An Inventory Model for Maximum Life Time Products under the Price and Stock Dependent Demand Rate

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ABSTRACT

Today's due to competitive business scenarios, the suppliers provide his/her retailers a discount in price of a product. To attract the customers to buy more products at one time, the retailer managed the demand rate depends on price and stock dependent which is very realistic in day to day life. The holding cost is assumed to be variable. For this we proposed a model to solve such types of problems to determine the optimal replenishment policy for non-instantaneous deteriorating products. Numerical example is provided to demonstrate the optimal total profit for discussed inventory model.

Keywords

Inventory, Non-instantaneous deterioration, Variable holding cost, price and Stock-dependent demand.

1. INTRODUCTION

In supermarkets, stores, malls, all of the customers not only focuses on the amount of stock but also observes the price of the items. The product in a large quantity and price discount in the supermarkets attracts many customers and generates the high demand. Therefore the effect of price and stock dependent demand rate cannot be ignored for determining the optimal order policy. Levin et al. (1972) discussed the big amount of consumer goods displayed in a supermarket would attract the customer to buy more products. Gupta and Vrat (1986) first proposed an inventory model for utilization environment to optimize the total inventory cost and considering the stock-dependent demand. Baker and Urban (1988) discussed an inventory model for a power-form inventory-level-dependent demand pattern. Datta and Pal (1990) gave a note on an inventory model with inventory level dependent demand rate.

Pal et al. (1993) extended the inventory model of Baker and Urban (1988) for perishable products that deteriorate at a constant rate. Bar-Lev et al. (1994) developed an extension of the inventory-level-dependent demand-type EOQ model. Padmanabhan and Vrat (1995) proposed an inventory model of deteriorating items with stock-dependent demand. Urban and Baker (1997) discussed the EOQ model in which the demand is a multivariate function of price, time, and level of inventory. Datta and Paul (2001) analyzed a multi-period EOQ model with stock-dependent and price-sensitive demand rate.

Other researchers related to this area such as Mandal and Phaujdar (1989), Giri et al. (1996 and 1998), Ray et al. (1998), etc. Chang (2004) developed an Inventory model with stock-dependent demand and nonlinear holding costs for deteriorating items. Soni and Shah (2008) developed the optimal ordering policy inventory model and stock-dependent demand under the progressive payment scheme environment. Goyal and Chang (2009) developed an inventory model with optimal ordering policy under the stock dependent demand rate.

Chang et al. (2010) discussed an optimal replenishment policy inventory model for non-instantaneous deteriorating items with stock-dependent demand rate. Lots of research work has been done in the area of inventory system with stock-dependent/multivariate demand rate. One may refer to the research work of Dye and Ouyang (2005), Alfares (2007), Singh et al. (2011a, b), Malik and Kumar (2011), Singh and Malik (2011), Sana (2011), Malik and Sharma (11), Sarkar (2012), Sarkar and Sarkar (2013).Sana, S.S., (2010) proposed an inventory model for perishable items under stock-dependent demand rate. Sharma et al. (2013) developed a non-instantaneous deterioration inventory model with inflation and stock-dependent demand. Gupta et al. (2013) proposed an optimal ordering policy for stock-dependent demand inventory model with non-instantaneous deteriorating items.

This paper deals with an inventory model with price and stock dependent demand rate under the variable holding cost. The optimal replenishment policy for non-instantaneous deteriorating items with maximum life time is discussed in this proposed model. The optimizing conditions of the existence and uniqueness of the optimal solutions are given. Here investigate the possible effects of a temporary price discount offered by a supplier on a retailer's replenishment policy for deteriorating items with linear and stock dependent demand rate.

2. NOTATION AND ASSUMPTIONS

Proposed inventory model used the following notations and assumptions:

- The demand rate is $D(t) = a + bt + cI(t) - dp$, Where a, b, c, d, p are positive constants and I(t) is the inventory level at time t.
- Shortages are not acceptable and lead time is zero.
- $\theta(t) = \dfrac{1}{1 + R - t}$ is the deterioration rate
- R is the maximum life time of an items
- A is the ordering cost per order
- $h(t) = h_1 + h_2 t$ is the inventory holding cost per unit time
- C_d is the deteriorating cost per unit
- C_p is the purchasing cost per unit

- p is the sales revenue cost per unit
- I_1 is the inventory level at time $[0, t_1]$ in which the product has no deterioration.
- I_2 is the inventory level at time $[t_1, T]$ in which due to demand and deterioration level go to zero level.
- t_1 is the time period in which product maintain their freshness.
- t_2 is the length in which occurrence of deterioration in product.
- TPF is the total profit function per unit time of the developed inventory system.

3. MATHEMATICAL MODEL

In the proposed inventory model, the time interval [0, t1], the inventory level (I1) decreases due to price and stock dependent demand rate. The inventory level (I2) drops to zero due to demand and the deterioration in the items during the interval is [t1, T]. Therefore the inventory level in the proposed model at any time t can be represented by the following differential equations:

$$\frac{dI_1(t)}{dt} = -\left(a + bt + cI_1(t) - dp\right) \qquad 0\leq t\leq t1 \qquad \ldots\ldots (1)$$

$$\frac{dI_2(t)}{dt} + \theta(t)\, I_2(t) = -\left(a + bt + cI_2(t) - dp\right) \qquad t1\leq t\leq T \qquad \ldots.\ (2)$$

with the boundary conditions $I_1(0) = Q$, $I_2(T) = 0$ respectively, solving above equations (1) and (2), we get

$$I_1(t) = Qe^{-ct} - n_3\left(e^{-ct} - 1\right) - \frac{b}{c}t \qquad \ldots..\ (3)$$

$$I_2(t) = \left\{\begin{array}{l} \frac{bc}{2}\left(t^2 - T^2\right) + n_1\left(t - T\right) \\ + n_2 \log\left(\frac{1+R-t}{1+R-T}\right) \end{array}\right\} e^{-ct}\,(1+R-t) \qquad \ldots\ldots(4)$$

Considering continuity of I(t) at t=t1, it follows from Eqns (3) and (4) that $I_1(t_1) = I_2(t_1)$

$$Q = n_3\left(1 - e^{ct_1}\right) - \frac{b}{c}t_1 e^{ct_1} + \left\{\begin{array}{l} \frac{bc}{2}\left(t_1^{\,2} - T^2\right) + n_1\left(t_1 - T\right) \\ + n_2 \log\left(\frac{1+R-t_1}{1+R-T}\right) \end{array}\right\}(1 + R - t_1) \qquad \ldots..(5)$$

Total profit per cycle contains the following components:

1) The ordering cost (OC) is = A. $\qquad \ldots.(6)$

2) The holding cost (HC) is

$$HC = \left(h_1 + h_2 t\right)\left(\int_0^{t_1} I_1(t)dt + \int_{t_1}^{T} I_2(t)dt\right)$$

$$= h_1\left[\frac{Q}{c}\left(1 - e^{-ct_1}\right) - \frac{n_3}{c}\left(1 - ct_1 - e^{-ct_1}\right) - \frac{b}{2c}t_1^2\right]$$

$$+ h_2\left[\frac{Q+n_3}{c^2}\left(1 - \left(ct_1 + 1\right)e^{-ct_1}\right) - \frac{n_3}{2}t_1^2 - \frac{b}{3c}t_1^3\right]$$

$$+ h_1\left[\begin{array}{l} \frac{m_1}{2}\left(T^2 - t_1^2\right) + \frac{m_2}{3}\left(T^3 - t_1^3\right) \\ + \frac{m_3}{4}\left(T^4 - t_1^4\right) + \frac{m_5}{5}\left(T^5 - t_1^5\right) \\ n_2\left(\begin{array}{l} \log(1+R-T)\left\{(1+R)T - \frac{R_1}{2}T^2 + \frac{c}{3}T^3\right\} \\ -\log(1+R-t_1)\left\{(1+R)t_1 - \frac{R_1}{2}t_1^2 + \frac{c}{3}t_1^3\right\} \\ -\frac{c}{9}\left(T^3 - t_1^3\right) - \frac{m_5}{2}\left(T^2 - t_1^2\right) - m_6\left(T - t_1\right) \\ -m_7 \log\left(\frac{1+R-T}{1+R-t_1}\right) \end{array}\right) \\ + \left\{\frac{bc}{2}T^2 + n_1 T + n_2 \log\left(1 + R - T\right)\right\} \times \\ \quad \left\{\left(1+R\right)\left(T - t_1\right) - \frac{R_1}{2}\left(T^2 - t_1^2\right) + \frac{c}{3}\left(T^3 - t_1^3\right)\right\} \end{array}\right]$$

$$+ h_2\left[\begin{array}{l} \frac{m_1}{3}\left(T^3 - t_1^3\right) + \frac{m_2}{4}\left(T^4 - t_1^4\right) \\ + \frac{m_3}{5}\left(T^5 - t_1^5\right) + \frac{m_4}{6}\left(T^6 - t_1^6\right) \\ n_2\left(\begin{array}{l} \log(1+R-T)\left\{\frac{(1+R)}{2}T^2 - \frac{R_1}{3}T^3 + \frac{c}{4}T^4\right\} \\ -\log(1+R-t_1)\left\{\frac{(1+R)}{2}t_1^2 - \frac{R_1}{3}t_1^3 + \frac{c}{4}t_1^4\right\} \\ -\frac{c}{16}\left(T^4 - t_1^4\right) - \frac{m_8}{3}\left(T^3 - t_1^3\right) - \frac{m_9}{2}\left(T^2 - t_1^2\right) \\ -m_{10}\left(T - t_1\right) - m_{11} \log\left(\frac{1+R-T}{1+R-t_1}\right) \end{array}\right) \\ + \left\{\frac{bc}{2}T^2 + n_1 T + n_2 \log\left(1 + R - T\right)\right\} \times \\ \quad \left\{\frac{(1+R)}{2}\left(T^2 - t_1^2\right) - \frac{R_1}{3}\left(T^3 - t_1^3\right) + \frac{c}{4}\left(T^4 - t_1^4\right)\right\} \end{array}\right] \qquad \ldots..(7)$$

3) The deterioration cost (DC) is

$$DC = C_d \int_{t_1}^{T} \theta(t)\, I_2(t)dt$$

$$= C_d \left[\begin{array}{l} u_3\left(T^3 - t_1^3\right) + \frac{n_1}{2}\left(T^2 - t_1^2\right) \\ -\frac{bc^2}{8}\left(T^4 - t_1^4\right) \\ n_2 \left(\begin{array}{l} \frac{c}{2}\left(T^2 - t_1^2\right) + \left(\frac{2T - cT^2}{2}\right)\log(1+R-T) \\ -\left(\frac{2t_1 - ct_1^2}{2}\right)\log(1+R-t_1) \\ -u_1(T-t_1) + u_2 \log\left(\frac{1+R-T}{1+R-t_t}\right) \end{array} \right) \\ + \left\{\frac{bc}{2}T^2 + n_1 T + n_2 \log(1+R-T)\right\} \times \\ \left\{(T-t_1) - \frac{c}{2}\left(T^2 - t_1^2\right)\right\} \end{array} \right]$$

….(8)

4) The purchasing cost (PC) is $CP = C_p \times Q$

$$= C_p \left(\begin{array}{l} n_3\left(1 - e^{ct_1}\right) - \frac{b}{c} t_1 e^{ct_1} \\ + \left\{\frac{bc}{2}\left(t_1^2 - T^2\right) + n_1(t_1 - T) + n_2 \log\frac{(1+R-t_1)}{1+R-T}\right\}(1+R-t_1) \end{array} \right)$$

….(9)

5) The sales revenue cost (SRC) is

$$SRC = p\int_0^T D(t)dt$$

$$= p \left[\begin{array}{l} \left(aT + \frac{b}{2}T^2 - dpT\right) + C\left\{ \begin{array}{l} \frac{Q}{c}\left(1 - e^{-ct_1}\right) \\ -\frac{n_3}{c}\left(1 - ct_1 - e^{-ct_1}\right) - \frac{b}{2c}t_1^2 \end{array} \right\} \\ +c \left(\begin{array}{l} \frac{m_1}{2}\left(T^2 - t_1^2\right) + \frac{m_2}{3}\left(T^3 - t_1^3\right) \\ + \frac{m_3}{4}\left(T^4 - t_1^4\right) + \frac{m_4}{5}\left(T^5 - t_1^5\right) \\ n_2 \left(\begin{array}{l} \log(1+R-T)\left\{(1+R)T - \frac{R_1}{2}T^2 + \frac{c}{3}T^3\right\} \\ -\log(1+R-t_1)\left\{(1+R)t_1 - \frac{R_1}{2}t_1^2 + \frac{c}{3}t_1^3\right\} \\ -\frac{c}{9}\left(T^3 - t_1^3\right) - \frac{m_5}{2}\left(T^2 - t_1^2\right) \\ -m_6\left(T - t_1\right) - m_7 \log\left(\frac{1+R-T}{1+R-t_1}\right) \end{array} \right) \\ + \left\{\frac{bc}{2}T^2 + n_1 T + n_2 \log(1+R-T)\right\} \times \\ \left\{(1+R)(T-t_1) - \frac{R_1}{2}\left(T^2 - t_1^2\right) + \frac{c}{3}\left(T^3 - t_1^3\right)\right\} \end{array} \right) \end{array} \right]$$

….(10)

Total profit TPF per unit time is

$$TPF = \frac{1}{T}\left[CSR - CO - CH - CD - CP - IP_1 + IE_1\right]$$

…. (11)

The total profit TPF is maximum if

$$\frac{dTPF}{dt_2} = 0$$

…. (12)

and

$$\frac{d^2TPF}{dt_2^2} < 0$$

…. (13)

4. NUMERICAL EXAMPLE

To illustrate the inventory model, following data is used on the basis of previous study: a=45, b=0.8, c=0.2, d=0.6, R=10, t1=2, A=80, Cp=30, h1=3.5, h2=0.6, Cd=1.7. Then, the optimal solution is T*=9.735, p* =77.052, TP* =169.0455.

5. CONCLUSION

This paper proposed an inventory model for non-instantaneous deteriorating items with price and stock dependent demand rate. Numerical example has been provided in this proposed inventory model to illustrate the results. This model is very helpful to vehicle showrooms and shopping malls etc. For future research, this inventory model can be extended in several ways, shortages, variable holding cost, stochastic demand rate, production dependent model, partial backlogging, inflation, permissible delay in payment, two warehouses etc.

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8. APPENDIX

$$n_1 = b + (a - b - dp)c,\ n_2 = a - dp + (1+R)n_1,$$

$$n_3 = \frac{(a-dp)c - b}{c^2},\ m_1 = n_1(1+R),$$

$$m_2 = \frac{bc(1+R) - 2n_1R_1}{2},\ m_3 = \frac{2n_1c - bcR_1}{2},$$

$$m_4 = \frac{bc^2}{2},\ m_5 = \frac{c(1+R)}{3} - \frac{R_1}{2},$$

$$m_6 = (m_5+1)(1+R), m_7 = m_6(1+R),$$

$$m_8 = \frac{c(1+R)}{4} - \frac{R_1}{3}, m_9 = (m_8 - \frac{1}{2})(1+R),$$

$$m_{10} = m_9(1+R),\ m_{11} = m_{10}(1+R),$$

$$u_1 = \frac{(1+R)c+2}{2}, u_2 = u_1(1+R),$$

$$u_3 = \frac{bc - 2n_1c}{6},\ R_1 = (1+R)c + 1.$$

9. AUTHOR PROFILE

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