# Generalized Directable Fuzzy Automata

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## ABSTRACT

In this paper, we introduce generalized directable fuzzy automaton and discuss their structural characterizations. We have shown that a generalized directable fuzzy automaton is an extension of a uniformly monogenically strongly directable fuzzy automaton by a uniformly monogenically trap-directable fuzzy automaton and obtain other equivalent conditions for a generalized directable fuzzy automaton.

### Keywords

Necks & Local necks of a fuzzy automaton, Uniformly monogenically directable fuzzy automaton, generalized directable fuzzy automaton.

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## 1. INTRODUCTION

Fuzzy set is a generalization of a classical set was introduced by Zadeh in 1965 [11]. This concept is applied in different discipline including medical sciences, artificial intelligence, pattern recognition and automata theory. Fuzzy ideas applied in automata was first proposed by Wee in 1967 [10]. Santos proposed fuzzy automata as a model of pattern recognition [9]. John N. Mordeson and D.S. Malik gave a detailed account of fuzzy automata and applications in their book 2002 [8].

The notion of a generalized directable automaton was introduced by T. Petkovic $et.al$ [13]. In this paper, we introduce a generalized directable fuzzy automaton and discuss their structural characterizations. We have shown that every finite fuzzy automaton can be uniquely represented as an extension of a direct sum of strongly connected fuzzy automata by a monogenically trap-directable fuzzy automaton and obtain a necessary condition for a fuzzy automaton to be strongly generalized directable. We have shown that a generalized directable fuzzy automaton is an extension of a uniformly monogenically strongly directable fuzzy automaton by a uniformly monogenically trap-directable fuzzy automaton and obtain other equivalent conditions for a generalized directable fuzzy automaton.

## 2. PRELIMINARIES

This section present basic concept and results to be used in the sequel.

Let $X$ denote a universal set. Then a fuzzy set $A$ in $X$ is set of ordered pairs: $A = \{(x,\ \mu_A(x)|x \in X\}$, $\mu_A(x)$ is called the membership function or grade of membership of $x$ in $A$ which maps $X$ to the membership space $[0, 1]$ [12].

A finite fuzzy automaton is a system of 5 tuples, $M = (Q,\ \Sigma,\ f_M,\ q_0,\ F)$,
where, $Q$ is set of states, $\Sigma$ is input symbols, $f_M$ is transition function from $Q \times \Sigma \times Q \to [0, 1]$, $q_0$ is an initial state and $q_0 \in Q$, and $F \subseteq Q$ set of final states. The transition in a fuzzy automaton is as follows:
$f_M(q_i, a, q_j) = \mu$, $0 \leq \mu \leq 1$, means that when $M$ is in state $q_i$ and reads the input $a$ will move to the state $q_j$ with weight function $\mu$.
$f_M$ can be extended to $Q \times \Sigma^* \times Q \to [0, 1]$ by,

$$f_M(q_i,\ \epsilon,\ q_j) = \begin{cases} 1 & \text{if } q_i = q_j \\ 0 & \text{if } q_i \neq q_j \end{cases}$$

$$f_M(q_i, w, q_m) = Max\{Min\{f_M(q_i, a_1, q_1), f_M(q_1, a_2, q_2), .., f_M(q_{m-1}, a_m, q_m)\}\}$$

for $w = a_1\ a_2\ a_3\ ...\ a_m \in \Sigma^*$, where $Max$ is taken over all the paths from $q_i$ to $q_m$ [4].

Throughout this paper, we consider a fuzzy automaton without initial state and final state and $M$ denotes $M = (Q, \Sigma, f_M)$, $f_M$ is transition function from $Q \times \Sigma \times Q \to [0, 1]$.

A fuzzy automaton $M$ is called deterministic if for each $a \in \Sigma$ and $q_i \in Q$, there exists a unique state $q_a$ such that $f_M(q_i,\ a,\ q_a) > 0$ otherwise it is called nondeterministic [3].

Let $M' = (Q',\ \Sigma,\ f_{M'})$, $Q' \subseteq Q$ and $f_{M'}$ is the restriction of $f_M$.
The fuzzy automaton $M'$ is called a subautomaton of $M$ if
(i) $f_{M'} : Q' \times \Sigma \times Q' \to [0, 1]$ and
(ii) For any $q_i \in Q'$ and $f_{M'}(q_i, u, q_j) > 0$ for some $u \in \Sigma^*$, then $q_j \in Q'$.

$M$ is said to be strongly connected if for every $q_i,\ q_j \in Q$, there exists $u \in \Sigma^*$ such that $f_M(q_i,\ u,\ q_j) > 0$. Equivalently, $M$ is strongly connected if it has no proper subautomaton [8].

Let $q_i \in Q$. The subautomaton of $M$ generated by $q_i$ is denoted by $\langle q_i \rangle$ and is given by $\langle q_i \rangle = \{\ q_j\ /\ f_M(q_i,\ u,\ q_j) > 0,\ u \in \Sigma^*\}$. It is called a least subautomaton of $M$ containing $q_i$ and it is also called a monogenic subautomaton of $M$. For any non-empty $H \subseteq Q$, the subautomaton of $M$ generated by $H$ is denoted by

$\langle H \rangle$ and is given by $\langle H \rangle = \{\ q_j\ /\ f_M(q_i,\ w,\ q_j) > 0,\ q_i \in H,\ w \in \Sigma^*\}$. It is called a least subautomaton of $M$ containing $H$. The least subautomaton of a fuzzy automaton $M$ is called the kernel of $M$ [6].

A state $q_j \in Q$ is called a neck of $M$, for every $q_i \in Q$ if there exists $u \in \Sigma^*$ such that $f_M(q_i,\ u,\ q_j) > 0$. In that case $q_j$ is also said to be a $u$-neck of $M$ and the word $u$ is called a directing word of $M$. If $M$ has a directing word, then we say that $M$ is a directable fuzzy automaton. The set of all necks of $M$ is denoted by $N(M)$ and the set of all directing words of $M$ is denoted by $DW(M)$. If $N(M) \neq \phi$, then $N(M)$ is a subautomaton of $M$ [6].

A state $q_j \in Q$ is called local neck of $M$ if it is neck of some directable subautomaton of $M$. The set of all local necks of $M$ is denoted by $LN(M)$ [6]. A state $q_i \in Q$ is called reversible if for every word $v \in \Sigma^*$, there exists a word $u \in \Sigma^*$ such that $f_M(q_i,\ vu,\ q_i) > 0$. The set of all reversible states of $M$ are called the reversible part of $M$. It is denoted by $R(M)$. $R(M)$ is non empty, then $R(M)$ is a subautomaton of $M$. If each state of a fuzzy automaton $M$ is reversible, then the fuzzy automaton $M$ is called reversible fuzzy automaton [6]. A fuzzy automaton $M$ is said to be a direct sum of its subautomata $M_\alpha,\ \alpha \in Y$, if $M = \cup_{\alpha \in Y} Q_\alpha$ and $Q_\alpha \cap Q_\beta = \phi$, for every $\alpha,\ \beta \in Y$ such that $\alpha \neq \beta$. $M$ is called monogenically directable if every monogenic subautomaton of $M$ is directable. $M$ is said to be monogenically strongly directable if every monogenic subautomaton of $M$ is strongly directable. $M$ is called monogenically trap-directable if every monogenic subautomaton of $M$ has a single neck [6].

Let $M$ be a fuzzy automaton. We say that $u \in \Sigma^*$ to be a common directing word of $M$ if $u$ is a directinng word of every monogenic subautomaton of $M$, i.e., $u \in DW(\langle q_i \rangle)$, for every $q_i \in Q$. The set all common directing words of $M$ will be denoted by $CDW(M)$. In other words, $CDW(M) = \cap_{q_i \in Q} DW(\langle q_i \rangle)$ [6]. A fuzzy automaton $M$ is called uniformly monogenically directable fuzzy automaton if every monogenic subautomaton of $M$ is directable and have at least one common directing word. $M$ is said to be uniformly monogenically strongly directable fuzzy automaton if every monogenic subautomaton of $M$ is strongly directable and have at least one common directing word. $M$ is called uniformly monogenically trap-directable if every monogenic subautomaton of $M$ has a single neck and have at least one common directing word.

A subset $I$ of a semigroup $S$ is called an ideal if $SIS \subseteq I$ [6].

Let $M$ be a fuzzy automaton. An equivalence relation $R$ on $Q$ in $M$ is called a congruence relation if for all $q_i, q_j \in Q$ and $a \in \Sigma$, $q_i\ R\ q_j$ implies that, then there exists $q_l, q_k \in Q$ such that $f_M(q_i, a, q_l) > 0$, $f_M(q_j, a, q_k) > 0$ and $q_l\ R\ q_k$ [1, 2].

Let $M$ be a fuzzy automaton. The quotient fuzzy automaton determined by the congruence $\cong$ is a fuzzy automaton $M/\cong\ = (Q/\cong, \Sigma, f_{M/\cong})$, where $Q/\cong = \{Q_i = [q_i]\}$ and $f_{M/\cong}(Q_1, a, Q_2) = Min\{f_M(q_1, a, q_2) > 0\ /\ q_1 \in Q_1,\ q_2 \in Q_2$ and $a \in \Sigma\}$ [7].

Let $M$ be a fuzzy automaton and $M' = (Q',\ \Sigma,\ f_{M'})$ be a subautomaton of $M$. A relation $R_{M'}$ on $M$ is defined as follows. For any $q_i, q_j \in Q$, we say that $(q_i, q_j) \in R_{M'}$ if and only if either $q_i = q_j$ or $q_i, q_j \in Q'$. This relation is clearly an equivalence relation and it is also congruence. This relation is called Rees congruence relation on $Q$ in $M$ determined by $M'$. A fuzzy automaton $M/M'$ is called Rees factor fuzzy automaton determined by the relation $R_{M'}$ and it is defined as $M/M' = (\ \overline{Q},\ \Sigma,\ f_{M/M'})$, where $\overline{Q} = \{\ [q_i]\ /\ q_i \in Q\}$ and $f_{M/M'} : \overline{Q} \times \Sigma \times \overline{Q} \to [0, 1]$.

Let $M$ be a fuzzy automaton. We say that two states $q_i, q_j \in Q$ are said to be mergeable or reducible if there exists a word $u \in \Sigma^*$ and $q_j \in Q$ such that $f_M(q_i, u, q_k) > 0 \Leftrightarrow f_M(q_j, u, q_k) > 0$ [5].

## 3. GENERALIZED DIRECTABLE FUZZY AUTOMATA

A fuzzy automaton $M$ is called a generalized directable fuzzy automaton if for every $v \in \Sigma^*$ and $q_i \in Q$, there exists a word $u \in \Sigma^*$ and $q_j \in Q$ such that $f_M(q_i, uvu, q_j) > 0 \Leftrightarrow f_M(q_i, u, q_j) > 0$. In this case the word $u$ is called generalized directing word of a fuzzy automaton $M$ and the set of all generalized directing words of $M$ are denoted by $GDW(M)$.

**Example**

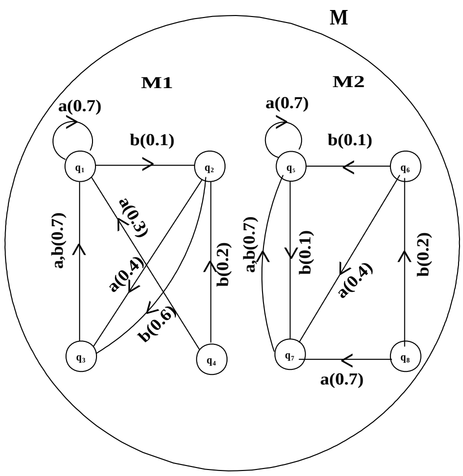


Fig - 3.1

In the above fuzzy automaton, for any $q_i \in Q$ and $v \in \Sigma^*$, $\exists\ aa \in \Sigma^*$ such that $f_M(q_i, aavaa, q_j) > 0 \Leftrightarrow f_M(q_i, aa, q_j) > 0$. In that case, the word $aa \in \Sigma^*$ is a generalized directing word of $M$.

**Note**

1) A directable fuzzy automaton always implies that generalized directable fuzzy automaton. The converse need not be true.

2) Consider the fuzzy automaton $M$ in Fig. 3.1 . The fuzzy automaton $M$ is generalized directable fuzzy automaton but not a directable fuzzy automaton.

THEOREM 3.1.. *Every finite fuzzy automaton can be uniquely represented as an extension of a direct sum of strongly connected fuzzy automata by a monogenically trap-directable fuzzy automaton.*

**Proof:** Let $M$ be a fuzzy automaton and $q_i \in Q$.

Let $M' = \{q_i\ /\ f_M(q_i, vu, q_i) > 0,\quad v, u \in \Sigma^*\}$. That is, $M'$ is set of all reversible part of $M$. $M'$ is a subautomaton of $M$. Since $M'$ is reversible part of $M$, $M'$ is a direct sum of strongly connected fuzzy automata. On the otherhand, every

Rees factor fuzzy automaton has a trap. Therefore, $M/M'$ is a monogenically trap-directable fuzzy automaton. Hence, every finite fuzzy automaton can be uniquely represented as an extension of a direct sum of strongly connected fuzzy automata by a monogenically trap-directable fuzzy automaton.

THEOREM 3.2.. *A fuzzy automaton $M$ is strongly generalized directable if and only if it is strongly connected and generalized directable.*

**Proof:** Let $M$ be a strongly generalized directable fuzzy automaton. It is clear that it is generalized directable. Now we will prove that it is strongly connected, it sufficies to show that for any $q_i, q_j \in Q$, there exists $u \in \Sigma^*$ such that $f_M(q_i, u, q_j) > 0$. Since $q_j \in N(M)$ $[N(M) = Q]$, $f_M(q_k, u, q_j) > 0$, for every $q_k \in Q$. Therefore, $f_M(q_i, u, q_j) > 0$. Thus, $M$ is strongly connected.
Conversely, let $M$ be strongly connected and generalized directable. Then $N(M) \neq \phi$, $N(M)$ is a subautomaton of $M$. But since $M$ is strongly connected, there is no proper subautomaton. Hence, $Q = N(M)$. Thus, $M$ is strongly generalized directable.

THEOREM 3.3.. *A fuzzy automaton $M$ is generalized directable if and only if it is an extension of a uniformly monogenically strongly directable fuzzy automaton $M'$ by a uniformly monogenically trap-directable fuzzy automaton $M''$.*
*In that case:*
*(i)* $DW(M'').CDW(M') \subseteq GDW(M) \subseteq DW(M'') \cap CDW(M)$;
*(ii)* $LN(M) = M'$.

**Proof:**
Let $M$ be a generalized directable fuzzy automaton.
Let $M' = \{q_j \ / \ f_M(q_i, u, q_j) > 0, \ \ q_i \in Q, \ \ u \in GDW(M)\}$ be a subautomaton of $M$. Now we have to show that $M'$ is a uniformly monogenically strongly directable fuzzy automaton.
Let $q_j \in M'$. That is, $f_M(q_i, u, q_j) > 0$, for some $q_i \in Q$ and $u \in GDW(M)$.
Then for every $v \in \Sigma^*$ we have that
$f_M(q_i, u, q_j) > 0 \Leftrightarrow f_M(q_i, uvu, q_j) > 0$
$\Leftrightarrow Min_{q_j \in Q} \{f_M(q_i, u, q_j), f_M(q_j, vu, q_j)\} > 0$
$\Rightarrow f_M(q_j, vu, q_j) > 0.$
By Lemma 4.1 [6], $q_j$ is a local neck and $\langle q_j \rangle$ is a strongly directable fuzzy automaton. Further $u \in CDW(\langle q_j \rangle)$.
Therefore, $M'$ is a uniformly monogenically strongly directable fuzzy automaton and $GDW(M) \subseteq CDW(M')$. ——(1)
Define Rees congruence on $M$. Then there exists a factor fuzzy automaton
$M'' = M/M'$ which is a uniformly monogenically trap-directable fuzzy automaton and $GDW(M) \subseteq DW(M'')$. ——(2)
From (1) and (2), $GDW(M) \subseteq CDW(M') \cap DW(M'')$. —-(3)
Conversely, let $M$ be an extension of a uniformly monogenically strongly directable fuzzy automaton $M'$ by a uniformly monogenically trap-directable fuzzy automaton $M''$. Consider an arbitrary $q_i \in Q$, $u_1 \in DW(M'')$,
$u_2 \in CDW(M')$ and $v \in \Sigma^*$.
Now, let $u = u_1 u_2 \in \Sigma^*$. Then $f_M(q_i, u_1, q_k) > 0$, $f_M(q_i, u_1 u_2 v u_1, q_k) > 0$ where $q_k \in \langle q_i \rangle$ for some strongly directable subautomaton $\langle q_i \rangle$ of $M'$.
Now,
$f_M(q_i, uvu, q_j) = f_M(q_i, u_1 u_2 v u_1 u_2, q_j) > 0$
$\Leftrightarrow Min_{q_k \in Q} \{f_M(q_i, u_1 u_2 v u_1, q_k), f_M(q_k, u_2, q_j)\} > 0$
$\Rightarrow f_M(q_k, u_2, q_j) > 0$
Now,
$f_M(q_i, u_1 u_2, q_j) =$
$Min_{q_k \in Q}\{f_M(q_i, u_1, q_k), f_M(q_k, u_2, q_j)\} > 0.$
$f_M(q_i, uvu, q_j) > 0 \Leftrightarrow f_M(q_i, u, q_j) > 0.$
Therefore, $M$ is a generalized directable fuzzy automaton and $u \in GDW(M)$.
Hence, $DW(M'').CDW(M') \subseteq GDW(M)$. ——(4)
From (3) and (4), $DW(M'').CDW(M') \subseteq GDW(M) \subseteq DW(M'') \cap CDW(M')$.
Now let us prove that $LN(M) = M'$.
By Lemma 4.3 [6], $M' \subseteq LN(M)$.
Conversely, let $q_i \in LN(M)$.
By Lemma 4.1 [6], for every $v \in \Sigma^*$, there exists $u \in \Sigma^*$ such that $f_M(q_i, vu, q_i) > 0$.
If we assume that $v \in DW(M'')$, then $f_M(q_i, v, q_k) > 0$, for some $q_k \in M'$.
Now,
$f_M(q_i, vu, q_i) > 0 \Leftrightarrow Min_{q_k \in Q}\{f_M(q_i, v, q_k),$
$f_M(q_k, u, q_i)\} > 0$
$\Rightarrow f_M(q_k, u, q_i) > 0$
$\Rightarrow q_i \in M'$[Since $M'$ is strongly connected].
Therefore, $LN(M) \subseteq M'$. Hence, $LN(M) = M'$.

THEOREM 3.4.. *Let $M$ be a fuzzy automaton. Then the following conditions are equivalent:*
*(i) $M$ is a generalized directable fuzzy automaton;*
*(ii) every strongly connected subautomaton of $M$ is directable;*
*(iii) every subautomaton of $M$ contains a direcable subautomaton;*
*(iv)* $(\forall q_i \in Q)(\exists u \in \Sigma^*)(\forall v \in \Sigma^*)(\exists w \in \Sigma^*)$ *such that*
$f_M(q_i, uvw, q_l) > 0 \Leftrightarrow f_M(q_i, uw, q_l) > 0,$ *for some* $q_l \in Q$.

**Proof:**
$(i) \Rightarrow (ii)$
Let $M$ be a generalized directable fuzzy automaton.
Let $M' = \{q_j \ / \ f_M(q_i, u, q_j) > 0, \ \ q_i \in Q, \ \ u \in GDW(M)\}$ be a subautomaton of $M$. Now we have to show that $M'$ is a strongly directable fuzzy automaton.
Let $q_j \in M'$. That is, $f_M(q_i, u, q_j) > 0$, for some $q_i \in Q$ and $u \in GDW(M)$.
Then for every $v \in \Sigma^*$ we have that
$f_M(q_i, u, q_j) > 0 \Leftrightarrow f_M(q_i, uvu, q_j) > 0$
$\Leftrightarrow Min_{q_j \in Q} \{f_M(q_i, u, q_j), f_M(q_j, vu, q_j)\} > 0$
$\Rightarrow f_M(q_j, vu, q_j) > 0.$
By Lemma 4.1 [6], $q_j$ is a local neck and $\langle q_j \rangle$ is a strongly directable fuzzy automaton.
$(ii) \Rightarrow (i)$
By Theorem 3.1, $M$ is an extension of a fuzzy automaton $M'$ by a monogenically trap-directable fuzzy automaton $M''$, where $M'$ is a direct sum strongly connected of fuzzy automata $M'_i$, $i \in [1, n]$.
By the hypothesis it follows that $M'_i$ is a directable fuzzy automaton, for every $i \in [1, n]$. Since $DW(M'_i)$ is an ideal of $\Sigma^*$, for each $i \in [1, n]$ and the intersection of any finite family of ideals is non-empty, then there exists $u \in \cap_{i=1}^{n} DW(M'_i)$.
According to Theorem 1.3 [14], the fuzzy automaton $M'$ is uniformly monogenically strongly directable and hence, by

Theorem 3.3, $M$ is a generalized directable fuzzy automaton.
$(ii) \Rightarrow (iii)$
Let $M''$ be any strongly connected directable subautomaton of $M$. Since $M''$ is strongly connected and directable, $N(M'') = M''$ which is a least subautomaton of $M$.
If $M'$ is any other subautomaton of $M$, then $M'' \subseteq M'$. Hence, $M'$ is the subautomaton of $M$ that contains a directable subautomaton $M''$.
$(iii) \Rightarrow (i)$
Consider an arbitrary $q_i \in Q$. By the hypothesis, the monogenic subautomaton $\langle q_i \rangle$ contains a directable subautomaton $M'$.
Therefore, there exists a $u_1 \in \Sigma^*$ such that $f_M(q_i, u_1, q_k) > 0$, for some $q_k \in M'$.
Let $u = u_1u_2$, where $u_2 \in DW(M')$ and let $v \in \Sigma^*$.
Now,$f_M(q_i, u, q_j) = f_M(q_i, u_1u_2, q_j)$
$= Min_{q_k \in M'} \{f_M(q_i, u_1, q_k), f_M(q_k, u_2, q_j)\}$ .
Since $f_M(q_i, u_1, q_k) > 0$ and $M$ is a deterministic fuzzy automaton, we have $f_M(q_k, u_2, q_j) > 0$.
Therefore, $f_M(q_i, u, q_j) > 0$. ——-(1)
Now,
$f_M(q_i, uvu, q_j) = Min_{q_j \in Q} \{f_M(q_i, u, q_j), f_M(q_j, vu, q_j)\}$ .
Since from (1), $f_M(q_i, u, q_j) > 0$ and $M$ is a deterministic fuzzy automaton,
we have $f_M(q_j, vu, q_j) > 0$.
Therefore, $f_M(q_i, uvu, q_j) > 0$. ——-(2)
From (1) and(2), we have
$(\forall q_i \in Q)(\exists u \in \Sigma^*)(\forall v \in \Sigma^*)$ such that $f_M(q_i, uvu, q_j) > 0 \Leftrightarrow f_M(q_i, u, q_j) > 0$.
$(i) \Rightarrow (iv)$
By the hypothesis, $(\forall\, q_i \in Q)(\exists\, u \in \Sigma^*)(\forall\, v \in \Sigma^*)$ such that $f_M(q_i, uvu, q_j) > 0 \Leftrightarrow f_M(q_i, u, q_j) > 0$.
Let $w = uu_1$ for some $u_1 \in \Sigma^*$.
Now,$f_M(q_i, uvw, q_l) = f_M(q_i, uvuu_1, q_l)$
$= Min_{q_j \in Q} \{f_M(q_i, uvu, q_j), f_M(q_j, u_1, q_l)\}$
By the hypothesis, $f_M(q_i, uvu, q_j) > 0$ and since $M$ is a deterministic fuzzy automaton $f_M(q_j, u_1, q_l) > 0$. Therefore, $f_M(q_i, uvw, q_l) > 0$. ——(3)
$f_M(q_i, uw, q_l) = f_M(q_i, uuu_1, q_l)$
$\Rightarrow Min_{q_j \in Q} \{f_M(q_i, uu, q_j), f_M(q_j, u_1, q_l)\}$.
Since $f_M(q_i, u, q_j) > 0$, we have $f_M(q_i, uu, q_j) > 0$ and therefore, $f_M(q_j, u_1, q_l) > 0$.
Hence, $f_M(q_i, uw, q_l) > 0$.——(4)
From (3)and (4), $f_M(q_i, uvw, q_l) > 0 \Leftrightarrow f_M(q_i, uw, q_l) > 0$.
Therefore, $(\forall\, q_i \in Q)(\exists\, u \in \Sigma^*)(\forall\, v \in \Sigma^*)(\exists\, w \in \Sigma^*)$ such that
$f_M(q_i, uvw, q_l) > 0 \Leftrightarrow f_M(q_i, uw, q_l) > 0$.
$(iv) \Rightarrow (ii)$
By the hypothesis, $(\forall\, q_i \in Q)(\exists\, u \in \Sigma^*)(\forall\, v \in \Sigma^*)(\exists\, w \in \Sigma^*)$ such that $f_M(q_i, uvw, q_l) > 0 \Leftrightarrow f_M(q_i, uw, q_l) > 0$ for some $q_l \in Q$.
Take an arbitrary strongly connected subautomaton $M'$ of $M$ and $q_i, q_k \in M'$.
Now, $f_M(q_i, u, q_j) > 0$ and $f_M(q_k, u, q_l) > 0$, for some $u \in \Sigma^*$ and $q_j, q_l \in M'$. Since $M'$ is strongly connected, there exists $u_1 \in \Sigma^*$ such that
$f_M(q_i, uu_1, q_l) > 0$.
For that $u_1$, there exists $u_2 \in \Sigma^*$ such that
$f_M(q_i, uu_1u_2, q_m) > 0 \Leftrightarrow f_M(q_i, uu_2, q_m) > 0$, for some $q_m \in M'$.
$f_M(q_i, uu_2, q_m) > 0 \Leftrightarrow f_M(q_i, uu_1u_2, q_m) > 0$.
$\Leftrightarrow Min_{q_l \in M'} \{f_M(q_i, uu_1, q_l), f_M(q_l, u_2, q_m)\} > 0$
$\Rightarrow f_M(q_l, u_2, q_m) > 0$.
Now,
$f_M(q_k, uu_2, q_m) = Min_{q_l \in M'}\{f_M(q_k, u, q_l),$
$f_M(q_l, u_2, q_m)\} > 0$.
Therefore, we have proved that $q_i$ and $q_k$ are mergeable. Hence ,$M'$ is a directable fuzzy automaton.

## 4. CONCLUSION

In this paper, we introduce generalized directable fuzzy automaton and discuss their structural characterizations. We have shown that every finite fuzzy automaton can be uniquely represented as an extension of a direct sum of strongly connected fuzzy automata by a monogenically trap-directable fuzzy automaton and obtain a necessary condition for a fuzzy automaton to be strongly generalized directable. Also, We have shown that a generalized directable fuzzy automaton is an extension of a uniformly monogenically strongly directable fuzzy automaton by a uniformly monogenically trap-directable fuzzy automaton and obtain other equivalent conditions for a generalized directable fuzzy automaton.

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