

MHD Convective Flow of non-Newtonian Fluid Through Porous Medium over an Oscillating Porous Plate with Suction

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ABSTRACT

This paper investigates the convective flow of an incompressible viscous electrically conducting second grade fluid through a porous medium occupying a semi-infinite region over an oscillating porous plate in presence of a transverse magnetic field with suction. The field equations for the velocity and temperature fields are solved analytically and the expressions for the velocity field, temperature field, skin friction and heat flux are obtained. The effects of permeability parameter, magnetic parameter/Hartmann number, Prandtl number, heat source parameter and suction parameter on the velocity field u are illustrated graphically. The influence of suction parameter, Prandtl number and heat source parameter S on temperature field is analyzed with the help of figures. Furthermore the effects of permeability parameter and magnetic parameter on skin friction are shown diagrammatically.

Keywords

Convective; Oscillating; Transverse Magnetic field; Suction; Porous Medium; Second Grade Fluid

1. INTRODUCTION

The study related to free convective flow in presence of heat source has drawn considerable attention of many researchers during last few decades because of its wide application in astrophysical sciences, cosmical studies etc. These types of flows play vital role in chemical engineering, aerospace technology etc. Unsteady oscillating flows have applications in many fields such as paper industry and many other technological fields.

Asghar et al. [1] have studied the flow of a non-Newtonian fluid induced due to the oscillations of a porous plate. Choudhury and Das [2] investigated visco-elastic MHD free convective flow through porous media in presence of radiation and chemical reaction with heat and mass transfer. Deka et al. [3] have discussed free convection effects on MHD flow past an infinite vertical oscillating plate with constant heat flux. Das et al. [4] have investigated mass transfer effects on free convective MHD flow of a viscous fluid bounded by an oscillating porous plate in the slip flow regime with heat source. A study has been done by Hayat et al. [5] on the flow of a visco-elastic fluid on an oscillating plate. Manna et al. [6] have discussed effects of radiation on unsteady MHD free convective flow past an oscillating vertical porous plate embedded in a porous medium with oscillatory heat flux. Shen et al. [7] investigated Rayleigh-Stokes problem for a heated generalized second grade fluid with fractional derivative model. Singh and Gupta [8] have studied MHD free convective flow of viscous fluid through a

porous medium bounded by an oscillating porous plate in slip flow regime with mass transfer.

In present problem a convective flow of an incompressible non-Newtonian fluid through a porous medium over an oscillating porous plate in presence of transverse magnetic field has been considered. It is observed that an increase in the parameter of permeability K_p leads to an increase in the velocity field u . The velocity field u decreases with an increase in magnetic field parameter M , Prandtl number P_r , heat source parameter S and suction parameter v_0 as shown in figures. It can be shown in the figure that the temperature field T decreases with an increase in suction parameter v_0 , Prandtl number P_r , heat source parameter S separately. Further it is observed that the period of oscillation of the stress curves increases with an increase in the parameter of permeability K_p .

2. ANALYSIS OF THE PROBLEM

Let us consider the convective flow of an incompressible viscous electrically conducting second grade fluid through a porous medium occupying a semi-infinite region over an oscillating porous plate in presence of transverse magnetic field B_0 . Let u and v are the velocity components in the x - and y -directions respectively and the direction of x - and y axes are taken respectively along and normal to the plate. Since the plate is infinite dimensional in x and z -directions, all the physical quantities are functions of y and t only. The Reynolds number is assumed to be very small and the induced magnetic field due to the flow is neglected with respect to the applied magnetic field. The pressure is assumed to be constant in the flow field. If v_0 be the suction/injection velocity at the plate then the equation of continuity is given by

$$\frac{\partial v}{\partial y} = 0 \quad (1)$$

Under the condition $y = 0$, $v = -v_0$ then the governing boundary layer equation is given by

$$\frac{\partial u}{\partial t} - v_0 \frac{\partial u}{\partial y} = \left(\nu + \alpha \frac{\partial}{\partial t} \right) \frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) - \frac{\nu}{K_0} u - \frac{\sigma B_0^2 u}{\rho} \quad (2)$$

$$\frac{\partial T}{\partial t} - v_0 \frac{\partial T}{\partial y} = \kappa \left(1 + \alpha \frac{\partial}{\partial t} \right) \frac{\partial^2 T}{\partial y^2} - S(T - T_\infty) \quad (3)$$

Where α is normal stress moduli, κ is thermal diffusivity, ν is kinematic viscosity, g is acceleration due to gravity, β is the volumetric coefficient of expansion for heat transfer, K_0 is permeability of the porous medium, ρ is the density, σ is electrical conductivity of the fluid, S is the heat source

parameter, T is the temperature, T_∞ is the temperature of the fluid far away from the plate.

The boundary conditions are given by

$$u = U_0 e^{i\omega t}, T = T_w \text{ at } y = 0$$

$$u \rightarrow 0, T \rightarrow T_\infty \text{ as } y \rightarrow \infty \quad (4)$$

The non-dimensional variables are introduced as follows

$$y' = U_0 \frac{y}{\nu}, u' = \frac{u}{U_0}, T' = \frac{T - T_\infty}{T_w - T_\infty}, t' = U_0^2 \frac{t}{\nu^2},$$

$$v'_0 = \frac{v_0}{U_0}, \omega' = \frac{v_0 \omega}{U_0^2}, S' = \frac{v_0 S}{U_0^2}, M = \frac{B_0}{U_0} \left(\frac{v_0 \sigma}{\rho} \right)^{\frac{1}{2}}, \alpha' = \alpha \frac{U_0^2}{\nu^2},$$

$$P_r = \frac{\nu}{k}, K_p = \frac{K_0 U_0^2}{\nu^2}, G_r = \nu g \beta \frac{T_w - T_\infty}{U_0^3}$$

Where M is Hartmann number/magnetic field parameter, P_r is Prandtl number, K_p is the permeability parameter, G_r is Grashof number for heat transfer.

The governing boundary layer equations (2) and (3) in terms of non-dimensional variables becomes (Dropping ' sign for convenience)

$$\frac{\partial u}{\partial t} - v_0 \frac{\partial u}{\partial y} = \left(1 + \alpha \frac{\partial}{\partial t} \right) \frac{\partial^2 u}{\partial y^2} + G_r T - \left(\frac{1}{K_p} + M^2 \right) u \quad (5)$$

$$\frac{\partial T}{\partial t} - v_0 \frac{\partial T}{\partial y} = \frac{1}{P_r} \left(1 + \alpha \frac{\partial}{\partial t} \right) \frac{\partial^2 T}{\partial y^2} - ST \quad (6)$$

Then the non-dimensional boundary conditions are given by

$$u = e^{i\omega t}, T = 1 \text{ at } y = 0$$

$$u \rightarrow 0, T \rightarrow 0 \text{ as } y \rightarrow \infty \quad (7)$$

3. SOLUTION OF THE PROBLEM

For solving the equations (5) and (6) the velocity and temperature fields are assumed to be of the following forms

$$u = u_0 + u_1 e^{i\omega t} \quad (8)$$

$$\text{And } T = T_0 + T_1 e^{i\omega t} \quad (9)$$

Where $u_i, T_i (i = 0,1)$ are functions of y only.

Substituting the expressions for u and T from (8) and (9) respectively in equations (5) and (6) and separating harmonic and non-harmonic terms the following equations are obtained

$$\frac{d^2 u_0}{dy^2} + v_0 \frac{du_0}{dy} - \left(\frac{1}{K_p} + M^2 \right) u_0 = -G_r T_0 \quad (10)$$

$$\frac{d^2 u_1}{dy^2} + \frac{v_0}{1 + i\alpha\omega} \frac{du_1}{dy} - \frac{\left(\frac{1}{K_p} + M^2 + i\omega \right)}{1 + i\alpha\omega} u_1 = \frac{-G_r T_1}{1 + i\alpha\omega} \quad (11)$$

$$\frac{d^2 T_0}{dy^2} + P_r v_0 \frac{dT_0}{dy} + P_r S T_0 = 0 \quad (12)$$

$$\frac{d^2 T_1}{dy^2} + \frac{P_r v_0}{1 + i\alpha\omega} \frac{dT_1}{dy} + \frac{P_r (S - i\omega)}{1 + i\alpha\omega} T_1 = 0 \quad (13)$$

The corresponding boundary conditions are given by

$$u_0 = 0, u_1 = 1 \text{ at } y = 0 \quad (14)$$

$$u_0 = 0, u_1 = 0 \text{ at } y \rightarrow \infty \quad (15)$$

$$T_0 = 1, T_1 = 0 \text{ at } y = 0 \quad (16)$$

$$T_0 = T_1 = 0 \text{ at } y \rightarrow \infty \quad (17)$$

The solutions of the equations (10) – (13) under the boundary conditions (14) to (17) are obtained as

$$u_0 = \frac{G_r}{m_1^2 + v_0 m_1 - \left(\frac{1}{K_p} + M^2 \right)} \left[e^{\frac{-v_0 - \sqrt{v_0^2 + 4 \left(\frac{1}{K_p} + M^2 \right)}}{2} y} - e^{m_1 y} \right] \quad (18)$$

$$u_1 = e^{\left\{ \frac{v_0}{1 + i\alpha\omega} - \sqrt{\left(\frac{v_0}{1 + i\alpha\omega} \right)^2 + \frac{4 \left(\frac{1}{K_p} + M^2 + i\omega \right)}{1 + i\alpha\omega}} \right\} y} \quad (19)$$

$$T_0 = e^{m_1 y} \quad (20)$$

$$T_1 = 0 \quad (21)$$

$$\text{Where } m_1 = \frac{-P_r v_0 - \sqrt{P_r^2 v_0^2 - 4 P_r S}}{2}$$

Therefore the equations (8) and (9) yield the velocity and temperature fields as

$$u = \frac{G_r}{m_1^2 + v_0 m_1 - \left(\frac{1}{K_p} + M^2 \right)} \left[e^{\frac{-v_0 - \sqrt{v_0^2 + 4 \left(\frac{1}{K_p} + M^2 \right)}}{2} y} - e^{m_1 y} \right]$$

$$+ e^{\frac{-v_0 - U - \alpha\omega V}{1 + \alpha^2 \omega^2} y} \cos \left(\omega t + \frac{\alpha\omega v_0 + \alpha\omega U - V}{1 + \alpha^2 \omega^2} y \right) \quad (22)$$

$$T = e^{m_1 y} \quad (23)$$

$$\text{Where } U = \frac{1}{\sqrt{2}} \sqrt{\sqrt{P^2 + Q^2} + P}, V = \frac{1}{\sqrt{2}} \sqrt{\sqrt{P^2 + Q^2} - P},$$

$$P = v_0^2 + 4 \left(\frac{1}{K_p} + M^2 - \alpha\omega^2 \right), Q = 4 \left(\omega + \frac{\alpha\omega}{K_p} + M^2 \alpha\omega \right)$$

The skin friction at the plate is given by

$$\tau = \left(\frac{\partial u}{\partial y} \right)_{y=0} = \frac{G_r}{m_1^2 + v_0 m_1 - \left(\frac{1}{K_p} + M^2 \right)} \left[\frac{-v_0 - \sqrt{v_0^2 + 4 \left(\frac{1}{K_p} + M^2 \right)}}{2} - m_1 \right]$$

$$- \frac{v_0 + U + \alpha\omega V}{1 + \alpha^2 \omega^2} \cos \omega t - \sin \omega t \frac{\alpha\omega v_0 + \alpha\omega U - V}{1 + \alpha^2 \omega^2} \quad (24)$$

The heat flux at the plate in terms of Nusselt number is given by

$$N_u = \left(\frac{\partial T}{\partial y} \right)_{y=0} = m_1 = \frac{-P_r v_0 - \sqrt{P_r^2 v_0^2 - 4 P_r S}}{2} \quad (25)$$

4. RESULTS AND DISCUSSION

In present paper a study on the free convective flow of an unsteady incompressible electrically conducting second grade fluid through a porous medium occupying semi-infinite region over a porous oscillating plate in presence of transverse magnetic field with suction has been made. The constitutive equation for the stress considered here is related to second grade fluid. The expressions for velocity field, temperature field are obtained analytically by solving the field equations. The skin friction and heat flux at the plate are found out as partial derivatives of velocity and temperature fields respectively. The effects of the flow parameters K_p, M, P_r, S, v_0 and α on velocity field u have been discussed with the help of **Figures 1-5** and **Figure 11** respectively. The effects of v_0 and P_r, S on temperature field T have been illustrated graphically with the help of **Figures 6-8**. Furthermore the influence of M on stress field τ is presented pictorially in **Figure 9**. It is observed that the stress field related to the flow changes with the variation of the flow parameters.

In **Figure 1** velocity field is depicted against y for different values of K_p . The figure shows that velocity field sharply

decreases from a finite value as y increases and approaches zero. By comparing the velocity curves in the figure it is observed that a growing permeability parameter K_p leads to accelerate the magnitude of velocity field at all points. **Figure 2** shows that the increase in Hartmann number M results in decrease in velocity field. This is expected, as the magnetic field has a retarding influence on the flow fields. In **Figure 3** velocity field is depicted against y for different values of Prandtl number P_r . The figure shows that the velocity field decreases with increase in P_r . **Figure 4** discusses the effects of heat source parameter S on the velocity field u . From the figure it can be seen that the velocity field u sharply decreases from a finite value as y increases near the boundary layer. The figure shows that increase in the parameter S leads to decrease in the magnitude of the velocity field. The velocity field is depicted against y for different values of suction velocity v_0 in **Figure 5**. The velocity field decreases with the increase in v_0 . The velocity decreases sharply from a definite velocity with increase in y and becomes zero finally. In **Figure 6** temperature field is plotted against y for different values of suction velocity v_0 . The temperature decreases sharply with the increase in y near the boundary layer and approaches zero. Temperature field T decreases with the increase in suction velocity v_0 very near to the plate. Temperature field is depicted against y for different values of P_r in **Figure 7**. From there it can be seen that an increase in y leads to sharp decrease in T and T approaches zero finally. As the Prandtl number takes higher values the temperature field T decreases very near to the plate. **Figure 8** shows the plot of temperature field T against y for different values of heat source parameter S . The temperature field T decreases sharply with increase in y near the boundary layer and approaches zero. T decreases as the heat source parameter S takes higher values near the plate. The skin friction τ is depicted against ω for different values of M in **Figure 9**. It is evident from the figure that the stress curves are oscillatory in nature. There are points of separation at which the stress τ becomes zero. The figure shows that for the flow of non-Newtonian fluid in presence of magnetic field the amplitude of the stress curve decreases with the increase in ω as shown in **Figure 9**. In **Figure 10** the velocity field is plotted against time t for different values of y . The figure shows that the velocity curves are oscillatory in nature. There are points about which the magnitude of amplitude of oscillation of the velocity curve changes its direction for different values of y periodically. The amplitude of oscillation of the velocity curve decreases for increasing values of y on the left of that points and then on the right of that points the direction of amplitude changes periodically and as y takes larger values the amplitude of oscillation of the velocity curve becomes zero. **Figure 11** shows the effects of normal stress moduli α on the velocity field u . There are sharp decrease in the velocity field u with the increase in y . It can be observed that as α increases the magnitude of the velocity field increases. The velocity curve for the flow of Newtonian fluid has also been plotted in the figure. It is also seen that there are negative velocities in the boundary layer and for large values of α , there is an incipient flow reversal near the plate. In **Table 1** the stress values for different values of magnetic field parameter M and frequency of oscillation ω with $G_r = 1, v_0 = 1, K_p = 10, t = 0.4, P_r = 0.5, S = 5, \alpha = 0.0$ are computed. From the figure it can be seen that the stress field τ oscillates with ω for fixed values of M and it is the case seen in **Figure 9**. It is also evident from the Table that the stress field is monotonic function of M for fixed value of ω .

5. FIGURES

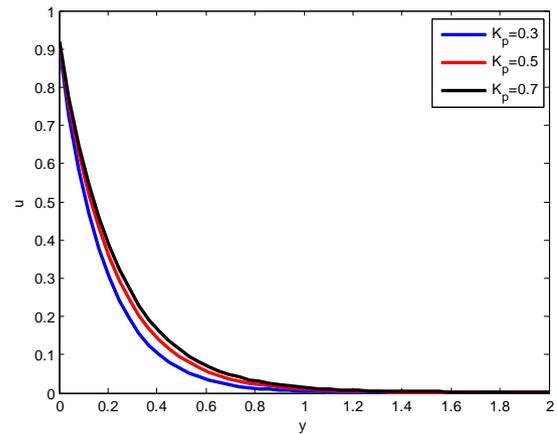


Figure 1. Velocity profile is depicted against y for different values of permeability parameter K_p with $G_r = 1, M = 1, \alpha = 0.3, v_0 = 1, \omega = 0.2, t = 0.4, P_r = 0.3, S = 0.5$

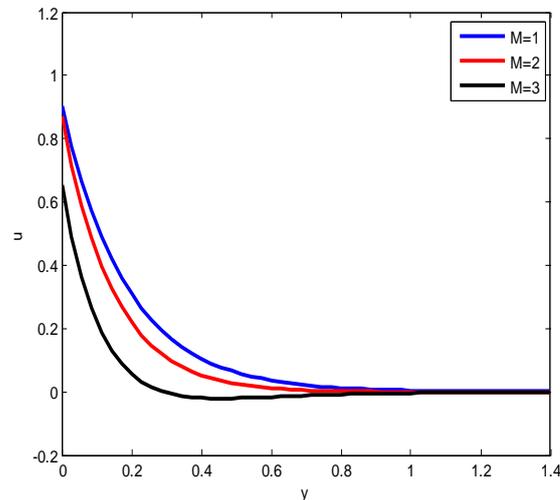


Figure 2. Velocity profile is depicted against y for different values of magnetic field parameter M with $G_r = 1, v_0 = 1, K_p = 0.3, \alpha = 0.3, \omega = 0.2, t = 0.4, P_r = 5, S = 0.5$

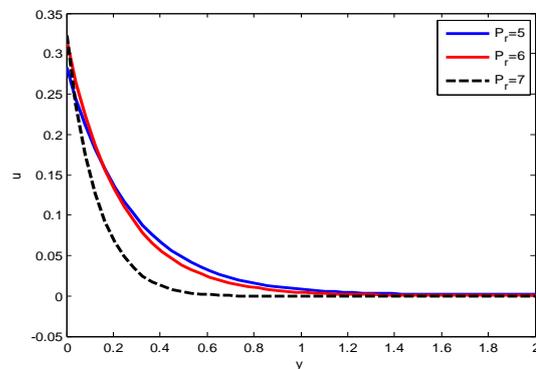


Figure 3. Velocity profile is depicted against y for different values of Prandtl number P_r with $G_r = 1, v_0 = 1, M = 1, \alpha = 0.3, K_p = 0.3, \omega = 0.2, t = 0.4, S = 0.5$

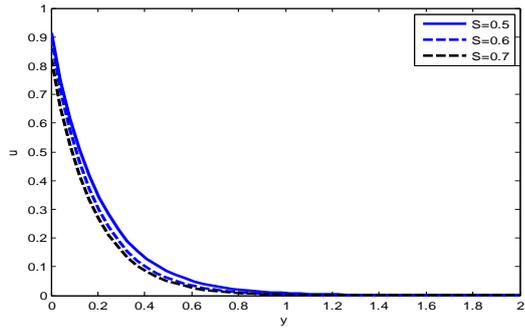


Figure 4. Velocity profile is depicted against y for different values of heat source parameter S with $G_r = 1, M = 1, \alpha = 0.3, v_0 = 1, K_p = 0.3, \omega = 0.2, t = 0.4, P_r = 5$

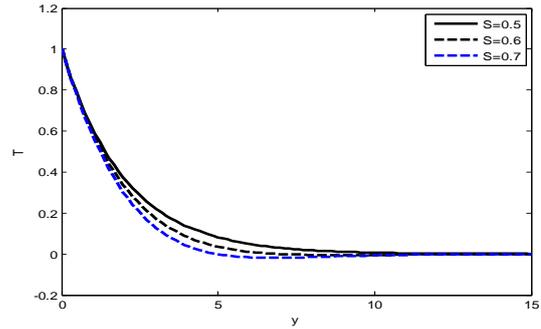


Figure 8. Temperature T is depicted against y for different values of heat source parameter S with $P_r = 0.5, v_0 = 2.0$

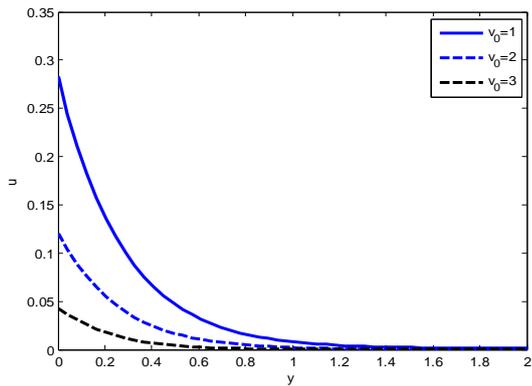


Figure 5. Velocity profile is depicted against y for different values of suction velocity v_0 with $G_r = 1, K_p = 0.3, M = 1, \alpha = 0.3, \omega = 0.2, t = 0.4, P_r = 5, S = 5.0$

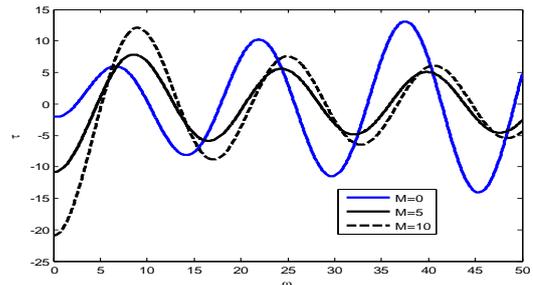


Figure 9. Skin friction τ is depicted against ω for different values of magnetic field parameter M with $K_p = 0.3, G_r = 1, t = 0.4, \alpha = 0.3, P_r = 0.5, S = 5, v_0 = 1.0$

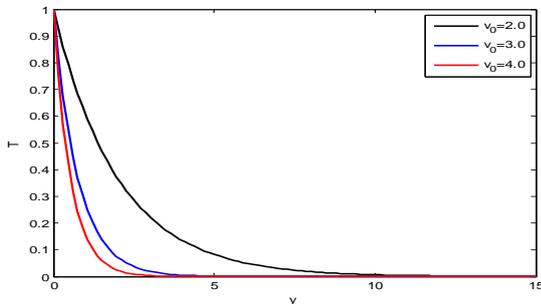


Figure 6. temperature T is depicted against y for different values of suction velocity v_0 with $P_r = 0.5, S = 0.5$

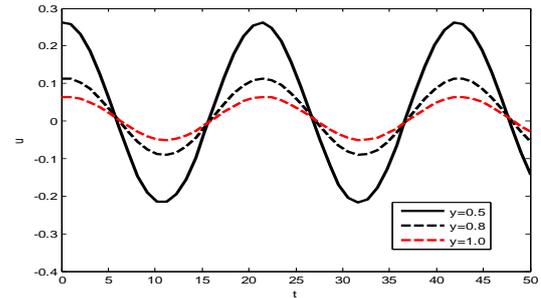


Figure 10 The velocity field is plotted against time t for different values of y with $G_r = 1.0, K_p = 3, M = 0.5, \alpha = 0.2, v_0 = 1.0, \omega = 0.5, P_r = 0.5, S = 5$

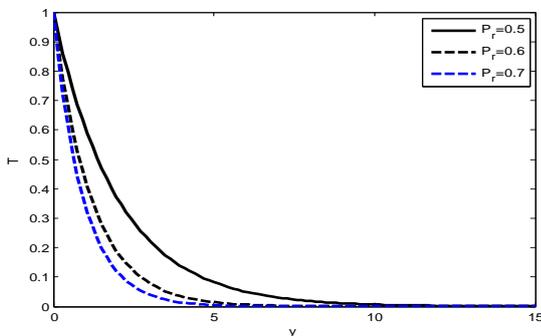


Figure 7. Temperature T is depicted against y for different values of Prandtl number P_r with $v_0 = 2.0, S = 0.5$

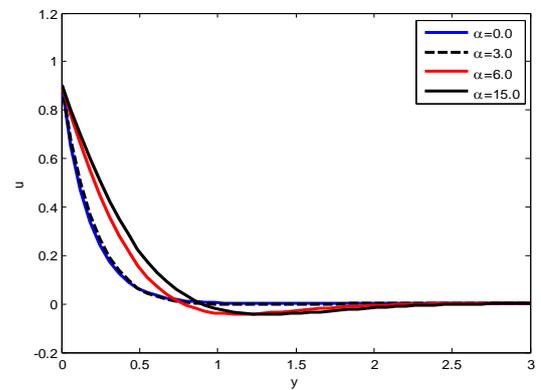


Figure 11. The velocity is depicted against y for different values of normal stress moduli α with $G_r = 1, M = 1, v_0 = 1, K_p = 0.3, \omega = 0.2, t = 0.4, P_r = 5, S = 0.5$

Table 1. Shear stress at the plate $y = 0$ with $G_r = 1, v_0 = 1, K_p = 10, t = 0.4, P_r = 0.5, S = 5, \alpha = 0.0$

$\omega \backslash M$	0	5	10	15
0	-2.0495	-10.8809	-20.9351	-30.9563
5	4.6893	5.7143	9.3098	13.2809
10	0.4363	6.0711	13.1109	19.8460
15	-7.6599	-11.6245	-20.5679	-30.0157
20	7.4103	5.5824	5.1403	5.9020

6. ACKNOWLEDGEMENTS

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