A Supply Chain Production Inventory Model for Deteriorationg Product with Stock Dependent Demand under Inflationary Environment and Partial Backlogging

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ABSTRACT
In this paper we have developed a two echelon supply chain production inventory model for deteriorating products having stock dependent demand under inflationary environment. This model is developed for finite time horizon. The shortages are allowed and partially backlogged. To make this study close to reality the production rate is assumed to be a function of demand rate. A numerical example and sensitivity analysis with respect to different associated parameter is also presented to illustrate the study.

Keywords
Deterioration, Inflation, Inventory, Stock dependent demand, Demand dependent production, Shortages, Partial backlogging

1. INTRODUCTION
In classical inventory models most of the researchers continued their study without considering the effect of inflation. Inflation plays a major role for long term business plans. The results obtained without considering the inflation misleads the results. So the effect of inflation cannot be ignored in the development of inventory models. Buzacott (1975) was the first who came forward with pioneer work and developed an inventory model with different pricing policies under inflationary environment. Datta and Pal (1991) developed a finite time horizon inventory model with linear time-dependent demand rate and shortages under the effects of inflation and time value of money. restriction of equal replenishment cycle and provided two solution procedures with and without shortages. After that, Ray and Chaudhari (1997) and Wee and Law (2001) all have included the inflation and time value of money in their models and investigated its effect on results. Singh and Singh (2012) developed an EPQ model with power form stock dependent demand under inflationary environment using genetic algorithm. Singh et al. (2013) presented an EOQ model with volume agility, variable demand rate, Weibull deterioration rate and with

the effect of inflation. Tayal et al. (2014) introduced a two echelon supply chain model for deteriorating items with effective investment in preservation technology under inflationary environment.

In the development of inventory models to assume the demand rate as a constant or time dependent is not enough for all type of business. There are so many products in our daily life for which the demand rate depends on displayed stock level. For these the demand rate may increase or decrease according to the available stock level. Gupta and Vrat (1986) first developed a model for stock dependent demand with the assumption that the demand for the products is a function of initial stock level and optimize the total cost in this model after this Padmanabhan and Vrat (1988) defined stock-dependent consumption rate in which the demand rate is a function of available stock level at that time. Kumar and Singh (2009) introduced a two warehouse inventory model with stock dependent demand rate for deteriorating items with shortages. Tyagi et al. (2014) presented an optimal replenishment policy for non-instantaneous deteriorating items with stock dependent demand and variable holding cost. Deterioration is also an important and necessary feature for the development of inventory models. In our daily life all the products deteriorate with time. Some products have a high rate of deterioration and some products deteriorate at a lower rate. Hou (2006) introduced an inventory model for deteriorating items with stock-dependent consumption rate and shortages under inflation and time discounting. Chang et al. (2010) developed an inventory model with stock and price dependent demand rate for deteriorating items based on limited shelf space. Tayal et al. (2014) presented a deteriorating production inventory problem with space restriction. Tayal et al. (2014) also introduced an inventory model for deteriorating items having seasonal demand and an option of an alternative market. But for many products deterioration do not occur instantly; the products start to deteriorate after some time. This type of products is known as non instantaneous deteriorating products. For such type of products an EPQ model for non-instantaneous deteriorating item with time dependent holding cost and exponential demand rate was developed by Tayal et al. (2014). After that Tayal et al. (2015) developed an integrated production inventory model for perishable products with trade credit period and investment in preservation technology. In this model a preservation technology is applied to reduce the existing rate of deterioration. Further stock out is also a realistic problem in inventory models. There are so many models in existing literature which assumed that the demand during stock out is completely backlogged or completely lost, but generally, both of these conditions are not possible. Considering this phenomenon Singh et al. (2010) presented an EOQ model with Pareto distribution for deterioration, Trapezoidal type demand and backlogging under trade credit policy. In this model the occurring shortages are partially backlogged. Tayal et al. (2014) presented a multi item inventory model for deteriorating items with expiration date and allowable shortages.

In the present model we have tried to combine all above mentioned realistic features in a single model. This is a production inventory model in which the production rate is taken as a function of demand rate and demand rate is taken as a function of available stock level to make the study close to reality. In this the shortages are allowed and occurring shortages are partially backlogged at a constant rate. The model is developed under inflationary environment. A numerical example and sensitivity analysis is also presented to illustrate the model.

1.2 Assumptions

The product considered in this model is deteriorating in nature.

1. The deterioration rate is a constant fraction of on hand inventory.
2. The model is developed for finite time horizon.
3. The demand for the products is stock dependent.
4. The production rate is also considered as a function of demand rate.
5. The shortages are allowed for retailer only and occurring shortages are partially backlogged.

1.2.1 Notations

- α positive constant
- β stock dependent parameter for demand, $0 \leq \beta \leq 1$
- T time horizon
- r rate of inflation
- K production coefficient, $K \geq 1$
- θ constant deterioration rate, $0 < \theta << 1$
- T_1 production period for vendor
- c_m production cost per unit for the vendor
- h_1 holding cost per unit for the vendor
- h_2 holding cost per unit for the retailer
- A_1 set up cost per production run for the vendor
- A_2 ordering cost per order for the retailer
- s shortage cost per unit for the retailer
- l lost sale cost per unit for the retailer
- Q initial inventory level for the retailer
- Q_2 backordered quantity during shortages
- p purchasing cost per unit for the retailer
- v the time at which inventory level becomes zero for the retailer

2. Mathematical Modelling

For Vendor

In this section we have developed a mathematical inventory model for the manufacturer. The production starts at t=0. The production occurs during [0,T1] and after that during [T1, T] the inventory depletes due to combined effect of demand and deterioration. This procedure is shown in figure 1.

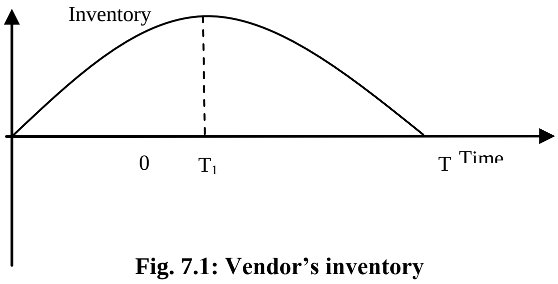

Fig. 7.1: Vendor's inventory

The differential equations governing the transition of the system during production and non production period is given as follow:

$$\frac{dI_v(t)}{dt} = (K-1)(\alpha + \beta I_v(t)) - \theta I_v(t) \quad 0 \leq t \leq T_1 \qquad (1)$$

$$\frac{dI_v(t)}{dt} = -(\alpha + \beta I_v(t)) - \theta I_v(t) \quad T_1 \leq t \leq T \qquad (2)$$

Boundary conditions are:

$I_v(0) = 0$ and $I_v(T) = 0$ … (3)

The solutions of these equations are given by:

$$I_v(t) = \frac{(K-1)\alpha}{\theta - \beta(K-1)}(1 - e^{-(\theta - K\beta + \beta)t}) \quad 0 \leq t \leq T_1 \quad (4)$$

$$I_v(t) = \frac{\alpha}{(\theta + \beta)}(e^{(\beta+\theta)(T-t)} - 1) \quad T_1 \leq t \leq T \quad (5)$$

For Retailer

The inventory time graph for the retailer is shown in figure 2. The inventory cycle starts at t=0. During the time period [0, v], the inventory level decreases due to demand and deterioration. At t=v the inventory level becomes zero and after it shortages occur. The occurring shortages are partially backlogged at a constant rate. The differential equations showing the behavior of the system are given as follow.

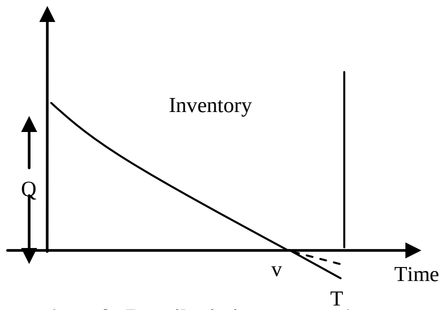

Fig.7. 2: Retailer's inventory time

$$\frac{dI_r(t)}{dt} = -(\alpha + \beta I_r(t)) - \theta I_r(t) \quad 0 \leq t \leq v \qquad (6)$$

$$\frac{dI_r(t)}{dt} = -\alpha - \theta I_r(t) \qquad v \leq t \leq T \qquad (7)$$

With boundary condition $I_r(v) = 0$ (8)

The solutions of these equations are given by:

$$I_r(t) = \frac{\alpha}{(\theta + \beta)}(e^{(\beta+\theta)(v-t)} - 1) \quad 0 \leq t \leq v \qquad (9)$$

$$I_r(t) = -\alpha(t - v), \qquad v \leq t \leq T \qquad \text{… (10)}$$

Now from equation (10):

$$Q = I_r(0) = \frac{\alpha}{(\beta+\theta)}(e^{(\beta+\theta)v} - 1) \quad \text{… (11)}$$

Total cost for the vendor will be the sum of production cost, holding cost and set up cost.

T.Cv = Production cost + Holding Cost + Set up cost … (12)

For the retailer total cost will be the sum of purchasing cost, ordering cost, holding cost, shortage cost and lost sale cost.

T.Cr = Purchasing cost + Holding cost + Ordering Cost + Shortage cost + Lost sale cost … (13)

2.1 Cost Analysis for Vendor

Production Cost

Present value of production cost-

$$P.C. = c_m \int_0^{T_1} K(\alpha + \beta I_v(t)) e^{-rt} dt$$

$$P.C. = c_m(K(\frac{\alpha}{r}(1-e^{-rT_1}) + \frac{\alpha\beta(K-1)}{(\theta-K\beta+\beta)}(\frac{(1-e^{-rT_1})}{r} + \frac{(e^{-(\theta-K\beta+\beta+r)T_1}-1)}{(\theta-K\beta+\beta+r)}))) \quad (14)$$

Holding Cost:

Present value of holding cost-

$$H.C. = h_1 \int_0^{T_1} I_v(t) e^{-rt} dt + h_1 \int_{T_1}^{T} I_v(t) e^{-r(T_1+t)} dt$$

$$H.C. = h_1 \frac{(K-1)\alpha}{(\theta-K\beta+\beta)}(\frac{e^{-(\theta-K\beta+\beta+r)T_1} - 1}{(\theta-K\beta+\beta+r)} + \frac{(1-e^{-rT_1})}{r}) +$$

(15)

Setup Cost

Present value of set up cost-

All set up is made at the beginning of cycle, so there will be no inflation at that time.

$$S.U.C. = A_1 \quad \text{… (16)}$$

Cost Analysis for Retailer

Purchasing Cost

Present value of purchasing cost-

Since the replenishment is made at the beginning of each cycle, so the present value of purchasing cost will be-

$$P.C. = (Q + Q_2 e^{-rT}) p \quad \text{… (17)}$$

Holding Cost

Present value of holding cost-

$$H.C_r = h_2 \int_0^{v} I_r(t) e^{-rt} dt$$

$$H.C_r = h_2 \frac{\alpha}{(\beta+\theta)(\beta+\theta+r)}(e^{(\theta+\beta)v} - e^{-rv}) + \frac{h_2\alpha}{r(\beta+\theta)}(e^{-rv} - 1) \quad (18)$$

Shortage Cost

Present value of shortage cost-

$$S.C_r = s \int_v^T \alpha e^{-rt} dt$$

$$S.C_r = \frac{s\alpha}{r}(e^{-rv} - e^{-rT}) \quad \text{… (19)}$$

Lost Sale Cost

Present value of lost sale cost-

$$L.S.C_r = l \int_v^T (1-\theta)\alpha e^{-rt} dt$$

$$L.S.C_r = \frac{l\alpha(1-\theta)}{r}(e^{-rv} - e^{-rT})$$

… (20)

$$T.C_S = T.C_v + T.C_r$$

$$T.A.C_S = \frac{1}{T}[T.C_v + T.C_r]$$

… (21)

2.2 Numerical Example

The following data is used to illustrate the model numerically.

$\alpha = 500\, units,\ r = 0.06,\ T = 10\, days,\ c_m = 5\, rs/unit,\ p = 12\, rs/unit,\ s = 6\, rs/unit,\ h_1 = 0.8\, rs/unit$

$h_2 = 0.7\, rs/unit,\ A_1 = 500\, rs,\ \theta = 0.01,\ \beta = 0.02,\ l = 7\, rs/unit$

For these input values the optimal value of v and T1 come out to be 8.67279 days and 2.79674 days respectively. Corresponding to these, the optimal value of T.A.C for the supply chain is Rs. 25350.

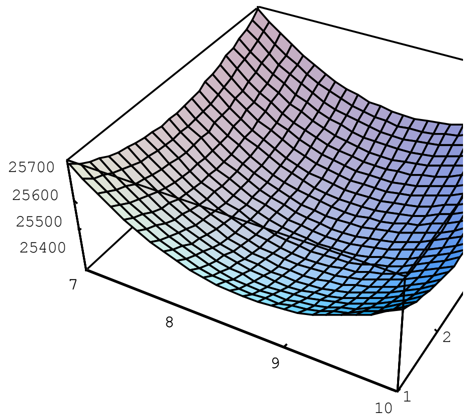

Fig. 7.3: Behaviour of the T.C. function

3 SENSITIVITY ANALYSIS

A sensitivity analysis with respect to different associated parameters is carried out to observe the change in T.A.C. with the change in these parameters. Table 1-

Table 7.1: Sensitivity Analysis with respect to demand parameter (α):

% variation in α	A	v	T_1	T.A.C.
-20%	400	8.67279	2.79674	20310
-15%	425	8.67279	2.79674	21570
-10%	450	8.67279	2.79674	22830
-5%	475	8.67279	2.79674	24090

0%	500	8.67279	2.79674	25350
5%	525	8.67279	2.79674	26610
10%	550	8.67279	2.79674	27869.9
15%	575	8.67279	2.79674	29129.9
20%	600	8.67279	2.79674	30389.9

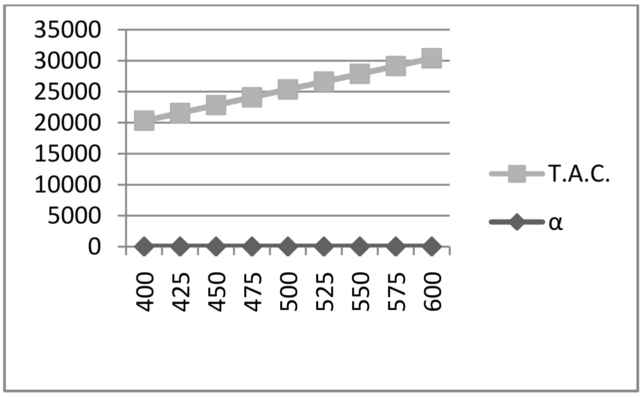

Fig. 7.4: T.A.C. v/s α

Table 7.2: Sensitivity Analysis with respect to r

% variation in r	**r**	**V**	**T_1**	**T.A.C.**
-20%	0.048	9.71663	2.7127	22238.2
-15%	0.051	9.44301	2.73433	22942.5
-10%	0.054	9.17723	2.75549	23692.3
-5%	0.057	8.9203	2.77626	24492.7
0%	0.06	8.67279	2.79674	25350
5%	0.063	8.43492	2.81699	26271
10%	0.066	8.20671	2.83709	27264.2
15%	0.069	7.98802	2.85708	28339.1
20%	0.072	7.7786	2.87702	29507.2

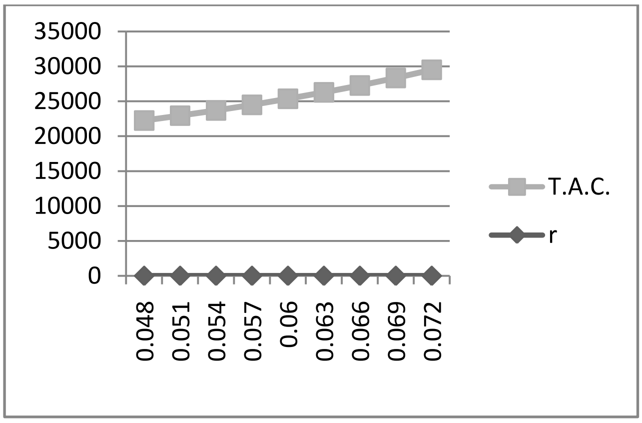

Fig. 7.5: T.A.C. v/s r

Table 7.3: Sensitivity Analysis with respect to deterioration parameter (θ):

% variation in θ	θ	v	T1	T.A.C.
-20%	0.008	9.09532	2.79017	24056.8
-15%	0.0085	8.9878	2.79184	24365.5
-10%	0.009	8.88155	2.7935	24683.7
-5%	0.0095	8.77654	2.79513	25011.6
0%	0.01	8.67279	2.79674	25350
5%	0.0105	8.57028	2.79833	25699.1
10%	0.011	8.46901	2.7999	26059.6
15%	0.0115	8.36898	2.80145	26432.1
20%	0.012	8.27018	2.80298	26817.2

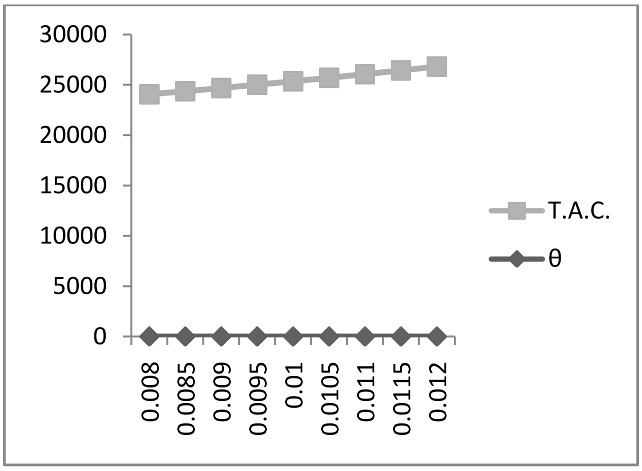

Fig. 7.6: T.A.C. v/s θ

Table 7.4: Sensitivity Analysis with respect to demand parameter (β):

% variation in β	**β**	**v**	**T_1**	**T.A.C.**
-20%	0.016	9.5419	2.79366	28497.2
-15%	0.017	9.31707	2.79455	27627.6
-10%	0.018	9.09728	2.79536	26817.1
-5%	0.019	8.88252	2.79609	26059.6
0%	0.02	8.67279	2.79674	25350
5%	0.021	8.46805	2.79732	24683.6
10%	0.022	8.26826	2.79782	24056.6
15%	0.023	8.07338	2.79825	23465.5
20%	0.024	7.88335	2.79862	22907.2

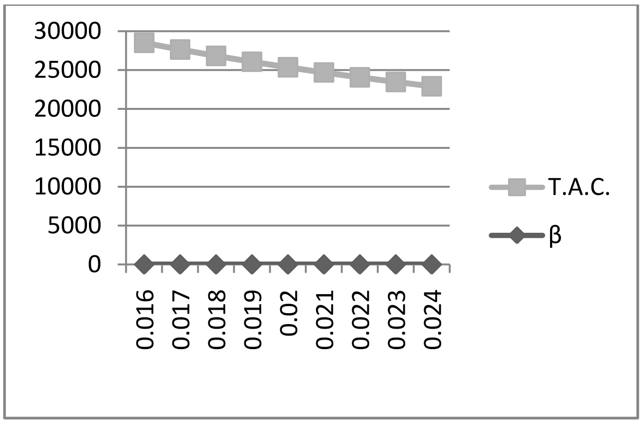

Fig. 7.7: T.A.C. v/s β

Table 7.5: Sensitivity Analysis with respect to holding cost (h_1):

% variation in h_1	h_1	v	T_1	T.A.C.
-20%	0.64	8.67279	1.87883	25145.2
-15%	0.68	8.67279	2.1374	25201.7
-10%	0.72	8.67279	2.37493	25254.3
-5%	0.76	8.67279	2.594	25303.5
0%	0.8	8.67279	2.79674	25350
5%	0.84	8.67279	2.98499	25393.9
10%	0.88	8.67279	3.16028	25435.7
15%	0.92	8.67279	3.32394	25475.6
20%	0.96	8.67279	3.47712	25573.8

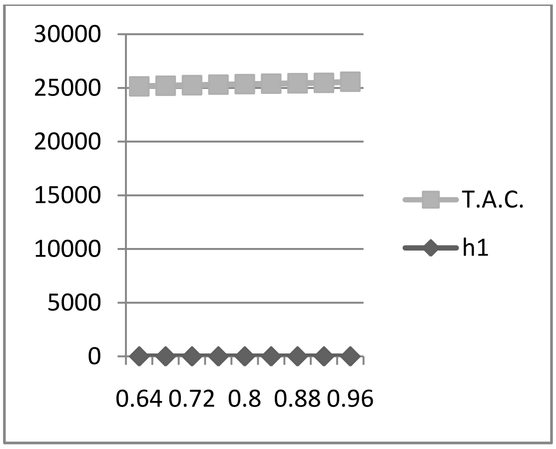

Fig. 7.8: T.A.C. v/s h_1

Table7.6: Sensitivity Analysis with respect to holding cost (h_2):

% variation in h_2	h_2	v	T_1	T.A.C.
-20%	0.56	8.11772	2.79674	21873.1
-15%	0.595	8.27259	2.79674	22747.5
-10%	0.63	8.41593	2.79674	23618.1
-5%	0.665	8.54897	2.79674	24485.15
0%	0.7	8.67279	2.79674	25350
5%	0.735	8.78831	2.79674	26211.7
10%	0.77	8.89635	2.79674	27071.1
15%	0.805	8.9976	2.79674	27928.4
20%	0.84	9.09268	2.79674	28889

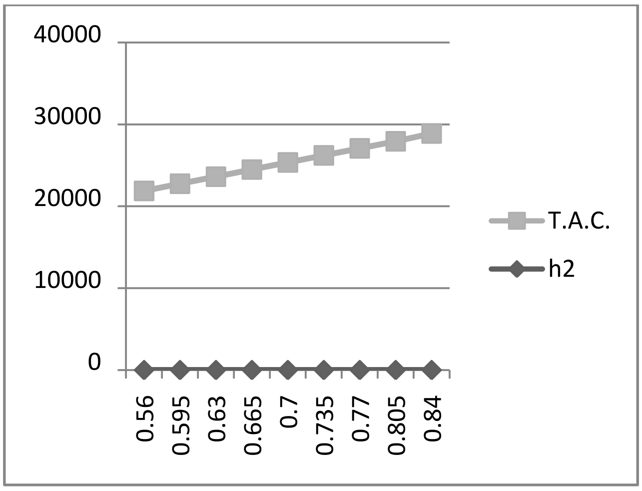

Fig. 7.9: T.A.C. v/s h_2

Table 7.7: Sensitivity Analysis with respect to m:

% variation in m	m	v	T_1	T.A.C.
-20%	4	8.867279	3.62081	25175.2
-15%	4.25	8.867279	3.40628	25222.7
-10%	4.5	8.867279	3.19761	25267.6
-5%	4.75	8.867279	2.99452	25310
0%	5	8.867279	2.79674	25350
5%	5.25	8.867279	2.60401	25387.4
10%	5.5	8.867279	2.41607	25422.4
15%	5.75	8.867279	2.2327	25455
20%	6	8.867279	2.05369	25485.2

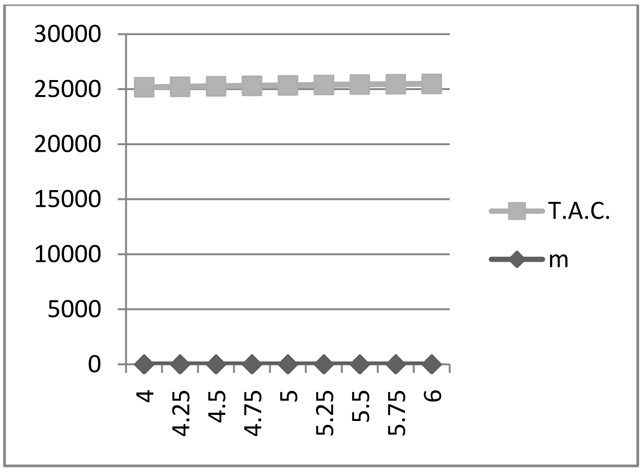

Fig. 7.10: T.A.C. v/s m

Table 7.8: Sensitivity Analysis with respect to p:

% variation in p	p	v	T1	T.A.C.
-20%	9.6	9.50203	2.79674	24110.5
-15%	10.2	9.29165	2.79674	24432.2
-10%	10.8	9.08334	2.79674	24746
-5%	11.4	8.87706	2.79674	25051.8
0%	12	8.67279	2.79674	25350
5%	12.6	8.47049	2.79674	25640.4
10%	13.2	8.27015	2.79674	25923.3
15%	13.8	8.07172	2.79674	26198.8
20%	14.4	7.87519	2.79674	26467

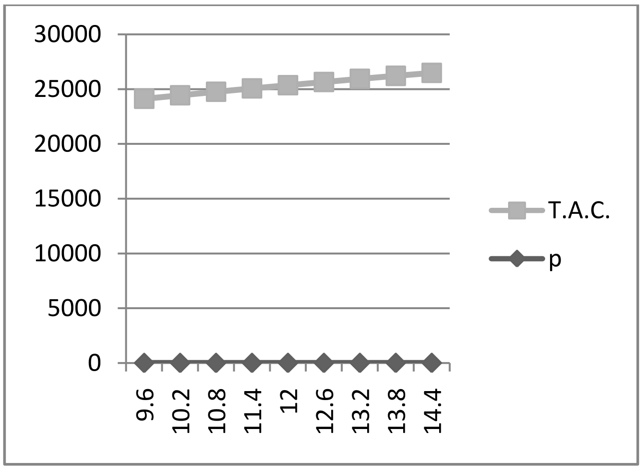

Fig. 7.11: T.A.C. v/s p

4 OBSERVATIONS

1. Table 1 shows the behavior of T.A.C. with the variation in demand parameter α. From this table it is observed that as the value of demand parameter α increases, the T.A.C. of the system also increases.

2. Table 2 shows the variation in inflation rate r and it is observed that with the increment in inflation rate r, T.A.C. of the system increases.

3. From table 3 we observe the behavior of T.A.C. with the variation in deterioration rate θ, and it is observed that with the increment in deterioration rate θ, the T.A.C. of the supply chain increases.

4. Table 4 lists the variation in demand parameter (β) and it is observed that an increment in β results an increment in T.A.C.

5. Table 5 and table 6 show the variation in holding cost h_1 and h_2 and it is observed that the increment in both h_1 and $h_{2,}$ increase the T.A.C. of the supply chain.

6. Table 7 shows the production cost m at different points and other variable unchanged. From this table it is observed that with the increment in m, T.A.C. of the system increases.

7. Table 8 lists the variation in purchasing cost (p) and it is observed that T.A.C. of the supply chain increases with the increment in purchasing cost (p).

5 CONCLUSION

This is a two echelon supply chain model for deteriorating items for vendor and buyer. The demand rate considered in this model is stock dependent. The production rate is taken as a function of demand rate. During stock out the occurring shortages are partially backlogged. These all factors together make the study close to reality. The inflation rate is also considered in the development of this model. So this model is very useful in real life situations. This model and demand pattern is applicable for seasonal products, fashion products and bakery products. The numerical example and sensitivity analysis is presented to illustrate this model and its significant features. This model has a further scope of extension with permissible delay period.

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