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Stochastic Analysis of a Repairable Cold Standby System Attacked by Poisson Shocks Considering Inspection and Post Repair

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ABSTRACT
This paper investigated the stochastic analysis of two-dissimilar unit cold standby system considering repair, inspection, post repair under Poisson shocks. The serverman, is called when the operative unit fails. The shocks can attack the operative unit. The repaired unit is sent for inspection to decide whether the repair is satisfactory. If the repair is found unsatisfactory, then the unit is again sent for post repair. Some reliability measures of the system such as system reliability, mean time to system failure (MTSF) and steady state availability are derived. Graphical representations are presented to illustrate the theoretical results.

General Terms
Applied stochastic processes

Keywords
Poisson shock, cold standby system, mean to system failure, steady state availability.

1. INTRODUCTION
Reliability theory is one of the most important branches of operations research and systems engineering. Any systems analysis in order to be complete, must give due consideration to system reliability. With remarkable advances made in electronics engineering, military and communication systems have become more sophisticated and when such systems fail, very serious situations arise. Thus in the present day context, high system reliability has become very important from the view point of both makers and the users.

The shock model has been extensively in the past. For example Qingtai Wu [11] studied the reliability analysis of a cold standby system attacked by shocks. A repairable system with an unreliable repair facility and one repairman who can take single vacation considered by Renbin et al.[5]. Abdul Ameer Al-Ali and Murari, K. [1] Developed a reliability model of a single unit system with the impact of random shocks. Haitao Liu, Xianyun Meng and Wenjuan Wu [6] Considered the cold standby sytem with repair of non-new and repairman vacation. [7] Considered a cold standby repairable deteriorating system consisting of three dissimilar components and one repairman. The geometric process, and the supplementary variable techniques, a group of partial differential equations of the system were presented, and other reliability indices are obtained.[2] analyzed some reliability indices of a cold standby system with an unreliable repair facility and one repairman who can take vacation under Poisson Shocks. By using the geometric process theory, the supplementary variable method and Laplace transform tool, availability and reliability of the system and other reliability indices are obtained. [9] considered a simple repairable system with a warning device and a repairman who can have delayed-multiple vacations. The asymptotic stability, especially the exponential stability of the system dynamic solution, is studied by using the strongly continuous semi group theory or C0 semi group theory. [3] discussed a single-unit system subject to random shocks. The impact of shocks may or may not be affected on this system. The single server who visits this system immediately to conduct maintenance and repair of the unit. [4] discussed the effect of human failures on the reliability of the system and determine the cost function, and some reliability indices were derived. [10] analyzed some reliability indices of a cold standby system consisting of two repairable units, a switch and a repairman who may not always be at the job site or take vacation. [12] studied a k-out-of-n : G system and a consecutive-k-out-ofn : F system, respectively, with R repairmen who can take multiple vacations and by using Markov model; the analytically solution of some reliability indices was discussed. [14] studied a deteriorating system with a repairman who can have multiple vacations. By means of the geometric process and the supplementary variable techniques, a group of partial differential equations of the system was presented, and some reliability indices were derived. [13] considered a deteriorating repairable system and a cold standby repairable system with two different components of different priority in use, both with one repairman who can take multiple vacations. The explicit expression of the expected cost rate was given, and an optimal replacement policy was discussed. [8] deal with the study of the stochastic analysis of a two-unit cold standby system consid ering hardware failure, human error failure and preventive maintenance.

The present paper, we consider a two-unit cold standby (non-identical) with inspection time under Poisson shocks. The arrival time of the shocks follow a homogeneous Poisson process and other distribution are arbitrary distribution. Finally, the effects of parameters on the system performance have been studied.

2. MODEL DESCRIPTION AND ASSUMPTIONS

1. The system consists of two non-identical units. Initially one unit is operating and the other is in standby case (cold standby).

2. The switch is perfect and instantaneously.

3. The system is subject to shocks. The arrivals of the shocks follow a Poisson process $\left[N(t), t \geq 0\right]$ with the intensity $\lambda > 0$. The magnitude of each shock $\hat{X}$, is a random variable with distribution function *F*.

4. When a shock arrives, it only affects the operating unit. The operating unit will fail when the magnitude of a shock exceeds a threshold. The threshold of unit *i* is a non-negative random variable τ_i with a distribution function Φ_i (i = 1, 2).

5. The repairman already is unavailable when two units are good. He is demanded when the failure occurs. If a unit fails when the other is being repaired, the newly failed unit must wait for repair and the system is down. If two units are waiting for repair when the repairman comes to the system, unit 1 has the priority to be repaired.

6. After the repair, a unit goes for inspection to decide whether the repair is satisfactory or not. If the repaired unit is found to be unsatisfactory then it is sent for post repair. The probability of having satisfactory repair is fixed.

7. Service discipline is a first-come, first-served (FCFS). A single perfect repair facility is available for repair, inspection and post repair.

8. Once the repairman completes his work, he leaves the system.

9. Shocks are assumed to be only cause of unit failure, and the system fails when both the units fail.

10. The repair, inspection, post repair and the demanding (waiting) time are assumed to be arbitrary.

11. All random variables are independent. At the beginning, the two units are new, one unit starts to work, the other unit is on cold standby and the repairman goes out the system. The units can be repaired as good as new.

3. NOTATIONS

H_i (i = 1, 2)	As unit's repair time. Their distributions are $\{\bar{H}_i(t) = e^{-\int_0^t \mu_i(y_i)dy_i}\}$; where $\mu_i(y_i)$ is hazard rate function.
V	As demanding (waiting) time. Its distribution is $\{\bar{V}(t) = e^{-\int_0^t \alpha(x)dx}\}$; where $\alpha(x)$ is hazard rate function.
A	As the inspection time. Its distribution is $\{\bar{A}(t) = e^{-\int_0^t \gamma(z)dz}\}$; where $\gamma(z)$ is hazard rate function.
U_i (i = 1, 2)	As post repair time. Their distributions are $\{\bar{U}_i(t) = e^{-\int_0^t k_i(t_i)dt_i}\}$; where $k_i(t_i)$ is hazard rate function.
*	Laplace transforms; $\{f^*(s) = L_s[f(x)] = \int_0^\infty f(x)e^{-xs}\,dx\,; s > 0\}$.
q	The probability that the repaired unit is unsatisfactory.
AV	The steady-state availability of system.
R(t)	The reliability of the system.
MTTF	Mean time to the system failure.
r_i	The probability that one shock causes unit *i* to fail, $(i = 1,2)$

4. SYSTEM ANALYSIS

With the model assumptions given in the preceding section, the failure probability of unit i $(i = 1,2)$, given the shock value $\hat{x}$, is $\Phi_i\,(\hat{x}) = P(\tau < \hat{x})$.Since the magnitude of a shock is a random variable $\hat{X}$, the conditional failure probability of unit i is a random $\Phi_i\,(\hat{x})$ with $(i = 1,2)$, respectively, and its probability distribution can be written by: $P_i(x) = P(\Phi_i\,(\hat{X}) \leq x) = P\left(\hat{X} \leq \Phi_i^{-1}\,(x)\right) = F\left(\Phi_i^{-1}\,(x)\right), 0 < x < 1, (i = 1,2)$. From assumption 2 and 3, we can see that, the probability that one shock causes unit i to fail is:

$$r_i = P(\hat{X} > \tau_i)$$
$$= \int_0^\infty P(\tau_i < \hat{x} | \hat{X} = \hat{x})dP(\hat{X} \leq \hat{x}) = \int_0^\infty \Phi_i\,(\hat{x})dF(\hat{x}). \quad (i = 1,2).$$

Let $S(t)$be the system state at time t, then

State 0: unit 1 is working and unit 2 is on cold standby.
State 1: unit 2 is working, unit 1 is waiting for repair and the repairman is calling.
State 2: unit 2 is working, unit 1 is being repaired.
State 3: the two units are waiting for repair because of the repairman does come.
State 4: unit 2 is working, unit 1 is being inspected.
State 5: unit 1 is under repair and unit 2 is waiting for repair.
State 6: unit 2 is working while unit 1 is being cold standby.
State 7: unit 2 is working and unit 1 is being post repaired.
State 8: unit 1 is being inspected and unit 2 is waiting for repair.
State9: unit 1 is being post repaired and unit 2 is waiting for repair.
State 10: unit 1 is working and unit 2 is being repaired.
State 11: unit 1 is working, unit 2 is waiting for repair and the repairman is calling.
State 12: unit 2 is under repair; unit 1 is waiting for repair.
State 13: unit 1 is working and unit 2 is being inspected.
State 14: unit 1 is working and unit 2 is being post repaired.
State 15: unit 2 is being inspected and unit 1 is waiting for repair.
State16: unit 2 is being post repaired and unit 1 is waiting for repair.

The state space is
$\Omega = \{0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16\}$,
where the up state set is W = {0,1,2,4,6,7,10,11,13,14} and the down state set D = {3,5,8,9,12,15,16}.

One can note that $\{S(t), t \geq 0\}$ is not a Markov process, where $S(t)$ represents all states of the system. So we introduce supplementary variable:

i). X(t) : if $S(t) = 1,3,11$.Then X(t) is the elapsed summoned time when the repairman is not exist at time t.

ii). Y_1 (t) : if $S(t) = 2,5$. Then Y_1 (t) is the elapsed repair time of unit 1 being repaired at time t.

iii). Y_2 (t) : if $S(t) = 10,12$. Then Y_2 (t) is the elapsed repair time of unit 2 being repaired at time t

iv). Z(t) if $S(t) = 4,8,13,15$.Then Z(t) is the elapsed inspection time at time t.

v). U_1 (t) : if $S(t) = 7,9$. Then U_1 (t) is the elapsed post repair time of unit 1 being post repaired at time t.

vi). U_2 (t): if $S(t) = 14,16$. Then U_2 (t) is the elapsed post repair time of unit 2 being post repaired at time t

Then $\{ (S(t), X(t), Y_1(t), Y_2(t), Z(t), U_1(t), U_2(t)), t \geq 0\}$ is a generalized Markov process. let:

$Q_i(t,x) = P(S(t) = i, X(t) \leq x), \quad (i = 1,3,11).$

$Q_i(t,y_1) = P(S(t) = i, Y_1(t) \leq y_1), \quad (i = 2,5).$

$Q_i(t,y_2) = P(S(t) = i, Y_2(t) \leq y_2), \quad (i = 10,12)$

$Q_i(t,z) = P(S(t) = i, Z(t) \leq z), \quad (i = 4,8,13,15).$

$Q_i(t,u_1) = P(S(t) = i, U_1(t) \leq u_1), \quad (i = 7,9).$

and,

$Q_i(t,u_2) = P(S(t) = i, U_2(t) \leq u_2). \quad (i = 14,16).$

where $P(B)$is probability of event B, consider:

$P_i(t,w) = \frac{d}{dw} Q_i(t,w)$

$; (i = \{1,2,3,4,5,7,8,9,10,11,12,13,14,15,16\}).$

We can express the process in a way considering the transitions in t and $t + \Delta t$.

It is easily to show that,

$$P_0(t+\Delta t) = P_0(t)(1 - r_1\lambda\Delta t) + \int_0^\infty k_2(u_2)P_{14}(t,u_2)du_2\Delta t + \int_0^\infty (1-q)\gamma(z)P_{13}(t,z)dz\Delta t + o(\Delta t), \quad (1)$$

then,

$$P_0(t+\Delta t) - P_0(t) = -r_1\lambda\Delta t P_0(t) + \int_0^\infty k_2(u_2)P_{14}(t,u_2)du_2\Delta t + \int_0^\infty (1-q)\gamma(z)P_{13}(t,z)dz\Delta t + o(\Delta t). \quad (2)$$

Dividing both sides of (2) .we get,

$$\lim_{\Delta t \to 0} \frac{P_0(t+\Delta t) - P_0(t)}{\Delta t} = = -r_1\lambda P_0(t) + \int_0^\infty k_2(u_2)P_{14}(t,u_2)du_2 + \int_0^\infty (1-q)\gamma(z)P_{13}(t,z)dz \quad ,$$

and this yields

$$\left(\frac{d}{dt} + r_1\lambda\right)P_0(t) = = \int_0^\infty k_2(u_2)P_{14}(t,u_2)du_2 + \int_0^\infty (1-q)\gamma(z)P_{13}(t,z)dz \quad (3)$$

By the same arguments, the following partial-differential equations can be obtained

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \alpha(x) + r_2\lambda\right)P_1(t,x) = 0 \ , \quad (4)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial y_1} + \mu_1(y_1) + r_2\lambda\right)P_2(t,y_1) = 0 \ , \quad (5)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \alpha(x)\right)P_3(t,x) = r_2\lambda P_1(t,x) + r_1\lambda P_{11}(t,x) \ , \quad (6)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial z} + \gamma(z) + r_2\lambda\right)P_4(t,z) = 0 \ , \quad (7)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial y_1} + \mu_1(y_1)\right)P_5(t,y_1) = r_2\lambda P_2(t,y_1) \ , \quad (8)$$

$$\left(\frac{d}{dt} + r_2\lambda\right)P_6(t) = \int_0^\infty k_1(u_1)P_7(t,u_1)du_1 + \int_0^\infty (1-q)\gamma(z)P_4(t,z)dz \ , \quad (9)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial u_1} + k_1(u_1) + r_2\lambda\right)P_7(t,u_1) = 0 \ , \quad (10)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial z} + \gamma(z)\right)P_8(t,z) = r_2\lambda P_4(t,z) \ , \quad (11)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial u_1} + k_1(u_1)\right)P_9(t,u_1) = r_2\lambda P_7(t,u_1) \ , \quad (12)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial y_2} + \mu_2(y_2) + r_1\lambda\right)P_{10}(t,y_2) = 0 \ , \quad (13)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \alpha(x) + r_1\lambda\right)P_{11}(t,x) = 0 \ , \quad (14)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial y_2} + \mu_2(y_2) + r_1\lambda\right)P_{12}(t,y_2) = r_1\lambda P_{10}(t,y_2) \ , \quad (15)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial z} + \gamma(z) + r_1\lambda\right)P_{13}(t,z) = 0 \ , \quad (16)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial u_2} + k_2(u_2) + r_1\lambda\right)P_{14}(t,u_2) = 0 \ , \quad (17)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial z} + \gamma(z)\right)P_{15}(t,z) = r_1\lambda P_{13}(t,z) \ , \quad (18)$$

and

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial u_2} + k_2(u_2)\right)P_{16}(t,u_2) = r_1\lambda P_{14}(t,u_2) \ . \quad (19)$$

Their boundary conditions are:-

$$P_1(t,0) = r_1\lambda P_0(t) + \delta(t) \ , \qquad \text{where, } \delta(t) = \begin{cases} 1, & t = 0 \\ 0, & t \neq 0 \end{cases} \quad (20)$$

$$P_2(t,0) = \int_0^\infty \alpha(x)P_1(t,x)dx + \int_0^\infty k_2(u_2)P_{16}(t,u_2)du_2 + \int_0^\infty (1-q)\gamma(z)P_{15}(t,z)dz \ , \quad (21)$$

$$P_3(t,0) = P_{12}(t,0) = 0, \quad (22)$$

$$P_4(t,0) = \int_0^\infty \mu_1(y_1)P_2(t,y_1)dy_1 \ , \quad (23)$$

$$P_5(t,0) = \int_0^\infty \alpha(x)P_3(t,x)dx \ , \quad (24)$$

$$P_7(t,0) = \int_0^\infty q\,\gamma(z)P_4(t,z)dz \ , \quad (25)$$

$$P_8(t,0) = \int_0^\infty \mu_1(y_1)P_5(t,y_1)dy_1 \ , \quad (26)$$

$$P_9(t,0) = \int_0^\infty q\,\gamma(z)P_8(t,z)dz \ , \quad (27)$$

$$P_{10}(t,0) = \int_0^\infty \alpha(x)P_{11}(t,x)dx + \int_0^\infty k_1(u_1)P_9(t,u_1)du_1 + \int_0^\infty (1-q)\gamma(z)P_8(t,z)dz \ , \quad (28)$$

$$P_{11}(t,0) = r_2\lambda P_6(t) \ , \quad (29)$$

$$P_{13}(t,0) = \int_0^\infty \mu_2(y_2)P_{10}(t,y_2)dy_2 \ , \quad (30)$$

$$P_{14}(t,0) = \int_0^\infty q\,\gamma(z)P_{13}(t,z)dz \ , \quad (31)$$

$$P_{15}(t,0) = \int_0^\infty \mu_2(y_2)P_{12}(t,y_2)dy_2 \ , \quad (32)$$

and,

$P_{16}(t,0) = \int_0^\infty q\,\gamma(z) P_{15}(t,z)dz$. (33)

The initial conditions are

$P_1(0,x) = \delta(x) = \begin{cases} 1, & x = 0 \\ 0, & x \neq 0 \end{cases}$, (34)

Otherwise is 0.

It is noticed that:

$$P_0(t) + \sum_{i=1}^{5} \int_0^\infty P_i(t,x)dx + P_6(t) + \sum_{j=7}^{16} \int_0^\infty P_j(t,x)dx = 1. \quad (35)$$

5. THE STEADY-STATE AVAILABILITY NORMAL

Define,

$P_i = \lim_{t\to\infty} P_i(t)$, i = {0,1,2, … ,16},

and,

$g_i(u) = \lim_{t\to\infty} P_i(t,u)$; i = {1,2,3,4,5,7,8,9,10,11,12,13,14,15,16}.

This follows the following relations:

$P_i = \int_0^\infty g_i(u)du$; (i = {1,2,3,4,5,7,8,9,10,11,12,13,14,15,16})

After taking the, $\lim_{t\to\infty}$ for both sides of Eq.(3-33), we can obtain the following equations:

$r_1\lambda P_0 = \int_0^\infty k_2(u_2) g_{14}(u_2)du_2 + \int_0^\infty (1-q)\gamma(z) g_{13}(z)dz$, (36)

$\left(\frac{d}{dx} + \alpha(x) + r_2\lambda\right) g_1(x) = 0$, (37)

$\left(\frac{d}{dy_1} + \mu_1(y_1) + r_2\lambda\right) g_2(y_1) = 0$, (38)

$\left(\frac{d}{dx} + \alpha(x)\right) g_3(x) = r_2\lambda g_1(x) + r_1\lambda g_{11}(x)$, (39)

$\left(\frac{d}{dz} + \gamma(z) + r_2\lambda\right) g_4(z) = 0$, (40)

$\left(\frac{d}{dy_1} + \mu_1(y_1)\right) g_5(y_1) = r_2\lambda g_2(y_1)$, (41)

$r_2\lambda P_6 = \int_0^\infty k_1(u_1) g_7(u_1)du_1 + \int_0^\infty (1-q)\gamma(z) g_4(z)dz$, (42)

$\left(\frac{d}{du_1} + k_1(u_1) + r_2\lambda\right) g_7(u_1) = 0$, (43)

$\left(\frac{d}{dz} + \gamma(z)\right) g_8(z) = r_2\lambda g_4(z)$, (44)

$\left(\frac{d}{du_1} + k_1(u_1)\right) g_9(u_1) = r_2\lambda g_7(u_1)$, (45)

$\left(\frac{d}{dy_2} + \mu_2(y_2) + r_1\lambda\right) g_{10}(y_2) = 0$, (46)

$\left(\frac{d}{dx} + \alpha(x) + r_1\lambda\right) g_{11}(x) = 0$, (47)

$\left(\frac{d}{dy_2} + \mu_2(y_2) + r_1\lambda\right) g_{12}(y_2) = r_1\lambda g_{10}(y_2)$, (48)

$\left(\frac{d}{dz} + \gamma(z) + r_1\lambda\right) g_{13}(z) = 0$, (49)

$\left(\frac{d}{du_2} + k_2(u_2) + r_1\lambda\right) g_{14}(u_2) = 0$, (50)

$\left(\frac{d}{dz} + \gamma(z)\right) g_{15}(z) = r_1\lambda g_{13}(z)$, (51)

$\left(\frac{d}{du_2} + k_2(u_2)\right) g_{16}(u_2) = r_1\lambda g_{14}(u_2)$, (52)

In this case the boundary condition can be put as follows:

$g_1(0) = r_1\lambda P_0$, (53)

$g_2(0) = \int_0^\infty \alpha(x) g_1(x)dx + \int_0^\infty k_2(u_2) g_{16}(u_2)du_2 + \int_0^\infty (1-q)\gamma(z) g_{15}(z)dz$, (54)

$g_3(0) = g_{12}(0) = 0$, (55)

$g_4(0) = \int_0^\infty \mu_1(y_1) g_2(y_1)dy_1$, (56)

$g_5(0) = \int_0^\infty \alpha(x) g_3(x)dx$, (57)

$g_7(0) = \int_0^\infty q\,\gamma(z) g_4(z)dz$, (58)

$g_8(0) = \int_0^\infty \mu_1(y_1) g_5(y_1)dy_1$, (59)

$g_9(0) = \int_0^\infty q\,\gamma(z) g_8(z)dz$, (60)

$g_{10}(0) = \int_0^\infty \alpha(x) g_{11}(x)dx + \int_0^\infty k_1(u_1) g_9(,u_1)du_1 + \int_0^\infty (1-q)\gamma(z) g_8(,z)dz$, (61)

$g_{11}(0) = r_2\lambda P_6$, (62)

$g_{13}(0) = \int_0^\infty \mu_2(y_2) g_{10}(y_2)dy_2$, (63)

$g_{14}(0) = \int_0^\infty q\,\gamma(z) g_{13}(z)dz$, (64)

$g_{15}(0) = \int_0^\infty \mu_2(y_2) g_{12}(y_2)dy_2$, (65)

and,

$g_{16}(0) = \int_0^\infty q\,\gamma(z) g_{15}(z)dz$. (66)

Solving Eq. (36-66) considering $P_i = \int_0^\infty g_i(u)du$; (i = {1,2,3,4,5,7,8,9,10,11,12,13,14,15,16})

We get the following steady-state probabilities:

$P_0 = \frac{c_0}{\lambda r_1}$, (67)

$P_1 = c_0\,\overline{V^*}(r_2\lambda)$, (68)

$P_2 = c_0\,(-1 + n + v^*[\lambda r_2])(\bar{H}_1)^*[\lambda r_2]$, (69)

$P_3 = c_0\,(\frac{1}{\alpha} - \bar{V}^*[\lambda r_2] + a^*[\lambda r_2](-1 + n + v^*[\lambda r_2])(\frac{1}{\alpha} - \bar{V}^*[\lambda r_1])(h_1)^*[\lambda r_2](1 - q + q(u_1)^*[\lambda r_2]))$, (70)

$P_4 = c_0\,(-1 + n + v^*[\lambda r_2])\bar{A}^*[\lambda r_2](h_1)^*[\lambda r_2]$, (71)

$P_5 = c_0\,\{\frac{1}{\mu_1}(n - a^*[\lambda r_2](-1 + v^*[\lambda r_1])(-1 + n + v^*[\lambda r_2])(h_1)^*[\lambda r_2](1 - q + q(u_1)^*[\lambda r_2])) - (-1 + n + v^*[\lambda r_2])(\bar{H}_1)^*[\lambda r_2]\}$, (72)

$P_6 = \frac{c_0}{\lambda r_2}(a^*[\lambda r_2](-1+n+v^*[\lambda r_2])(h_1)^*[\lambda r_2](1-q+q(u_1)^*[\lambda r_2]))$, (73)

$P_7 = c_0\, q a^*[\lambda r_2](h_1)^*[\lambda r_2](\bar{U}_1)^*[\lambda r_2](-1+n+v^*[\lambda r_2])$, (74)

$P_8 = \frac{1}{\gamma}c_0\,(n-(-1+n+v^*[\lambda r_2])(h_1)^*[\lambda r_2](\gamma\bar{A}^*[\lambda r_2]+a^*[\lambda r_2](-1+v^*[\lambda r_1])(1-q+q(u_1)^*[\lambda r_2])))$, (75)

$P_9 = \frac{qc_0}{k_2}(n-a^*[\lambda r_2](-1+n+v^*[\lambda r_2])(h_1)^*[\lambda r_2]((-1+v^*[\lambda r_1])(1-q+q(u_1)^*[\lambda r_2])+k_2(\bar{U}_1)^*[\lambda r_2]))$, (76)

$P_{10} = nc_0\,(\bar{H}_2)^*[\lambda r_1]$, (77)

$P_{11} = c_0\, a^*[\lambda r_2](-1+n+v^*[\lambda r_2])\bar{V}^*[\lambda r_1](h_1)^*[\lambda r_2](1-q+q(u_1)^*[\lambda r_2])$, (78)

$P_{12} = nc_0\,(\frac{1}{\mu_2}-(\bar{H}_2)^*[\lambda r_1])$, (79)

$P_{13} = nc_0\,\bar{A}^*[\lambda r_1](h_2)^*[\lambda r_1]$, (80)

$P_{14} = nqc_0\, a^*[\lambda r_1](h_2)^*[\lambda r_1](\bar{U}_2)^*[\lambda r_1]$, (81)

$P_{15} = nc_0\,(\frac{1}{\gamma}-\bar{A}^*[\lambda r_1](h_2)^*[\lambda r_1])$, (82)

and, $P_{16} = nqc_0\,(\frac{1}{k_2}-a^*[\lambda r_1](h_2)^*[\lambda r_1](\bar{U}_2)^*[\lambda r_1])$. (83)

where,

$$c_0 = \frac{\alpha\gamma\lambda k_2 r_1 r_2\mu_1\mu_2}{d},$$

$d = q\alpha\gamma\lambda r_1 r_2\mu_1\mu_2\{2n-a^*[\lambda r_2](-1+v^*[\lambda r_1])(-1+n+v^*[\lambda r_2])(h_1)^*[\lambda r_2](1-q+q(u_1)^*[\lambda r_2]))+k_2(\alpha\gamma r_2\mu_1\mu_2+r_1(\alpha\gamma\mu_1\mu_2 a^*[\lambda r_2](-1+n+v^*[\lambda r_2])(h_1)^*[\lambda r_2](1-q+q(u_1)^*[\lambda r_2])+\lambda r_2(\alpha\gamma\mu_2(n-a^*[\lambda r_2](-1+v^*[\lambda r_1])(-1+n+v^*[\lambda r_2])(h_1)^*[\lambda r_2](1-q+q(u_1)^*[\lambda r_2]))+\mu_1(n\alpha\gamma+\mu_2(2n\alpha+\gamma+a^*[\lambda r_2](-\alpha-\gamma+\alpha v^*[\lambda r_1])(-1+n+v^*[\lambda r_2])(h_1)^*[\lambda r_2](-1+q-q(u_1)^*[\lambda r_2]))))))\}$,

and

$$n = (\frac{1}{a^*[\lambda r_1](h_2)^*[\lambda r_1](1-q+q(u_2)^*[\lambda r_1])}).$$

Hence, the steady-state availability of the system can be given as

$$\text{Av} = P_0+P_1+P_2+P_4+P_6+P_7+P_{10}+P_{11}+P_{13}+P_{14}$$

$\text{Av} = \frac{c_0}{\lambda r_1 r_2}(r_1 a^*[\lambda r_2](-1+n+v^*[\lambda r_2])(h_1)^*[\lambda r_2](1-q+q(u_1)^*[\lambda r_2])+r_2(1+\lambda r_1(\bar{V}^*[\lambda r_2]+(-1+n+v^*[\lambda r_2])\bar{A}^*[\lambda r_2](h_1)^*[\lambda r_2]+n\bar{A}^*[\lambda r_1](h_2)^*[\lambda r_1]-(\bar{H}_1)^*[\lambda r_2]+n(\bar{H}_1)^*[\lambda r_2]+v^*[\lambda r_2](\bar{H}_1)^*[\lambda r_2]+n(\bar{H}_2)^*[\lambda r_1]+a^*[\lambda r_2](-1+n+v^*[\lambda r_2])(h_1)^*[\lambda r_2](\bar{V}^*[\lambda r_1](1-q+q(u_1)^*[\lambda r_2])+q(\bar{U}_1)^*[\lambda r_2])+nqa^*[\lambda r_1](h_2)^*[\lambda r_1](\bar{U}_2)^*[\lambda r_1])))$, (84)

6. RELIABILITY OF THE SYSTEM

Making use the method similar to that in Sec. (4) ,the following partial-differential equations can be obtained:

$$\left(\frac{d}{dt}+r_1\lambda\right)L_0(t) = \int_0^\infty k_2(u_2)L_{14}(t,u_2)du_2 + \int_0^\infty(1-q)\gamma(z)L_{13}(t,z)dz\ , \quad (85)$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\alpha(x)+r_2\lambda\right)L_1(t,x) = 0\ , \quad (86)$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial y_1}+\mu_1(y_1)+r_2\lambda\right)L_2(t,y_1) = 0, \quad (87)$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial z}+\gamma(z)+r_2\lambda\right)L_4(t,z) = 0, \quad (88)$$

$$\left(\frac{d}{dt}+r_2\lambda\right)L_6(t) = \int_0^\infty k_1(u_1)L_7(t,u_1)du_1 + \int_0^\infty(1-q)\gamma(z)L_4(t,z)dz\ , \quad (89)$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial u_1}+k_1(u_1)+r_2\lambda\right)L_7(t,u_1) = 0, \quad (90)$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial y_2}+\mu_2(y_2)+r_1\lambda\right)L_{10}(t,y_2) = 0, \quad (91)$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\alpha(x)+r_1\lambda\right)L_{11}(t,x) = 0, \quad (92)$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial z}+\gamma(z)+r_1\lambda\right)L_{13}(t,z) = 0, \quad (93)$$

and, $\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial u_2}+k_2(u_2)+r_1\lambda\right)L_{14}(t,u_2) = 0$, (94)

Their boundary conditions are:-

$L_1(t,0) = r_1\lambda L_0(t)+\delta(t)$, (95)

$L_2(t,0) = \int_0^\infty \alpha(x)L_1(t,x)dx$, (96)

$L_4(t,0) = \int_0^\infty \mu_1(y_1)L_2(t,y_1)dy_1$, (97)

$L_7(t,0) = \int_0^\infty q\,\gamma(z)L_4(t,z)dz$, (98)

$L_{10}(t,0) = \int_0^\infty \alpha(x)L_{11}(t,x)dx$, (99)

$L_{11}(t,0) = r_2\lambda L_6(t)$, (100)

$L_{13}(t,0) = \int_0^\infty \mu_2(y_2)L_{10}(t,y_2)dy_2$, (101)

$L_{14}(t,0) = \int_0^\infty q\,\gamma(z)L_{13}(t,z)dz$, (102)

The initial conditions are

$$L_1(0,x) = \delta(x) = \begin{cases}1, & x=0\\ 0, & x\neq 0\end{cases}, \quad (103)$$

Otherwise is 0

Taking Laplace transform with respect to t to the equations(85-103), and solving for

$L_i{}^*(s)$; $(i = 0,1,2,4,6,7,10,11,13,14)$.

We find,

$L_0{}^*(s) = \frac{C_1}{\lambda r_1}$, (104)

$L_1{}^*(s,x) = e^{-x(s+\lambda r_2)}C_1\bar{V}[x]$, (105)

$L_2{}^*(s,x) = e^{-x(s+\lambda r_2)}C_1\bar{H}_1[x]v^*[s+\lambda r_2]$, (106)

$L_4{}^*(s,x) = C_1\, e^{-sx-x\lambda r_2}\bar{A}[x]v^*[s+\lambda r_2](h_1)^*[s+\lambda r_2]$, (107)

$L_6{}^*(s) = C_1\frac{a^*[s+\lambda r_2]v^*[s+\lambda r_2](h_1)^*[s+\lambda r_2](1-q+q(u_1)^*[s+\lambda r_2])}{s+\lambda r_2}$, (108)

$L_7{}^*(s,x) = C_1 e^{-sx-x\lambda r_2}q\bar{U}_1[x]a^*[s+\lambda r_2]v^*[s+\lambda r_2](h_1)^*[s+\lambda r_2]$, (109)

$L_{10}{}^*(s,x) = C_1\frac{1}{s+\lambda r_2}e^{-x(s+\lambda r_1)}\lambda r_2\bar{H}_2[x]a^*[s+\lambda r_2]v^*[s+\lambda r_1]v^*[s+\lambda r_2](h_1)^*[s+\lambda r_2](1-q+q(u_1)^*[s+\lambda r_2])$, (110)

$L_{11}{}^*(s,x) = C_1 \frac{e^{-x(s+\lambda r_1)}\lambda r_2 \bar{V}[x] a^*[s+\lambda r_2] v^*[s+\lambda r_2](h_1)^*[s+\lambda r_2](1-q+q(u_1)^*[s+\lambda r_2])}{s+\lambda r_2}$, (111)

$L_{13}{}^*(s,x) = C_1 \frac{1}{s+\lambda r_2} e^{-x(s+\lambda r_1)}\lambda r_2 \bar{A}[x] a^*[s+\lambda r_2] v^*[s+\lambda r_1] v^*[s+\lambda r_2](h_1)^*[s+\lambda r_2](h_2)^*[s+\lambda r_1](1-q+q(u_1)^*[s+\lambda r_2])$, (112)

$L_{14}{}^*(s,x) = C_1 \frac{1}{s+\lambda r_2} e^{-x(s+\lambda r_1)} q\lambda r_2 \bar{U}_2[x] a^*[s+\lambda r_1] a^*[s+\lambda r_2] v^*[s+\lambda r_1] v^*[s+\lambda r_2](h_1)^*[s+\lambda r_2](h_2)^*[s+\lambda r_1](1-q+q(u_1)^*[s+\lambda r_2])$, (113)

Where, C_1:-

$$C_1^{-1} = 1 - \frac{1}{(s+\lambda r_1)(s+\lambda r_2)}\lambda^2 r_1 r_2 a^*[s+\lambda r_1] a^*[s+\lambda r_2] v^*[s+\lambda r_1] v^*[s+\lambda r_2](h_1)^*[s+\lambda r_2](h_2)^*[s+\lambda r_1](1-q+q(u_1)^*[s+\lambda r_2])(1-q+q(u_2)^*[s+\lambda r_1])$$

Since, the reliability of the system is

$$R(t) = L_0(t) + \int_0^\infty L_1(\mathrm{t,x})\mathrm{dx} + \int_0^\infty L_2(\mathrm{t,x})\mathrm{dx} + \int_0^\infty L_4(\mathrm{t,x})\mathrm{dx} + L_6(\mathrm{t}) + \int_0^\infty L_7(\mathrm{t,x})\mathrm{dx} + \int_0^\infty L_{10}(\mathrm{t,x})\mathrm{dx} + \int_0^\infty L_{11}(\mathrm{t,x})\mathrm{dx} + \int_0^\infty L_{13}(\mathrm{t,x})\mathrm{dx} + \int_0^\infty L_{14}(\mathrm{t,x})\mathrm{dx}$$

(114)

Then,

$$R^*(s) = L_0{}^*(s) + \int_0^\infty L_1{}^*(\mathrm{s,x})\mathrm{dx} + \int_0^\infty L_2{}^*(\mathrm{s,x})\mathrm{dx} + \int_0^\infty L_4{}^*(\mathrm{s,x})\mathrm{dx} + L_6{}^*(\mathrm{s}) + \int_0^\infty L_7{}^*(\mathrm{s,x})\mathrm{dx} + \int_0^\infty L_{10}{}^*(\mathrm{s,x})\mathrm{dx} + \int_0^\infty L_{11}{}^*(\mathrm{s,x})\mathrm{dx} + \int_0^\infty L_{13}{}^*(\mathrm{s,x})\mathrm{dx} + \int_0^\infty L_{14}{}^*(\mathrm{s,x})\mathrm{dx}$$

which lead to

$R^*(s) = C_1(\bar{V}^*[s+\lambda r_2] + \frac{1}{(s+\lambda r_1)(s+\lambda r_2)} v^*[s+\lambda r_2]((s+\lambda r_1)(s+\lambda r_2)(\bar{A}^*[s+\lambda r_2](h_1)^*[s+\lambda r_2] + (\bar{H}_1)^*[s+\lambda r_2]) + a^*[s+\lambda r_2](h_1)^*[s+\lambda r_2](s-qs+qs((u_1)^*[s+\lambda r_2] + s(\bar{U}_1)^*[s+\lambda r_2]) + \lambda r_2(s\bar{V}^*[s+\lambda r_1](1-q+q(u_1)^*[s+\lambda r_2]) + sv^*[s+\lambda r_1](1-q+q(u_1)^*[s+\lambda r_2])(\bar{A}^*[s+\lambda r_1](h_2)^*[s+\lambda r_1] + (\bar{H}_2)^*[s+\lambda r_1]) + qs(\bar{U}_1)^*[s+\lambda r_2] + a^*[s+\lambda r_1]v^*[s+\lambda r_1](h_2)^*[s+\lambda r_1](1-q+q(u_1)^*[s+\lambda r_2])(1-q+q(u_2)^*[s+\lambda r_1] + qs(\bar{U}_2)^*[s+\lambda r_1])) + \lambda r_1(1-q+q(u_1)^*[s+\lambda r_2] + qs(\bar{U}_1)^*[s+\lambda r_2] + \lambda r_2(\bar{V}^*[s+\lambda r_1](1-q+q(u_1)^*[s+\lambda r_2]) + q(\bar{U}_1)^*[s+\lambda r_2] + v^*[s+\lambda r_1](1-q+q(u_1)^*[s+\lambda r_2])((\bar{H}_2)^*[s+\lambda r_1] + (h_2)^*[s+\lambda r_1](\bar{A}^*[s+\lambda r_1] + qa^*[s+\lambda r_1](\bar{U}_2)^*[s+\lambda r_1])))))))$ (115)

The mean time to the system failure *(MTTF)* can be calculated on:

$MTTF = R^*(0) = (\bar{V}^*[\lambda r_2] + \frac{1}{\lambda^2 r_1 r_2} v^*[\lambda r_2](\lambda^2 r_1 r_2(\bar{A}^*[\lambda r_2](h_1)^*[\lambda r_2] + (\bar{H}_1)^*[\lambda r_2]) + a^*[\lambda r_2](h_1)^*[\lambda r_2](\lambda r_2 a^*[\lambda r_1] v^*[\lambda r_1](h_2)^*[\lambda r_1](1-q+q(u_1)^*[\lambda r_2])(1-q+q(u_2)^*[\lambda r_1]) + \lambda r_1(1-q+q(u_1)^*[\lambda r_2] + \lambda r_2(\bar{V}^*[\lambda r_1](1-q+q(u_1)^*[\lambda r_2]) + q(\bar{U}_1)^*[\lambda r_2] + v^*[\lambda r_1](1-q+q(u_1)^*[\lambda r_2])((\bar{H}_2)^*[\lambda r_1] + (h_2)^*[\lambda r_1](\bar{A}^*[\lambda r_1] + qa^*[\lambda r_1](\bar{U}_2)^*[\lambda r_1]))))))) / (1 - a^*[\lambda r_1] a^*[\lambda r_2] v^*[\lambda r_1] v^*[\lambda r_2](h_1)^*[\lambda r_2](h_2)^*[\lambda r_1](1-q+q(u_1)^*[\lambda r_2])(1-q+q(u_2)^*[\lambda r_1]))$, (116)

7. SPECIAL CASE

Studying the following special cases:

- Case 1: if($r_1 = r_2 = 0$), then it means that shocks do not impair on the working unit and the units will never fail.
- Case2: if ($r_1 = r_2 = 1$) and$\{P(X=0)=1\}$, then each shock will cause the working unit to fail and the system becomes un-repairable system.
- Case3: if ($r_1 = r_2 = 1$) and$\{q = 0\}$, then each shock will cause the working unit to fail and the units do not need post repaired.

8. NUMERICAL EXAMPLE AND STUDY OF SYSTEM BEHAVIOR THROUGH GRAPHS

Let $\alpha(x) = \alpha, \gamma(z) = \gamma, \mu_i(y_i) = \mu_i$and $\mathrm{k_i(t_i) = k_i}$; where $(i = 1{,}2)$.

We plot the steady- state availability and mean time to system failure for the system model. We show that

- In Fig (1), the steady- state availability is decreasing if the probabilities that one shock causes unit 1,2 to fail (r_1, r_2) are increasing.
- In Fig(2), the steady- state availability is increasing if the repair rate of unit1,2(μ_1, μ_2) are increasing.
- In Fig(3), the steady- state availability is increasing if .the recall repairman rate α,and inspection rate γ *are* increasing.
- In Fig(4), the steady- state availability is increasing if the post repair rate of unit1,2(k_1, k_2) are increasing.
- In Fig (5), the mean time to system failure is decreasing if the probabilities that one shock causes unit 1,2 to fail (r_1, r_2) are increasing.
- In Fig(6), the mean time to system failure is increasing if the repair rate of unit1,2(μ_1, μ_2) are increasing.
- In Fig(7), the mean time to system failure is increasing if .the recall repairman rate α,and inspection rate γ *are* increasing.
- In Fig(8), the mean time to system failure is increasing if the post repair rate of unit1,2(k_1, k_2) are increasing.

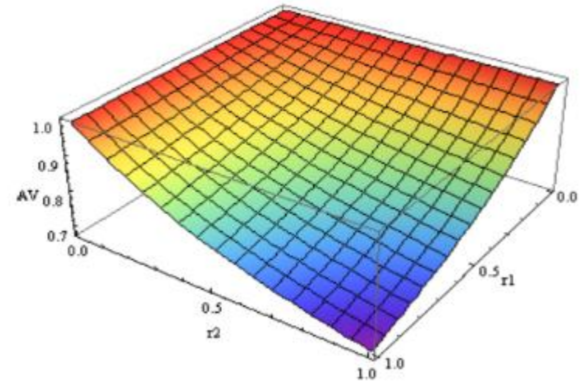

Figure (1)

S.S.Availabilty vs(r_1, r_2)

where($\mu_1 = 0.3, \mu_2 = 0.4, \gamma = 0.8, k_1 = 0.7, k_2 = 0.6, \alpha = 0.9, q = 0.5and\lambda = 0.2$).

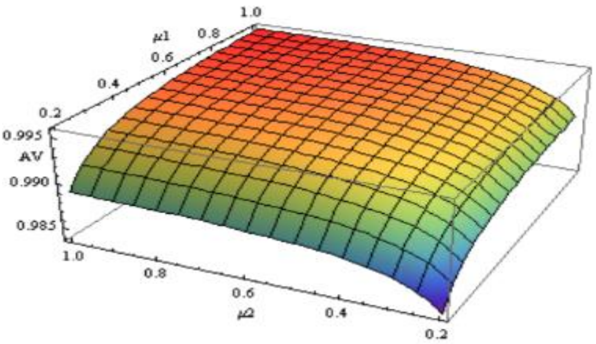

Figure (2)

S.S.Availabilty vs. repair rate of unit1,2(μ_1, μ_2)

Where ($r_1 = 0.1, r_2 = 0.1, \lambda = 0.2, \gamma = 0.8, k_1 = 0.7, k_2 = 0.6$, $\alpha = 0.9$and $q = 0.5$).

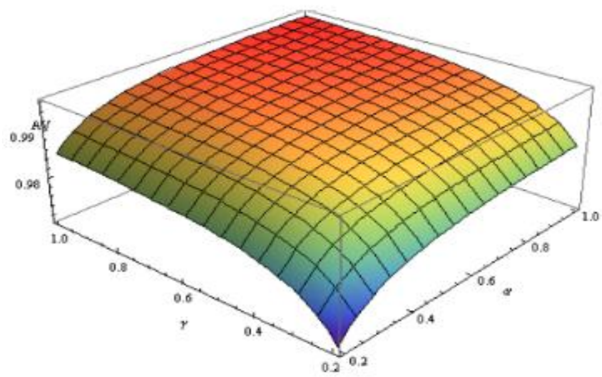

Figure (3)

S.S.Availabilty vs. recall repairman rate α,and inspection rate γ.

Where($r_1 = 0.1, r_2 = 0.1, \lambda = 0.2, \mu_1 = 0.8, \mu_2 = 0.9, k_1 = 0.7, k_2 = 0.6$ and $q = 0.5$).

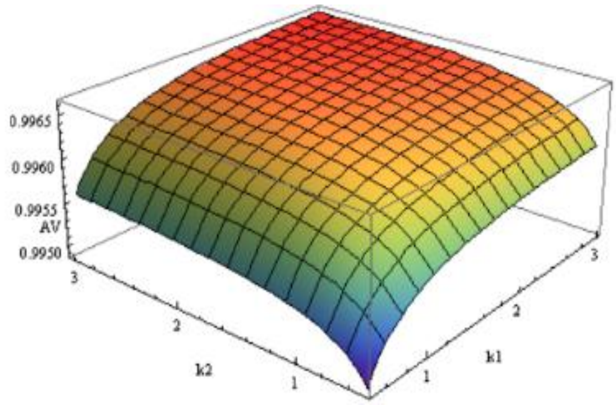

Figure (4)

S.S.Availabilty vs. post repair rate of unit1,2(k_1, k_2)

Where($r_1 = 0.1, r_2 = 0.1, \lambda = 0.2, \mu_1 = 0.8, \mu_2 = 0.9, \gamma = 0.8, \alpha = 0.9$ $andq = 0.5$).

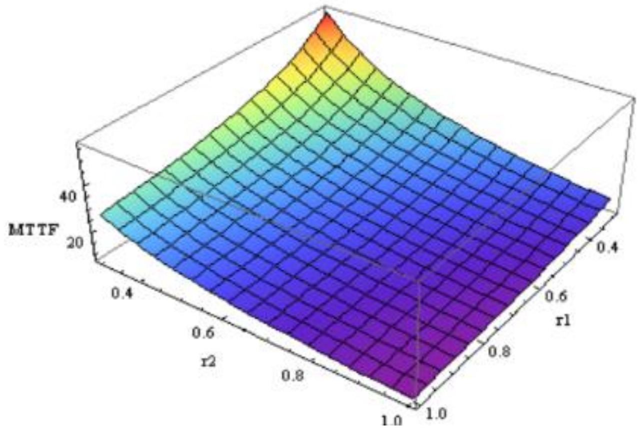

Figure (5)

MTTF vs(r_1, r_2)

where($\mu_1 = 0.3, \mu_2 = 0.4, \gamma = 0.8, k_1 = 0.7, k_2 = 0.6, \alpha = 0.9, q = 0.5and\lambda = 0.2$).

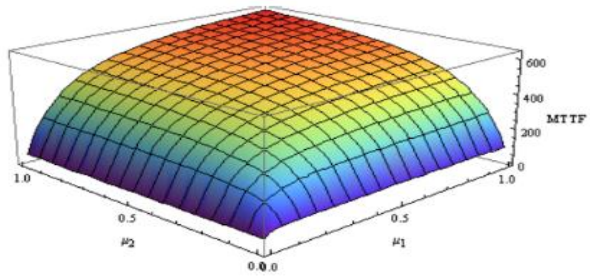

Figure (6)

MTTF vs. repair rate of unit1,2(μ_1, μ_2)

where($r_1 = 0.1, r_2 = 0.1, \lambda = 0.2, \gamma = 0.8, k_1 = 0.7, k_2 = 0.6, \alpha = 0.9andq = 0.5$).

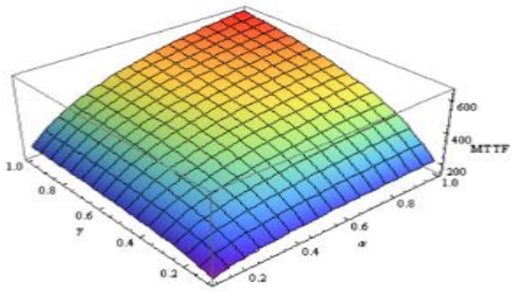

Figure (7)

MTTF vs. recall repairman rate α,and inspection rate γ.

Where($r_1 = 0.1, r_2 = 0.1, \lambda = 0.2, \mu_1 = 0.8, \mu_2 = 0.9, k_1 = 0.7, k_2 = 0.6$ and $q = 0.5$).

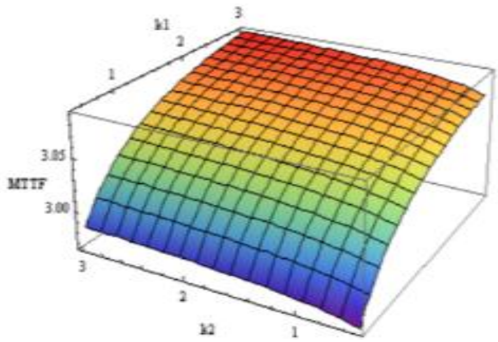

Figure (8)

MTTF vs. post repair rate of unit1,2(k_1, k_2)

Where($r_1 = 0.1, r_2 = 0.1, \lambda = 0.2, \mu_1 = 0.8, \mu_2 = 0.9, \gamma = 0.8, \alpha = 0.9\ and q = 0.5$).

9. CONCLUSIONS

In this paper some reliability measures of the system such as, the mean time to system failure (MTTF) and the steady state availability of the system under Poisson shocks with inspection and post repair using the supplementary variable technique and Laplace transform are successfully obtained. The followings are noticed

1. The MTTF and $Av(\infty)$ decrease by increasing of r_1 and r_2.
2. The MTTF and $Av(\infty)$ increase by increasing of μ_1 and μ_2.
3. The MTTF and $Av(\infty)$ increase by increasing of α and γ.
4. The MTTF and $Av(\infty)$ increase by increasing of $k_1$and k_2.

Based on the results obtained for a particular case, it is concluded that the system model can be more reliable and profitable to use by increasing the repair rates, recall repair repairman rate, inspection rate and post repair rates. The system can be in its best condition when the probabilities r_1, r_2 decrease.

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