eq.36 the equivalent generalized eigen – problem becomes

$$q^H(\bar{\boldsymbol{V}}_1^H - z\bar{\boldsymbol{V}}_0^H) = 0^H \quad (38)$$

Left multiplying Eq.38 by $\overline{\boldsymbol{V}_0}$, we have

$$q^H(\bar{\boldsymbol{V}}_1^H\bar{\boldsymbol{V}}_0 - z\bar{\boldsymbol{V}}_0^H\bar{\boldsymbol{V}}_0) = 0^H \quad (39)$$

Using the values of the generalized eigenvalues, z, of Eq.39, angles of arrival can be estimated as

$$\theta_k = Sin^{-1}\left(\frac{\Im(ln(z_k))}{\beta d}\right) \quad (40)$$

Where $\Im(ln(z_k))$ is the imaginary part of $ln(z_k)$.

4. SIMULATION RESULTS

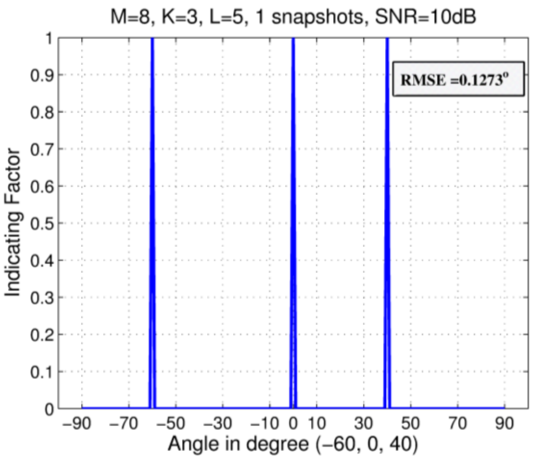

Fig. 5: DOA estimation by TFBMP with one snapshot

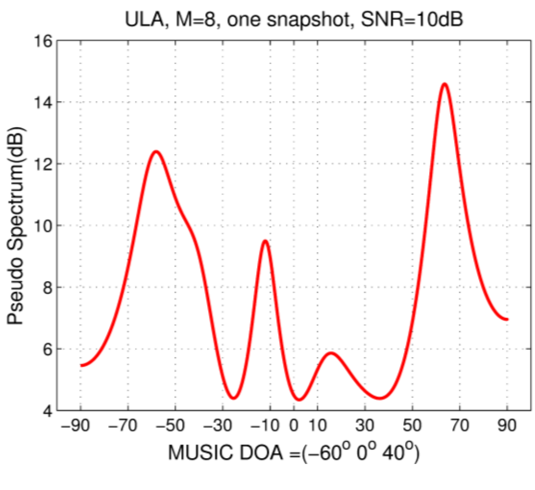

Fig. 6: DOA estimation by MUSIC with one snapshot

The proposed architecture of DF system is simulated using Matlab/Simulink in order to examine its performance. In this paper, it is assumed that there are three incoherent signals at 1 GHz, 1.1 GHz and 1.2 GHz impinging on the 8 element ULA with $d = \frac{\lambda}{2}$ in AWGN channel. In theory, the TFBMP can extract the DOA information of incoming signals with only one snapshot if the number of signal is known. In this work, we need to note that the estimated DOAs in the simulation are the numerical values as in Eq.40. Therefore, in order to demonstrate visually the results, we illustrate the DOA in XOY plane, in which the X – Axis is the DOA of incoming signal and the Y – Axis is the indicating factor. This factor is set to 1 corresponding to the estimated DOA in X - Axis. The simulation result presented in the Figure 5 shows that the DOA at -60^o, 0^o and 40^o of incoming signals can be estimated accurately by the TFBMP with only one snapshot. With the same number of snapshot, the DOA could not be estimated by MUSIC as shown in Figure 6.

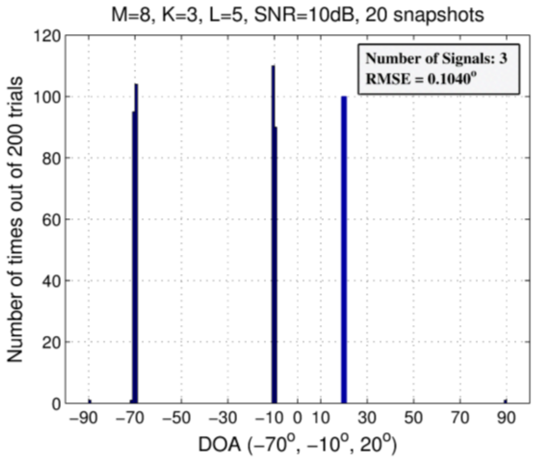

Fig. 7: DOA estimation by the proposed system with 20 snapshots using TFBMP

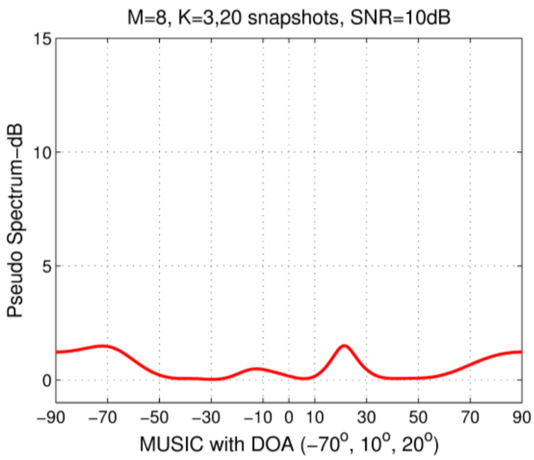

Fig. 8: DOA estimation by the proposed system with 20 snapshots using MUSIC

However, with only one snapshot, the number of incoming signals could not be determined. Because of this reason, in this paper, the signals are sampled at 500 MHz sampling frequency and 20 data snapshots will be processed as in Equation (19). The simulation result presented in the Fig 7 indicates that the number of incoming signals and their DOA information (-70^0, -10^0 and 20^0) are accurately found by the proposed system with TFBMP meanwhile they could not be estimated by the System I in which the signal digitized directly with the same sampling frequency but without passing the power divider and 90° phase shifter. This is due to the sampling frequency does not satisfy the Nyquist law. Moreover, in the similar situation, the DOA information could also not be estimated by the MUSIC algorithm as in Fig.8. This fact is due to the correlation matrix R_{xx} is not Toeplitz with with very small finite number of snapshots.

Obiviously, with the above analysis, it can be seen that the proposed system using TFBMP has successfully estimated the DOA of incoming signals with very small number of snapshots. Moreover, the proposed system in which the Hilbert transform is performed by using a 90° phase shifter is simply implemented even though for wideband application requiring a 90° phase shifter in the frequency band.

5. CONCLUSIONS

We have developed in this paper the robust architecture for DF system. The power of this system is that it can produce the DOA information with the digital data sampled at very low frequency. By this way, the computational complexity reduces significantly. Therefore, it is possible to work with millimeter wave in digital domain. Moreover, in comparison with system 2, our new system has simpler structure with only one phase shifter. With structure like this, the system is cheaper and is one of the practical ways to deploy in the real application.

6. ACKNOWLEDGMENTS

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