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Modified Eccentric Connectivity Index and Polynomial of Corona Product of Graphs

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ABSTRACT

The eccentric connectivity index of a graph is defined as the sum of the products of eccentricity with the degree of vertices over all vertices of the graph, and the modified eccentric connectivity index of a graph is defined as the sum of the products of eccentricity with the total degree of neighbouring vertices over all vertices of the graph. In this study, we find eccentric connectivity index and modified eccentric connectivity index and their respective polynomial versions of corona product of two graphs. Finally, we calculate the eccentric connectivity index and modified eccentric connectivity index of some important classes of chemically interesting molecular graphs by specializing the components of corona product of graphs.

Keywords

Eccentricity, topological index, eccentric connectivity index, modified eccentric connectivity index, corona product

1. INTRODUCTION

Topological indices are real numbers derived from the molecular graph structure for correlation of chemical structure with various physical properties, chemical reactivity or biological activity and two graphs G and H have the equal value of a particular topological index if $G \cong H$. In chemistry, biochemistry and nanotechnology different topological indices are found to be useful in isomer discrimination, structure-property relationship, structure-activity relationship, pharmaceutical drug design and so forth. Various topological indices have been defined in chemical literature and various applications and mathematical properties of these indices have been found. The concept of topological indices was started when the American chemist Harold Wiener introduced the first topological index, named as Wiener index [1], in 1947 for investigating boiling points of alkanes. Suppose G be a simple connected graph and $V(G)$ and $E(G)$ respectively denote the vertex set and edge set of G. Let, for any vertex $v \in V(G)$, $d_G(v)$ denotes its degree, that is the number of neighbor of v and $N(v)$ denotes the set of vertices which are the neighbors of the vertex v, so that $|N(v)| = d_G(v)$. Also let

$$\delta_G(v) = \sum_{u \in N(v)} d_G(u)$$

i.e., sum of degrees of the neighbor vertices of G. If $u, v \in V(G)$, then $d_G(u, v)$ is the minimum number of edges in the (u, v)-paths in G. Let $\varepsilon_G(v)$ denotes the eccentricity of a vertex v and is defined as the largest distance from v to any other vertex of G. Various vertex eccentricity and degree based topological indices are found in literature which is used to understand various properties of chemical compounds in theoretical chemistry. The total eccentricity index $\zeta(G)$ of a graph is defined as the sum of eccentricity of all the vertices of G [2, 3]. The eccentric connectivity index of a graph G is one of the most popular topological indices and is defined as [4]

$$\xi^c(G) = \sum_{v \in V(G)} d_G(v)\varepsilon_G(v).$$

For various applications and for more results related to chemical applications and mathematical properties of this index, we refer our reader to [5, 6, 7, 8, 9]. One modified version of eccentric connectivity index is defined as [10]

$$\xi_c(G) = \sum_{v \in V(G)} \delta_G(v)\varepsilon_G(v).$$

In [11] and [10], the modified eccentric connectivity polynomial for three infinite classes of fullerenes and one-pentagonal carbon nanocones was computed. In [12], some exact relations for the modified eccentric connectivity polynomial of different graph operations were derived. Different upper and lower bounds are obtained and some study on different generalized thorn graphs for this modified eccentric connectivity index is done by the present authors in [13] and [14]. The first Zagreb index of G denoted by $M_1(G)$ is one of the oldest vertex degree based topological indices introduced in [15] by Gutman and Trinajstić and is defined as

$$M_1(G) = \sum_{i=1}^{n} d_G(u_i)^2 = \sum_{i=1}^{n} \delta_G(u_i).$$

Let G_1 and G_2 be two simple connected graphs with n_i number of vertices and e_i number of edges respectively, for $i \in \{1, 2\}$. The corona product $G_1 \circ G_2$ of these two graphs is obtained by taking one copy of G_1 and n_1 copies of G_2; and by joining each vertex of the i-th copy of G_2 to the i-th vertex of G_1, where $1 \le i \le n_1$. The corona product of G_1 and G_2 has total number of $(n_1n_2 + n_1)$ vertices and $(e_1 + n_1e_2 + n_1n_2)$ edges. Clearly, the corona product operation of two graphs is not commutative. Different topological indices such as Wiener-type Indices [16], Szeged, vertex PI, first and second Zagreb indices [17], weighted PI index [18], etc. of the corona product of two graphs have already been studied.

Corona product of graphs appears in chemical literature as plerographs of the hydrogen suppressed molecular graphs known as kenographs. In this study, we find eccentric connectivity index and modified eccentric connectivity index and their respective polynomial versions of corona product of two graphs. Furthermore, by specializing the components of corona product of graphs we calculate the eccentric connectivity index and modified eccentric connectivity index of t-thorny graph, sunlet graph, bottleneck graph, suspension of graphs and some classes of bridge graphs.

2. MAIN RESULTS

Let the vertices of G_1 are denoted by $V(G_1) = \{u_1, u_2, ..., u_{n_1}\}$ and the vertices of the i-th copy of G_2 are denoted by $V(G_{2,i}) = \left\{v_1^i, v_2^i, ..., v_{n_2}^i\right\}$ for $i = 1, 2, ..., n_1$. Thus the vertex set and edge set of $G_1 \circ G_2$ are given by $V(G_1 \circ G_2) = V(G_1) \bigcup_{i=1}^{n_1} V(G_{2,i})$ and $E(G_1 \circ G_2) = E(G_1) \bigcup_{i=1}^{n_1} E(G_{2,i}) \bigcup \left\{(u_i, v_j^i) : u_i \in V(G_1), v_j^i \in V(G_{2,i})\right\}$.
First we start with following important lemma.

LEMMA 1. *The degree, eccentricity and neighborhood degree sum of the vertices of $G_1 \circ G_2$ are given as follows.*

(a) If $u_i \in V(G_1)$, then
- *(i)* $d_{G_1\circ G_2}(u_i) = d_{G_1}(u_i) + n_2$,
- *(ii)* $\varepsilon_{G_1\circ G_2}(u_i) = \varepsilon_{G_1}(u_i) + 1$, *and*
- *(iii)* $\delta_{G_1\circ G_2}(u_i) = \delta_{G_1}(u_i) + n_2 d_{G_1}(u_i) + 2m_2 + n_2$.

(b) If $v_j^i \in V(G_{2,i})$, where the i-th copy of G_2 is denoted by $G_{2,i}$, $1 \le i \le |V(G_1)|$, then
- *(i)* $d_{G_1\circ G_2}(v_j^i) = d_{G_2}(v_j) + 1$,
- *(ii)* $\varepsilon_{G_1\circ G_2}(v_j^i) = \varepsilon_{G_1}(u_i) + 2$, *and*
- *(iii)* $\delta_{G_1\circ G_2}(v_j^i) = \delta_{G_1}(v_j) + d_{G_2}(v_j) + d_{G_1}(u_i) + n_2$.

In the following, we start by computing the eccentric connectivity index of corona product of two graphs.

THEOREM 1. *The eccentric connectivity index of $G_1 \circ G_2$ is given by*

$$\xi^c(G_1\circ G_2) = \xi^c(G_1) + 2(n_2+m_2)\zeta(G_1) + 2m_1 + 4n_1m_2 + 3n_1n_2. \tag{1}$$

Proof. From the definition of corona product of graphs, the eccentric connectivity index of $G_1 \circ G_2$ is given by

$$\begin{aligned}\xi^c(G_1 \circ G_2) &= \sum_{i=1}^{n_1} d_{G_1\circ G_2}(u_i)\varepsilon_{G_1\circ G_2}(u_i) \\ &+ \sum_{i=1}^{n_1}\sum_{j=1}^{n_2} d_{G_1\circ G_2}(v_j^i)\varepsilon_{G_1\circ G_2}(v_j^i).\end{aligned}$$

Now, using $(a)(i)$ and $(a)(ii)$ of Lemma 1 we have

$$\begin{aligned}&\sum_{i=1}^{n_1} d_{G_1\circ G_2}(u_i)\varepsilon_{G_1\circ G_2}(u_i) \\ &= \sum_{i=1}^{n_1} \{d_{G_1}(u_i) + n_2\}\{\varepsilon_{G_1}(u_i) + 1\} \\ &= \sum_{i=1}^{n_1} d_{G_1}(u_i)\varepsilon_{G_1}(u_i) + n_2\sum_{i=1}^{n_1}\varepsilon_{G_1}(u_i) + \sum_{i=1}^{n_1} d_{G_1}(u_i) + n_1n_2 \\ &= \xi^c(G_1) + n_2\zeta(G_1) + 2m_1 + n_1n_2.\end{aligned}$$

Also, using $(b)(i)$ and $(b)(ii)$ of Lemma 1, we get

$$\begin{aligned}&\sum_{i=1}^{n_1}\sum_{j=1}^{n_2} d_{G_1\circ G_2}(v_j^i)\varepsilon_{G_1\circ G_2}(v_j^i) \\ &= \sum_{i=1}^{n_1}\sum_{j=1}^{n_2}\{d_{G_2}(v_j) + 1\}\{\varepsilon_{G_1}(u_i) + 2\} \\ &= \sum_{i=1}^{n_1}\varepsilon_{G_1}(u_i)\sum_{j=1}^{n_2} d_{G_2}(v_j) + n_2\sum_{i=1}^{n_1}\varepsilon_{G_1}(u_i) \\ &\quad + 2n_1\sum_{j=1}^{n_2} d_{G_2}(v_j) + 2n_1n_2 \\ &= (n_2 + 2m_2)\zeta(G_1) + 4n_1m_2 + 2n_1n_2.\end{aligned}$$

Adding the above two, we get the desired result. □
Luo and Wu computed the eccentric connectivity index of corona product of graphs as a particular case of cluster product of graphs in [19], which coincides with Theorem 1. Also, Došlić et al. in [20] computed the Theorem 1, when calculating the eccentric connectivity index of different graph operations.
The eccentric connectivity polynomial [21] and total eccentricity polynomial [12] of G are respectively defined as

$$ECP(G, x) = \sum_{i=1}^{n} d_G(v_i)x^{\varepsilon_G(v_i)}$$

and

$$\theta(G, x) = \sum_{i=1}^{n} x^{\varepsilon_G(v_i)}$$

respectively. It is easy to see that the eccentric connectivity index and the total eccentricity index of a graph can be obtained from the corresponding polynomials by evaluating their first derivatives at $x = 1$. In the following result we use the same reasoning of Theorem 1 to derive exact relation of eccentric connectivity polynomial of corona product of two graphs in terms of eccentric connectivity polynomial and total eccentricity polynomial of the first graph.

THEOREM 2. *The eccentric connectivity polynomial of $G_1\circ G_2$ is given by*

$$ECP(G_1\circ G_2, x) = xECP(G_1, x) + x\left((1 + x)n_2 + 2m_2x\right)\theta(G_1, x). \tag{2}$$

In the next paragraph, we give an exact expression for the modified eccentric connectivity index of corona product of graphs.

THEOREM 3. *The modified eccentric connectivity index of $G_1 \circ G_2$ is given by*

$$\begin{aligned}\xi_c(G_1 \circ G_2) &= \xi_c(G_1) + 2n_2\xi^c(G_1) + (n_2 + n_2^2 + 4m_2)\zeta(G_1) \\ &+ M_1(G_1) + (\zeta(G_1) + 2n_1)M_1(G_2) \\ &+ 6n_1m_2 + 6n_2m_1 + n_1n_2(2n_2 + 1). \end{aligned}\tag{3}$$

Proof. According to definition of corona product of graphs the modified eccentric connectivity index of $G_1 \circ G_2$ is given by

$$\begin{aligned}\xi_c(G_1 \circ G_2) &= \sum_{i=1}^{n_1} \delta_{G_1 \circ G_2}(u_i)\varepsilon_{G_1 \circ G_2}(u_i) \\ &+ \sum_{i=1}^{n_1}\sum_{j=1}^{n_2} \delta_{G_1 \circ G_2}(v_j^i)\varepsilon_{G_1 \circ G_2}(v_j^i).\end{aligned}$$

Now, using $(a)(ii)$ and $(a)(iii)$ of Lemma 1, we have

$$\begin{aligned}&\sum_{i=1}^{n_1} \delta_{G_1 \circ G_2}(u_i)\varepsilon_{G_1 \circ G_2}(u_i) \\ =& \sum_{i=1}^{n_1} \{\delta_{G_1}(u_i) + n_2 d_{G_1}(u_i) + 2m_2 + n_2\}\{\varepsilon_{G_1}(u_i) + 1\} \\ =& \sum_{i=1}^{n_1} \delta_{G_1}(u_i)\varepsilon_{G_1}(u_i) + n_2 \sum_{i=1}^{n_1} d_{G_1}(u_i)\varepsilon_{G_1}(u_i) \\ & +(2m_2+n_2)\sum_{i=1}^{n_1}\varepsilon_{G_1}(u_i) + \sum_{i=1}^{n_1}\delta_{G_1}(u_i) \\ & +n_2\sum_{i=1}^{n_1} d_{G_1}(u_i) + (2m_2+n_2)n_1 \\ =& \ \xi_c(G_1) + n_2\xi^c(G_1) + (2m_2+n_2)\zeta(G_1) + M_1(G_1) \\ & +2n_2m_1 + n_1(2m_2+n_2).\end{aligned}$$

Similarly using $b(ii)$ and $b(iii)$ of Lemma 1, we get

$$\begin{aligned}&\sum_{i=1}^{n_1}\sum_{j=1}^{n_2} \delta_{G_1 \circ G_2}(v_j^i)\varepsilon_{G_1 \circ G_2}(v_j^i) \\ =& \sum_{i=1}^{n_1}\sum_{j=1}^{n_2} \{\delta_{G_1}(v_j) + d_{G_2}(v_j) + d_{G_1}(u_i) + n_2\}\{\varepsilon_{G_1}(u_i) + 2\} \\ =& \sum_{i=1}^{n_1}\varepsilon_{G_1}(u_i)\sum_{j=1}^{n_2}\delta_{G_2}(v_j) + \sum_{i=1}^{n_1}\varepsilon_{G_1}(u_i)\sum_{i=1}^{n_2} d_{G_2}(v_j) \\ & +n_2^2\sum_{i=1}^{n_1}\varepsilon_{G_1}(u_i) + 2\sum_{i=1}^{n_1}\sum_{j=1}^{n_2}\delta_{G_2}(v_j) + 2\sum_{i=1}^{n_1}\sum_{j=1}^{n_2} d_{G_2}(v_j) \\ & +n_2\sum_{i=1}^{n_1} d_{G_1}(u_i)\varepsilon_{G_1}(u_i) + 2n_2\sum_{i=1}^{n_1} d_{G_1}(u_i) + 2n_1n_2^2 \\ =& \ \zeta(G_1)M_1(G_2) + 2m_2\zeta(G_1) + n_2\xi^c(G_1) + {n_2}^2\zeta(G_1) \\ & +2n_1M_1(G_2) + 4n_1m_2 + 4n_2m_1 + 2n_1{n_2}^2.\end{aligned}$$

The expression for the modified eccentric connectivity index of $G_1 \circ G_2$ follows by summing the above two expressions. □

The modified eccentric connectivity polynomial of G is defined as [11, 10, 12]

$$MECP(G,x) = \sum_{i=1}^{n} \delta_G(v_i)x^{\varepsilon_G(v_i)}$$

so that, the modified eccentric connectivity index of a graph can be obtained from this polynomial by evaluating its first derivative at $x = 1$. In the following we give the modified eccentric connectivity polynomial of $G_1 \circ G_2$ using the similar arguments as Theorem 3.

THEOREM 4. *The modified eccentric connectivity polynomial of $G_1 \circ G_2$ is given by*

$$\begin{aligned}MECP(G_1 \circ G_2, x) &= xMECP(G_1,x) + x(1+x)n_2ECP(G_1,x) \\ &+x(xM_1(G_2) + 2(1+x)m_2 \\ &+n_2(n_2x+1))\theta(G_1,x).\end{aligned} \quad (4)$$

3. COROLLARIES AND EXAMPLES

For any given graph G, the t-thorny graph or the t-fold bristled graph is obtained by attaching t-vertices of degree one to each vertex of G. This graph was introduced by Gutman in [22] and can be obtained as corona product of G and complement of complete graph on t-vertices, i.e., $\overline{K_t}$. Let G be a connected graph with n vertices and m edges. In the following we find the eccentric connectivity index of t-thorny graphs from eccentric connectivity index of corona product of graphs. For different topological indices of thorny graphs interested reader may refer to [21, 23, 14].

COROLLARY 1. *The eccentric connectivity index of G^t is given by*

$$\xi^c(G^t) = \xi^c(G) + 2t\zeta(G) + 2m + 3nt. \quad (5)$$

Proof. This result is a straight forward application of Theorem 1. □

Total eccentricity indices of $P_n (n \geq 1)$ and $C_n (n \geq 3)$, i.e., path and cycle of order n respectively, are given by

$$\zeta(P_n) = \begin{cases} \frac{3}{4}n^2 - \frac{1}{2}n, & \text{when } n \text{ is even} \\ \frac{3}{4}n^2 - \frac{1}{2}n - \frac{1}{4}, & \text{when } n \text{ is odd,} \end{cases}$$

$$\zeta(C_n) = \begin{cases} \frac{1}{2}n^2, & \text{when } n \text{ is even} \\ \frac{1}{2}n(n-1), & \text{when } n \text{ is odd.} \end{cases}$$

and the eccentric connectivity indices are given by

$$\xi^c(P_n) = \begin{cases} \frac{1}{2}\left(3n^2 - 6n + 4\right), & \text{when } n \text{ is even} \\ \frac{3}{2}(n-1)^2, & \text{when } n \text{ is odd,} \end{cases}$$

$$\xi^c(C_n) = \begin{cases} n^2, & \text{when } n \text{ is even} \\ n(n-1), & \text{when } n \text{ is odd.} \end{cases}$$

Then from (5) the following result follows.

EXAMPLE 1. *The eccentric connectivity index of t-thorny graph of C_n and P_n are given by*

(i)

$$\xi^c(C_n \circ \overline{K_t}) = \begin{cases} n^2(t+1) + n(3t+2), & \textit{when n is even} \\ n(n-1)(t+1) + n(3t+2), & \textit{when n is odd.} \end{cases}$$

(ii)

$$\xi^c(P_n \circ \overline{K_t}) = \begin{cases} \frac{3}{2}n^2(t+1) + 2nt - n, & \textit{when n is even} \\ \frac{3}{2}n^2(t+1) + 2nt - n \\ -\frac{1}{2}t - \frac{1}{2}, & \textit{when n is odd,} \end{cases}$$

Similar to eccentric connectivity index, the modified eccentric connectivity index of t-thorny graph is obtained from (3) as follows.

COROLLARY 2. *The modified eccentric connectivity index of G^t is given by*

$$\xi_c(G^t) = \xi_c(G)+2t\xi^c(G)+t(t+1)\zeta(G)+M_1(G)+6mt+nt+2nt^2. \quad (6)$$

The modified eccentric connectivity index of $P_n (n \geq 4)$ and $C_n (n \geq 3)$ are respectively given by

$$\xi_c(P_n) = \begin{cases} 3n^2 - 8n + 8, & \text{when } n \text{ is even} \\ 3n^2 - 8n + 7, & \text{when } n \text{ is odd,} \end{cases}$$

$$\xi_c(C_n) = \begin{cases} 2n^2, & \text{when } n \text{ is even} \\ 2n(n-1), & \text{when } n \text{ is odd.} \end{cases}$$

Then the following result follows from (6).

EXAMPLE 2. *The modified eccentric connectivity index of t-thorny graph of C_n and P_n are given by*

(i)

$$\xi_c(C_n \circ \overline{K_t}) = \begin{cases} \frac{1}{2}n^2t(t+1) \\ +2n^2(t+1) + 2nt^2 \\ +7nt + 4n, & \text{when } n \text{ is even} \\ \frac{1}{2}n(n-1)t(t+1) \\ +2n(n-1)(t+1) \\ +2nt^2 + 7nt + 4n, & \text{when } n \text{ is odd.} \end{cases}$$

(ii)

$$\xi_c(P_n \circ \overline{K_t}) = \begin{cases} \frac{3}{4}n^2t^2 + \frac{3}{2}nt^2 + \frac{15}{4}n^2t \\ +\frac{1}{2}nt + 3n^2 - 4n \\ -2t + 2, & \text{when } n \text{ is even} \\ \frac{3}{4}n^2t^2 + \frac{3}{2}nt^2 + \frac{15}{4}n^2t \\ +\frac{1}{2}nt + 3n^2 - 4n - \frac{1}{4}t^2 \\ -\frac{13}{4}t + 1, & \text{when } n \text{ is odd.} \end{cases}$$

It can be easily checked that the formula for eccentric connectivity index and modified eccentric connectivity index of t-thorny graph coincides with the results directly derived in [21] and [14] respectively.

A particular thorny graph, the n-sunlet graph is obtained by attaching n pendent edges to the cycle C_n, so that it contains $2n$ vertices and edges. Let it be denoted by SL_n and thus $SL_n = C_n \circ \overline{K_1}$. So using (1) and (3), the following result follows.

EXAMPLE 3. *The eccentric connectivity index and modified eccentric connectivity index of SL_n ($n \geq 3$),is given by*

(i)

$$\xi^c(SL_n) = \begin{cases} 2n^2 + 5n, & \text{when } n \text{ is even} \\ 2n(n-1) + 5n, & \text{when } n \text{ is odd.} \end{cases}$$

(ii)

$$\xi_c(SL_n) = \begin{cases} 5n^2 + 13n, & \text{when } n \text{ is even} \\ 5n^2 + 8n, & \text{when } n \text{ is odd.} \end{cases}$$

A bistar $B_{n,n}$ is obtained by joining the center vertices of two copies of $K_{1,n}$. Thus the star graph S_n on n vertices and the bistar $B_{n,n}$ on $(2n+2)$ vertices are corona product of K_1 and $\overline{K_{n-1}}$, and P_2 and $\overline{K_n}$ respectively.

EXAMPLE 4. *The eccentric connectivity index and modified eccentric connectivity index of star graph is given by $\xi^c(K_1 \circ \overline{K_{n-1}}) = 3(n-1)$ and $\xi_c(K_1 \circ \overline{K_{n-1}}) = 2n^2 - 3n + 1$.*

EXAMPLE 5. *The eccentric connectivity index and modified eccentric connectivity index of bistar $B_{n,n}$ is given by $\xi^c(P_2 \circ \overline{K_n}) = 10n + 4$ and $\xi_c(P_2 \circ \overline{K_n}) = 6n^2 + 14n + 4$.*

The suspension of a graph G is defined as corona product of K_1 and G. So from (1) and (3) the following result follows.

COROLLARY 3. *The eccentric connectivity index and modified eccentric connectivity index of suspension of G are respectively given by $\xi^c(K_1 \circ G) = 3n + 4m$ and $\xi_c(K_1 \circ G) = 2M_1(G) + 2n^2 + n + 6m$.*

EXAMPLE 6. *The wheel graph W_n on $(n+1)$ vertices is the suspension of C_n. So its eccentric connectivity index and modified eccentric connectivity index are given by $\xi^C(K_1 \circ C_n) = 7n$ and $\xi_c(K_1 \circ C_n) = 2n^2 + 15n$.*

EXAMPLE 7. *The fan graph F_n on $(n+1)$ vertices is the suspension of P_n. So its eccentric connectivity index and modified eccentric connectivity index are given by $\xi^c(K_1 \circ P_n) = 7n - 4$ and $\xi_c(K_1 \circ P_n) = 2n^2 + 15n - 18$.*

The bottleneck graph B can be obtained by the corona product of K_2 and G. So from (1) and (3) the following result follows.

COROLLARY 4. *The eccentric connectivity index and modified eccentric connectivity index of the graph B is given by*

(i) $\xi^c(B) = 10n + 12m + 4$

(ii) $\xi_c(B) = 6M_1(G) + 6n^2 + 14n + 20m + 4$.

Let $v_1, v_2, ..., v_n$ be the vertices of a set of finite pair wise disjoint graphs $G_1, G_2, ..., G_n$ respectively. Then the bridge graph with respect to the vertices $v_1, v_2, ..., v_n$ is denoted by $B(G_1, G_2, ..., G_n; v_1, v_2, ..., v_n)$ which is obtained by joining the vertices v_i and v_{i+1} of G_i and G_{i+1} by an edge, for all $i = 1, 2, ..., (n-1)$. In particular, if $G_1 \cong G_2 \cong ... \cong G_n \cong G$ and $v_1 = v_2 = ... = v_n = v$, then the bridge graph is denoted by $G_n(G, v)$. Now we consider three particular type of bridge graphs as in [24, 2, 18], named as $B_n = G_n(P_3, v)$, $T_{m,k} = G_m(C_k, u)$ and $J_{n,m+1} = G_n(P_3, v)$. According to definition of corona product of graphs the bridge graphs $B_n = P_n \circ \overline{K_2}$, $T_{m,3} = P_m \circ K_2$ and $J_{n,m+1} = P_n \circ C_m$. So using, (1) we obtain the eccentric connectivity index of these bridge graphs.

EXAMPLE 8. *(i) For $n \geq 2$,*

$$\xi^c(B_n) = \begin{cases} \frac{9}{2}n^2 + 3n, & \text{when } n \text{ is even} \\ \frac{9}{2}n^2 + 3n - \frac{3}{2}, & \text{when } n \text{ is odd.} \end{cases}$$

(ii) For $m \geq 2$,

$$\xi^c(T_{m,3}) = \begin{cases} 6m^2 + 6m, & \text{when } m \text{ is even} \\ 6m^2 + 6m - 2, & \text{when } m \text{ is odd.} \end{cases}$$

(iii) For $n \geq 2$ and $m \geq 3$,

$$\xi^c(J_{n,m+1}) = \begin{cases} 3n^2(m + \frac{1}{2}) + 5mn - n, & \text{when } n \text{ is even} \\ 3n^2(m + \frac{1}{2}) + 5mn - n \\ -m - \frac{1}{2}, & \text{when } n \text{ is odd.} \end{cases}$$

Similarly, using (3), the modified eccentric connectivity index of the bridge graphs $P_n \circ \overline{K_2}$, $P_m \circ K_2$ and $P_n \circ C_m$ are obtained as follows.

EXAMPLE 9. *(i) For* $n \geq 3$,

$$\xi_c(B_n) = \begin{cases} \frac{27}{2}n^2 + 3n - 2, & \text{when } n \text{ is even} \\ \frac{27}{2}n^2 + 3n - \frac{13}{2}, & \text{when } n \text{ is odd.} \end{cases}$$

(ii) For $m \geq 3$,

$$\xi_c(T_{m,3}) = \begin{cases} 18m^2 + 10m - 2, & \text{when } m \text{ is even} \\ 18m^2 + 10m - 8, & \text{when } m \text{ is odd.} \end{cases}$$

(iii) For $n \geq 3$ *and* $m \geq 3$,

$$\xi_c(J_{n,m+1}) = \begin{cases} 3n^2 + 3mn^2 + 2nm^2 \\ +15nm - 4n - 2m + 2 \\ +(\frac{3}{4}n^2 - \frac{1}{2}n)(m^2 + 9m), & \text{if } n \text{ is even} \\ 3n^2 + 3mn^2 + 2nm^2 \\ +15nm - 4n - 3m + 1 \\ +(\frac{3}{4}n^2 - \frac{1}{2}n - \frac{1}{4})(m^2 + 9m), & \text{if } n \text{ is odd.} \end{cases}$$

4. CONCLUSION

In this study, we have computed the eccentric connectivity index and modified eccentric connectivity index and their respective polynomials for corona product of graphs. Also, we apply our results to calculate these indices for some classes of graphs such as t-thorny graph, sunlet graph, bottleneck graph, suspension of graphs and bridge graphs by considering corona product of specific graphs. Nevertheless, there are still many classes of graphs which are not covered in this study. Various chemical graphs and nano-structure may also be considered and evaluation of these indices in terms of degeneracy, discriminating power and intercorrelation for such classes of graphs can be further investigated in this direction.

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