An Inventory Model with Partial Backordering, Weibull Distribution Deterioration under Two Level of Storage

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ABSTRACT
This paper mainly presents a two warehouse inventory model for deteriorating items which follows the weibull deterioration rate under assumption that the deterioration rates are different in the both warehouses but deterioration cost is same in the both warehouses. The holding cost is variable and taken as linear function of time and demand is taken to be constant with the time. Salvages value is associated with the deteriorated units of inventories and Shortages are allowed in the OW and partially backlogged at the next replenishment cycle.

Keywords
Weibull distributed deterioration, partial backlogging, salvages value and Variable holding cost.

1. INTRODUCTION
Now days in the present market scenario of explosion of choice due to cut-throat competition, no company can bear a stock-out situation as a large number of alternative products are available with additional features. Furthermore, there is no cut-and- dried formula by means of which one can determine the demand exactly. Despite having considerable cost, firms have to keep an inventory of the various types of goods for their smooth functioning mainly due to geographical specialization, periodic variation and gap in demand and supply. When a firm needs an inventory, it must be stored in such a way that the physical attributes of inventory items can be preserved as well as protected. Thus, inventory produces the need for warehousing. Traditionally, a warehouse is typically viewed as a place where inventory items are stored. Warehouse is an essential limb of an industrial unit. In the existing literature, it was found that the classical inventory models generally deal with a single storage facility. The basic assumption in these models was that the management had storage with unlimited capacity. However, it is not true (e.g., in a supermarket, the storage space of showroom is very limited) in the field of inventory control. Due to attractive price discount during bulk purchase or some problems in frequent procurement or very high demand of items, management decides to purchase a huge quantity of items at a time. These items cannot be stored in the existing storage (owned warehouse, OW) with limited capacities. So, for storing the excess items, one (sometime more than one) warehouse is hired on rental basis. The rented warehouse RW is located near the OW or little away from it. Usually, the holding cost in RW is greater than the OW. Further, the items of RW are transported to OW, in bulk fashion to meet the customers demand until the stock level of RW is emptied. In the classical inventory models, found in the existing literature are that the life time of an item is infinite while it is in storage. But the effect of deterioration plays an important role in the storage of some commonly used physical goods like fruits, vegetables etc. In these cases, a certain fraction of these goods are either damaged or decayed and are not in a condition to satisfy the future demand of costumers as fresh units. Deterioration in these units is continuous in time and is normally proportional to on-hand inventory. Over a long time a good number of works have been done by some authors for controlling inventories in which a constant or a variable part of the on hand inventory gets deteriorated per unit of time. In many models it is assumed that the products are deteriorated constantly (i.e. deterioration rate is assumed to be constant) with time but in certain models, rate of deterioration taken probability dependent distribution rate i.e. deterioration depend on the quality of product i.e. some product deteriorated very fast and some are slow, so many researchers worked taking different probability distribution rate as the rate of deterioration. In general, in formulating inventory models, two factors of the problem have been of growing interest to the researchers, one being the deterioration of items and the other being the variation in the demand rate with time.Donaldson [1977] developed an optimal algorithm for solving classical no-shortage inventory model analytically with linear trend in demand over fixed time horizon. Dave, U. (1989) proposed a deterministic lot-size inventory model with shortages and a linear trend in demand. Goswami and Chaudhuri [1991]discussed different types of inventory models with linear trend in demand. Hariga (1995). Mandal and Maiti [1999] discussed an inventory of damageable items with variable replenishment rate and deterministic demand. Balkhi and Benkherouf [2004] developed an inventory model for deteriorating items with stock dependent and time varying demand rates over a finite planning horizon. Yang [2004] provided a two warehouse inventory model for a single item with constant demand and shortages under inflation. Zhou and Yang [2005] studied stock-dependent demand without shortage and deterioration with quantity based transportation cost. Wee et al. [2005] considered two-warehouse model with constant demand and weibull distribution deterioration under inflation. Mahapatra, N. K. and Maiti, M. [2005] presented the multi objective and single objective inventory models of stochastically deteriorating items are developed in which demand is a function of inventory level and selling price of the commodity. Panda et al. [2007] considered and EOQ model with ramp-type demand and Weibull distribution deterioration. Ghosh and Chakrabarty [2009] suggested an order-level inventory model with two levels of storage for deteriorating items. Sarala Pareek and Vinod Kumar [2009] developed a deterministic inventory model for deteriorating items with salvage value and shortages. Skouri, Konstantaras, Papachristos, and Ganas [2009] developed an inventory models with ramp type demand rate, partial backlogging and Weibell`s deterioration rate. Mishra and Singh [2010] developed a deteriorating inventory model for waiting time partial backlogging when demand is time dependent and deterioration rate is constant. Kuo-Chen Hung [2011]gave an inventory model with generalized type demand, deterioration

and backorder rates Mishra & Singh [2011]developed a deteriorating inventory model for time dependent demand and holding cost with partial backlogging.Vinod Kumar Mishra [2012] made the paper of Sarala Pareek & Vinod Kumar [2009] and Mishra & Singh [2011] more realistic by considering that the salvage value is incorporated to the deteriorated items and holding cost is linear function of time and developed an inventory model for deteriorating items with time dependent deterioration rate in which demand rate is constant. Shortages are allowed and fully backlogged. Deteriorating items have salvage value.

Many authors have discussed Inventory model with single storage facility and Weibull distribution deterioration rate. An inventory model for a deteriorating item having two separate warehouses, one is an own warehouse (OW) and the other rented warehouse (RW) with Weibull deterioration rate has been considered and a rented warehouse is used when the ordering quantity exceeds the limited capacity of the owned warehouse. The holding costs at RW are higher than OW. In this study, it is assumed that the rate of deterioration in both warehouses is same as in the modern era the preservation facilities is the better in both warehouse and the holding cost was different and linearly depend on time. The demand rate is taken to be constant and shortages are allowed and partially backlogged. The aim of this model is to find an optimal order quantity and to minimize the total inventory cost. Numerical example will be presented to validate the model.

2. ASSUMPTIONS AND NOTATION

The mathematical model of the two-warehouse inventory problem is based on the following assumption and notations.

2.1 Assumptions

1. Demand rate is constant and known.

2. The lead time is zero or negligible and initial inventory level is zero.

3. The replenishment rate is infinite.

4. Shortages are allowed and partially backordered.

5. Deterioration rate is time dependent and follows a two parameter weibull distribution where $\alpha > 0$denote scale parameter and $\beta > 1$ denote the shape parameter.

6. The salvage value $\gamma(0 \leq \gamma < 1)$ is associated to deteriorated units during the cycle time.

7. The holding cost is a linear function of time and is higher in RW than OW.

8. The deteriorated units cannot be repaired or replaced during the period under review.

9. Deterioration occurs as soon as items are received into inventory.

2.2 Notation

The following notation is used throughout the paper:

d Demand rate (units/unit time) which is constant

W Capacity of OW

α Scale parameter of the deterioration rate in OW and $0<\alpha<1$

β Shape parameter of the deterioration rate in OW and $\beta>0$.

μ Scale parameter of the deterioration rate in RW, $\alpha > \mu$

η Shape parameter of the deterioration rate in RW

F Fraction of the demand backordered during the stock out period

C_o Ordering cost per order

C_d Deterioration cost per unit of deteriorated item in both ware-houses

$H_o = bt_1$; Holding cost per unit per unit time in OW during T_1 time period and $b>0$

$H_o = bt_2$; Holding cost per unit per unit time in OW during T_2 time period and $b>0$

$H_R = at_1$; Holding cost per unit per unit time in RW during T_1 time period such that $H_R > H_o$

C_s Shortage cost per unit per unit time

L_c Shortage cost for lost sales per unit

Q_o The order quantity in OW

Q_R The order quantity in RW

Q_M Maximum ordered quantity after a complete time period T

I_k Maximum inventory level in RW

T_1 Time with positive inventory in RW

T_1+T_2 Time with positive inventory in OW

T_3 Time when shortage occurs in OW

T Length of the cycle, $T = T_1+T_2+T_3$

$I_i^O(ti)$ Inventory level in OW at time ti, $0 \leq ti \leq Ti$,$i = 1,2,3$

$I^R(t_1)$ Inventory level in RW at time t_1, $0 \leq t_1 \leq T_1$

T_C^I The present value of the total relevant inventory cost per unit time

The rate of deterioration is given as follows:

t Time to deterioration, $t > 0$

Instantaneous rate of deterioration in OW

$Z(t)=\alpha\beta t^{\beta-1}$where $0 \leq \alpha < 1$

Instantaneous rate of deterioration in RW

R(t)=�$\eta t^{\eta-1}$ where $\eta > 0$,

3. MATHEMATICAL DEVELOPMENT OF MODEL

Figure-1,representing Own Ware-House Inventory System can be divided into three part depicted by T_1, T_2 and T_3. For each replenishment a portion of the replenished quantity is used to backlog shortage, while the rest enters into the system. W units of items are stored in the OW and the rest is kept into the RW. The inventory level in RW inventory system has been depicted graphically in Figure-2.The inventory in RW is supplied first to reduce the inventory cost due to more holding cost as compared to OW. Stock in the RW during time interval T_1depletes due to demand and deterioration until it reaches zero. During the time interval, the inventory in OW decreases due to deterioration only. The stock in OW depletes due to the combined effect of demand and deterioration during time T_2. During the time

intervalT_3, both warehouses are empty, and part of the shortage is backordered in the next replenishment.

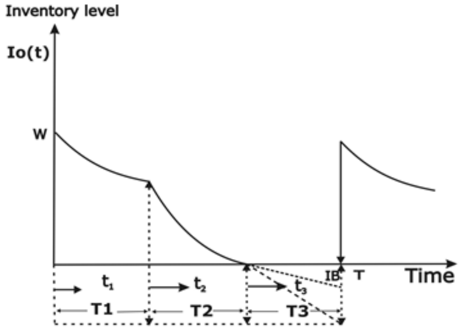


Figure 1.Graphical representation for the OW inventory system

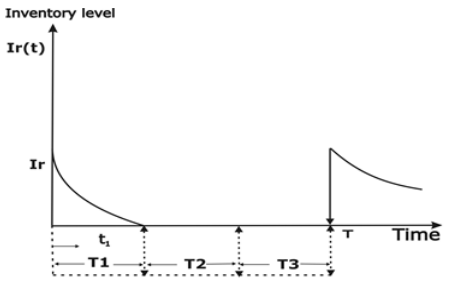


Figure 2.Graphical representation for the RW inventory system

The rate of change of inventory during positive stock in RW and time period T_1 can be represented by the differential equation

$$\frac{dI^R(t_1)}{dt_1} = -\mu\eta t_1^{\eta-1} I^R(t_1) - d \quad ; \quad 0 \le t_1 \le T_1 \quad (3.1)$$

Solution of above equation with B.C. $I^R(0) = I^r$ is

$$I^{R1}(t_1) = (I^r - d\int_0^{t_1} e^{\mu u^\eta} du) e^{-\mu t_1^\eta}; \quad 0 \le t_1 \le T_1 \quad (3.2)$$

Where $I^r = d\int_0^{T_1} e^{\mu u^\eta} du = d\sum_{m=0}^{\infty} \frac{\mu^m T_1^{m\eta+1}}{m!(m\eta+1)}$

$$\approx d\left(T_1 + \frac{\mu T_1^{\eta+1}}{\eta+1}\right) \quad (3.3)$$

The rate of change of inventory during positive stock in OW and time period $T_1+T_2+T_3$can be represented by the following differential equation

$$\frac{dI_1^O(t_1)}{dt_1} = -\alpha\beta t_1^{\beta-1} I_1^O(t_1); \quad 0 \le t_1 \le T_1 \quad (3.4)$$

$$\frac{dI_2^O(t_2)}{dt_2} = -\alpha\beta t_1^{\beta-1} I_2^O(t_2) - d; \quad 0 \le t_2 \le T_2 \quad (3.5)$$

Shortages starts during the stock out time period T_3 in OW and can be represented by the differential equation

$$\frac{dI_3^O(t_3)}{dt_3} = -Fd; \quad 0 \le t_3 \le T_3 \quad (3.6)$$

Solution of above differential equation with boundary conditions, $I_1^O(0)$ =W, $I_1^O(T_1)$ =W$e^{-\alpha T_1^\beta}$=$I_2^O(0)$ and $I_3^O(0)$=0 can be given as

$$I_1^O(t_1) = We^{-\alpha t_1^\beta}; \quad 0 \le t_1 \le T_1 \quad (3.7)$$

$$I_2^O(t_2) = (We^{-\alpha T_1^\beta} - d\int_0^{t_2} e^{\alpha u^\beta} du)\, e^{-\alpha t_2^\beta}; \quad 0 \le t_2 \le T_2 \quad (3.8)$$

$$I_3^O(t_3) = -Fdt_3; \quad 0 \le t_3 \le T_3 \quad (3.9)$$

The amount of inventory deteriorated during time period T_1 in RW is denoted and given as

$$D^R = \int_0^{T_1} R(t) I^R \, dt_1$$

$$= \mu d\left(T_1 + \frac{\mu T_1^{\eta+1}}{\eta+1}\right) T_1^\eta \approx \mu\, dT_1^{\eta+1} \quad (3.10)$$

Cost of deteriorated items in RW is denoted and given as

$$CD^R = C_d \mu\, d\, T_1^{\eta+1}$$
(3.11)

The amount of inventory deteriorated during time period T_1+T_2 in OW is denoted and given as

$$D^O = \int_0^{T_1} Z(t)\, Wdt_1 + (We^{-\alpha T_1^\beta} - \int_0^{T_2} d\ dt_2)$$

$$= W(\alpha T_1^\beta + e^{-\alpha T_1^\beta}) - dT_2$$
(3.12)

Cost of deteriorated items in OW is denoted and given as

$$CD^O = C_d\{W(\alpha T_1^\beta + e^{-\alpha T_1^\beta}) - dT_2\}$$
(3.13)

The maximum ordered quantity is denoted and given as

$$M_Q = d\left(T_1 + \frac{\mu T_1^{\eta+1}}{\eta+1}\right) + W + Fd\frac{T_3^2}{2}$$
(3.14)

The inventory holding cost in OW during time period T_1+T_2 is denoted by IH^O and given as

$$IH^O = \int_0^{T_1} H_O\, I_1^O(t_1)\, dt_1 + \int_0^{T_2} H_O\, I_2^O(t_2)\, dt_2$$

$$= \left[bW\left\{\frac{T_1^2}{2} - \frac{\alpha T_1^{\beta+2}}{\beta+2}\right\} + bW\left\{\frac{T_2^2}{2}\left(1 - \alpha T_1^\beta\right) - \frac{\alpha T_2^{\beta+2}}{\beta+2}\right\} - bd\left\{\frac{T_2^3}{3} - \frac{\alpha\beta T_2^{\beta+3}}{(\beta+1)(\beta+3)}\right\}\right]$$
(3.15)

Shortages occurs during time period T_3 due to non-availability of stock in OW, which is denoted by S_H and can be given as follows

$$S_H = \int_0^{T_3} \{-I_3^O(t_3)\}\, dt_3$$

$$= Fd\frac{T_3^2}{2} \quad (3.16)$$

Shortages cost of inventory short is denoted and given as

$$CS_H = C_S Fd\frac{T_3^2}{2}$$
(3.17)

Lost sales occurs during shortages period in OW due to partial backlogging and the amount of sale lost is denoted by L_S and given as follows

$$L_S = \int_0^{T_3} (1-F) d\ dt_3$$

$$= (1-F) dT_3 \quad (3.18)$$

Cost of lost sales is denoted by CL_S and is given as

$$CL_S = L_S(1-F)dT_3 \quad (3.19)$$

The inventory holding cost in RW during time period T_1 is denoted by IH^R and given as

$IH^R = \int_0^{T_1} at_1 \, I^R(t_1)\, dt_1$

$= ad\left\{\frac{T_1^3}{6} + \frac{\square\eta T_1^{\eta+3}}{2(\eta+2)(\eta+3)}\right\}$ (3.20)

The salvages cost of deteriorated units per unit time is denoted by SV and given as

$SV = \gamma\left[\mu\, dT_1^{\eta+1} + W(\alpha T_1^{\beta} + e^{-\alpha T_1^{\beta}}) - dT_2\right]$ (3.21)

The present value of the total inventory cost during the cycle denoted by T_C^I is the sum of ordering cost (OC) per cycle, the inventory holding cost (IH^R) per cycle in RW, the inventory holding (IH^O) per cycle in OW, Deterioration cost per cycle in RW, Deterioration cost per cycle in OW, the shortages cost (CS_H) in OW, the lost sales cost (CLS) and minus the salvages value of deteriorated units i.e.

$T_C^I(T_1, T_2, T_3) = \frac{1}{T}\left[OC + IH^R + IH^O + CD^R + CD^O + CS_H + CL_S - SV\right]$

$$= \frac{1}{T}\left[O_c + ad\left\{\frac{T_1^3}{6} + \frac{\square\eta T_1^{\eta+3}}{2(\eta+2)(\eta+3)}\right\} + \left[bW\left\{\frac{T_1^2}{2} - \frac{\alpha T_1^{\beta+2}}{\beta+2}\right\} + bW\left\{\frac{T_2^2}{2}\left(1 - \alpha T_1^{\beta}\right) - \frac{\alpha T_2^{\beta+2}}{\beta+2}\right\} - bd\left\{\frac{T_2^3}{3} - \frac{\alpha\beta T_2^{\beta+3}}{(\beta+1)(\beta+3)}\right\}\right] + C_d\mu\, d\, T_1^{\eta+1} + C_d\left\{W\left(\alpha T_1^{\beta} + e^{-\alpha T_1^{\beta}}\right) - dT_2\right\} + C_S F d\frac{T_3^2}{2} + L_S(1-F)dT_3 - \gamma\left[\mu\, dT_1^{\eta+1} + W(\alpha T_1^{\beta} + e^{-\alpha T_1^{\beta}}) - dT_2\right]\right]$$ (3.22)

The optimal problem can be formulated as

Minimize: $T_C^I(T_1, T_2, T_3)$

Subject to: $T_1 \geq 0$, $T_2 \geq 0$, $T_3 \geq 0$

To find the optimal solution of the equation the following condition must be satisfied

$$\frac{\partial T_C^I(T_1, T_2, T_3)}{\partial T_1} = 0; \quad \frac{\partial T_C^I(T_1, T_2, T_3)}{\partial T_2} = 0; \quad \frac{\partial T_C^I(T_1, T_2, T_3)}{\partial T_3} = 0$$ (3.23)

Solving equation (1.22) respectively for T_1, T_2, T_3, we can obtain $\check{T}_1, \check{T}_2, \check{T}_3$, T* and with these values we can find the total minimum inventory cost from equation (3.22).

4. ONE WARE HOUSE SYSTEM

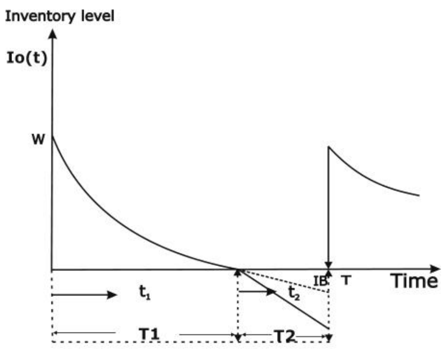


Figure3.Graphical representation for a single ware-house inventory system

Figure shows the graphical representation of one ware-house inventory system. Now considering the one warehouse inventory system we derive the inventory level during time periods T_1 and T_2 which are represented by differential equation

$\frac{dI_1^O(t_1)}{dt_1} = -\alpha\beta t_1^{\beta-1}W - d; \quad 0 \leq t_1 \leq T_1$ (4.1)

Solution of above differential equation with boundary conditions, $I_1^O(0) = W$

$I_1^O(t_1) = W(1 - \alpha t_1^{\beta}) - d\, t_1; \quad 0 \leq t_1 \leq T_1$ (4.12)

Shortages occur during the time period [0 T_2].The present worth shortages cost is

$S_C = C_S\{\int_0^{T_2}(Fdt_2)\, dt_2$

$= \frac{C_s\, Fd}{2} T_2^2$ (4.2)

Loss of sales occur during T_2time period .The OW present worth lost sales cost is given as

$CL_S = L_S \{\int_0^{T_2}(1-F)d\, dt_2\}$

$= L_C(1-F)d\, T_2$ (4.3)

Cost of deteriorated units in time interval [0 T_1] is given as

$CD^R = C_d\alpha W\, d\, T_1^{\beta}$ (4.4)

The Maximum order quantity per order is

$M_Q = W + \frac{Fd}{2}T_2^2$ (4.5)

Salvages value of deteriorated units per unit time is

$SV = \gamma W\alpha\, T_1^{\beta}$ (4.6)

Inventory holding cost during time period T_1 is

$IH^O = \int_0^{T_1} H_O\, I_1^O(t_1)\, dt_1$

$= bW\left\{\frac{T_1^2}{2} - \frac{\alpha T_1^{\beta+2}}{\beta+2}\right\}$ (4.7)

Noting that $T = T_1 + T_2$,the total present value of the total relevant cost per unit time during the cycle is the sum of the ordering cost ,holding cost shortages cost ,lost sales cost minus salvages value of deteriorated units i.e.

$$T_C^I(T_1, T_2) = \frac{1}{T}\left[O_c + bW\left\{\frac{T_1^2}{2} - \frac{\alpha T_1^{\beta+2}}{\beta+2}\right\} + C_d\alpha W\, d\, T_1^{\beta} + C_S F d\frac{T_2^2}{2} + L_C(1-F)dT_2 - \gamma W\alpha\, T_1^{\beta}\right]$$ (4.8)

The optimal problem can be formulated as

Minimize: $T_C^I(T_1, T_2)$

Subject to: $T_1 \geq 0$, $T_2 \geq 0$;

To find the optimal solution of the equation the following condition must be satisfied

$$\frac{\partial T_C^I(T_1, T_2)}{\partial T_1} = 0; \quad \frac{\partial T_C^I(T_1, T_2)}{\partial T_2} = 0;$$ (4.9)

5. NUMERICAL EXAMPLE

The Optimal replenishment policy to minimize the total present value inventory cost is derived by using the methodology given in the preceding section. The following set of parameters is assumed to analyse and validate the model

The values of parameter should be taken in a proper unit. The fixed values of set {a,b,C,C_d,Cs,L_s,F,α,β,μ, η,d,W, γ} taken as {25,20,100,10,25,10,0.8,0.05,1.8,1.8,0.02,400,100,8}. We have computed the value of decision variables using equations 3.23 and 4.9 for the two models and then the value of inventory cost for the corresponding model is calculated using equations 3.22 and 4.8.The computational results are shown in table-1.

5.1 Numerical results

The decision variable so obtained for the models are as follows:

Table-1

Decision Variables	Value obtained for Two Ware-house model	Value obtained for One Ware-house system
$\check{T}_1$	0.4003150	0.0426341
$\check{T}_2$	3.5014900	0.0426341
$\check{T}_3$	0.0859854	-----------
T*	3.9877904	0.1668881
T_C^{I*}	1487.8800	1794.0300

We conclude from the above numerical result as follows

1.From table-1, when all the given conditions and constraints are satisfied, the optimal solution is obtained. In this example the minimal present value of total relevant inventory cost per unit time in an appropriate unit is 1487.88, while the respective optimal values of decision variables Ť1, Ť2, Ť3 and T* are 0.4003150, 3.5014900, 0.0859854and 3.9877904 respectively.

2. When there is only single ware-house with limited capacity W units is considered then the minimal present value of total relevant inventory cost per unit time in an appropriate unit is 1794.03 while the respective optimal time period of positive and negative inventory level are 0.0426341, 0.0426341and 0.1668881 respectively.The total relevant inventory cost incurs an increase of 306.15 as compared with two ware-house system. The system has no space to store excess units and the total relevant inventory cost is higher due to holding cost and shortages cost.

3. When there is complete backlogging (i.e. F=1), the minimal value of the total relevant inventory cost is 2122.66 while the respective values of decision variables $\check{T}_1$, $\check{T}_2$, $\check{T}_3$ and T* are 6.65684, 1.78748, 0.212266 and 8.656586 respectively. The total relevant inventory cost incurs an increase of 634.78 as compared to partial backlogging model under two ware-house systems.

4. The graphical representation of convex in Figure-4 for the total relevant inventory cost in the two ware-house system and in Figure-5 for the total relevant inventory cost in the one ware-house system shows that there exist a point where the total relevant inventory cost is minimum.

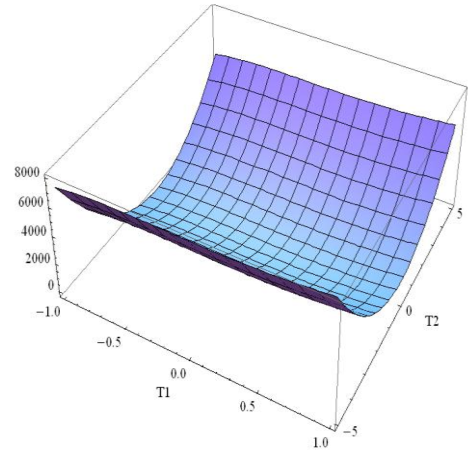


Figure-4 Graphical representation of T_C^{I*} (When $\check{T}_3$*= 0.0859854) for Two ware-house system

6. SENSITIVITY ANALYSIS

In order to study the effects of parameters after the optimal solution. Sensitivity analysis is performed for the numerical example given at point 6.0, we found that the optimal values of $\check{T}_1$,$\check{T}_2$,$\check{T}_3$ and T_C^{I*}. For a fixed subset S={a,b,C,C_d,Cs,L_s,F,α,β,μ,ŋ, d, W, γ }.The base column of S is S={25,20,100,10,25,10,0.8,0.05,1.8,1.8,0.02,400,100,8}. The optimal values of $\check{T}_{1*}$,$\check{T}_{2*}$,$\check{T}_{3*}$andT_C^I are derived when one of the parameters in the subset S increases by 10 % and all other parameters remain unchanged. The result of the sensitivity analysis are shown in the Table-2 with corresponding graphical representation. The change in the total relevant inventory cost given as percentage Change in Cost (PCC) is as PCC=$\frac{T_C^{IC}-T_C^{I*}}{T_C^{I*}}\times 100$

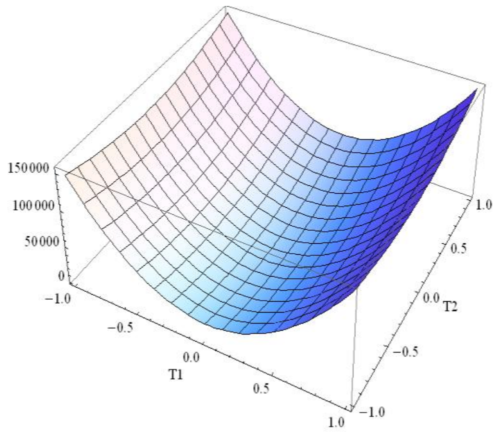
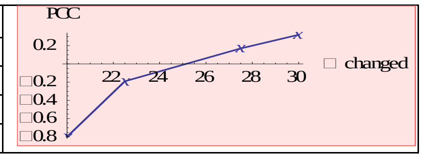


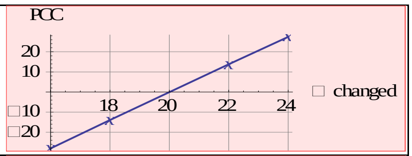
Figure-5. Graphical representation convex of T_C^{I*} for Oneware-house system

Table-2. Sensitivity analysis when the parameter is changed by 10%

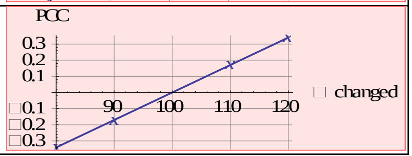
A	$\check{T}_{1*}$	$\check{T}_{2*}$	$\check{T}_{3*}$	T_C^{IC}	PCC(%)
30	0.3732	3.4991	0.0866	1492.72	0.32
27.5	0.3859	3.5002	0.0863	1490.43	0.17
22.5	0.4169	3.5029	0.0856	1485.08	-0.19
20	0.4362	3.6668	0.0853	1475.85	-0.80



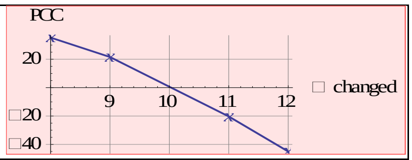
B	$\check{T}_{1*}$	$\check{T}_{2*}$	$\check{T}_{3*}$	T_C^{IC}	PCC(%)
24	0.4515	3.5220	0.1370	1896.03	27.43
22	0.4268	3.5123	0.1116	1692.90	13.78
18	0.3714	3.4894	0.0607	1280.54	-13.94
16	0.3393	3.4756	0.0338	1070.75	-28.04



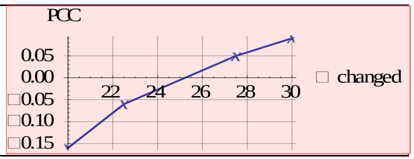
C_o	$\check{T}_{1*}$	$\check{T}_{2*}$	$\check{T}_{3*}$	T_C^{IC}	PCC(%)
120	0.4008	3.5002	0.0863	1492.90	0.34
110	0.4013	3.4989	0.0866	1490.39	0.17
90	0.3998	3.5027	0.0857	1485.38	-0.17
80	0.3993	3.5040	0.0853	1482.87	-0.34



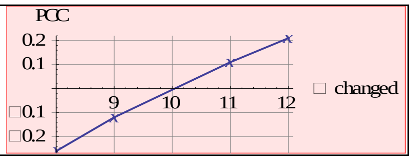
C_d	$\check{T}_{1*}$	$\check{T}_{2*}$	$\check{T}_{3*}$	T_C^{IC}	PCC(%)
12	0.2515	3.4328	0.0024	818.951	-44.96
11	0.3307	3.4653	0.0448	1185.25	-20.34
9	0.4636	3.5402	0.1262	1809.35	21.61
8	0.5224	3.5807	0.1655	2012.73	35.28



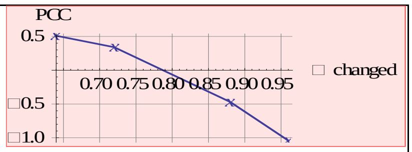
C_s	$\check{T}_{1*}$	$\check{T}_{2*}$	$\check{T}_{3*}$	T_C^{IC}	PCC(%)
30	0.4006	3.5009	0.0718	1489.18	0.09
27.5	0.4005	3.5012	0.0782	1488.56	0.05
22.5	0.4002	3.5019	0.0954	1487.06	-0.06
20	0.3999	3.5024	0.1072	1486.04	-0.12



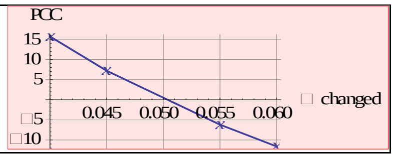
L_s	$\check{T}_{1*}$	$\check{T}_{2*}$	$\check{T}_{3*}$	T_C^{IC}	PCC(%)
12	0.4009	3.4999	0.0664	1490.95	0.21
11	0.4006	3.5067	0.0762	1489.51	0.11
9	0.3995	3.5024	0.0958	1486.09	-0.12
8	0.3996	3.5034	0.1055	1484.05	-0.26



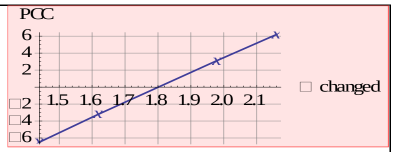
F	$\check{T}_{1*}$	$\check{T}_{2*}$	$\check{T}_{3*}$	T_C^{IC}	PCC(%)
0.96	0.3973	3.5092	0.1367	1472.48	-1.04
0.88	0.3989	3.5050	0.1137	1480.88	-0.47
0.72	0.4013	3.4989	0.0518	1492.99	0.34
0.64	0.4018	3.4977	0.0087	1495.50	0.51



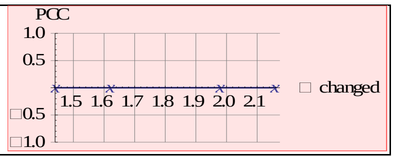
α	$\check{T}_{1*}$	$\check{T}_{2*}$	$\check{T}_{3*}$	T_C^{IC}	PCC(%)
0.060	0.3653	3.1908	0.0644	1315.15	-11.61
0.055	0.3818	3.3357	0.0744	1395.59	-6.20
0.045	0.4215	3.6935	0.0940	1595.21	7.21
0.040	0.4459	3.9198	0.1153	1722.08	15.74



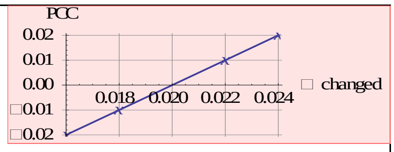
β	$\check{T}_{1*}$	$\check{T}_{2*}$	$\check{T}_{3*}$	T_C^{IC}	PCC(%)
2.16	0.4159	3.6657	0.0975	1580.07	6.20
1.98	0.4083	3.5846	0.0918	1534.49	3.13
1.62	0.3921	3.4163	0.0800	1440.17	-3.19
1.44	0.3835	3.3288	0.0739	1391.29	-6.49



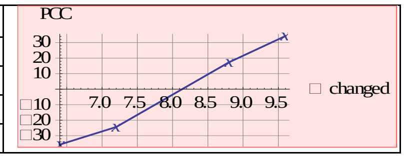
η	$\check{T}_{1*}$	$\check{T}_{2*}$	$\check{T}_{3*}$	T_C^{IC}	PCC(%)
2.16	0.4003	3.5015	0.0859	1487.88	0.00
1.98	0.4003	3.5015	0.0859	1487.88	0.00
1.62	0.4003	3.5015	0.0859	1487.88	0.00
1.44	0.4003	3.5015	0.0859	1487.88	0.00



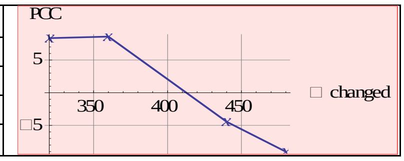
μ	$\check{T}_{1*}$	$\check{T}_{2*}$	$\check{T}_{3*}$	T_C^{IC}	PCC(%)
0.024	0.3997	3.5013	0.0806	1488.20	0.02
0.022	0.4000	3.5014	0.0860	1488.04	0.01
0.018	0.4006	3.5016	0.0859	1487.72	-0.01
0.016	0.4009	3.5017	0.0859	1487.56	-0.02



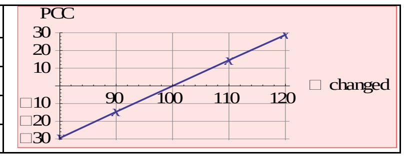
ϒ	$\check{T}_{1*}$	$\check{T}_{2*}$	$\check{T}_{3*}$	T_C^{IC}	PCC(%)
9.6	0.4994	3.5643	0.1499	1998.77	34.33
8.8	0.4515	3.5323	0.0118	1745.65	17.32
7.2	0.3452	3.4725	0.0531	1124.59	-24.41
6.4	0.2846	3.4452	0.0195	955.95	-35.75



D	$\check{T}_{1*}$	$\check{T}_{2*}$	$\check{T}_{3*}$	T_C^{IC}	PCC(%)
480	0.3475	3.4884	0.0410	1353.68	-9.02
440	0.3726	3.4945	0.0616	1421.80	-4.44
360	0.4452	3.5176	0.1525	1615.73	8.59
320	0.4669	3.5195	0.1519	1611.91	9.34



W	$\check{T}_{1*}$	$\check{T}_{2*}$	$\check{T}_{3*}$	T_C^{IC}	PCC(%)
120	0.4556	3.5297	0.1396	1917.06	28.85
110	0.4289	3.5164	0.1129	1703.62	14.50
90	0.3690	3.4854	0.0587	1269.66	-14.67
80	0.3342	3.4649	0.0311	1048.43	-29.54



We conclude from the sensitivity analysis of the model from the Table-2 as follows:

(1) The value of PCC is the highly sensitive to the parameters W (capacity of own ware-house), ϒ (Salvages value incurred on deteriorated items),b(Holding cost of inventory in OW and is directly proportional to these values.

(2) The Value of PCC is the highly sensitive to the C_d (Cost of deterioration) and sensitive to d (Demand of inventory)α (Scale parameter of the deterioration rate in RW), and is indirectly proportional to these values.

(3) The value of PCC is slightly sensitive to the values of β(The shape parameter of deterioration rate in RW,μ (scale parameter of deterioration rate in OW),C_s(Cost of shortages),L_s(Cost of lost sale),C_o (Ordering cost) ,a (Holding cost in RW) and is directly proportional to these values.

(4) The value of PCC is slightly sensitive to the values of F (Amount of shortages backlogged and sensitive to and is indirectly proportional to it.

(5) The value of PCC is not sensitive to the value of η (Shape parameter of deterioration rate in OW).

(6) The graphical representation of the changes in the value of PCC with corresponding change in the one parameter keeping others unchanged is shown in the above table.

7. CONCLUSIONS

In this paper, an inventory model is presented to determine the optimal replacement cycle for two warehouse inventory problem under varying rate of deterioration and partial backlogging .The model assumes that the capacity of distributors' warehouse is limited. The optimization technique is used to derive the optimum replenishment policy i.e. to minimize the total relevant cost of the inventory system. A numerical example is presented to illustrate the model validity. When there is single ware house is assumed in the inventory system then the total relevant cost per unit time of the system are higher than the two warehouse model. This model is is most useful for the instant deteriorating items under weibull distribution deterioration rate as inventory cost depending on demand is indirectly proportional to demand. Further this paper can be enriched by incorporating other types of time dependent demand and another extension of this modelcan be done for a bulk release pattern. In practice now days pricing and advertising also have effect on the demand rate and must be taken into consideration.

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