Fibonacci Graceful Labeling of some Star Related Graphs

R. Uma
Dept. of Mathematics
Sree Saraswathi Thyagaraja College,
(Autonomous),
Pollachi, India

D. Amuthavalli
Research Scholar, Dept. of Mathematics
Sree Saraswathi Thyagaraja college,
(Autonomous), Pollachi.

ABSTRACT
The function $f: V(G) \longrightarrow \{0,1,2,\dots,F_q\}$ (where F_q is the q^{th} Fibonacci number) is said to be Fibonacci graceful if the induced edge labeling $f^*: E(G) \longrightarrow \{F_1, F_2, \dots, F_q\}$ defined by $f^*(uv) = |f(u) - f(v)|$ is bijective. In this paper an analysis is made on Comb, Subdivision of Bistar $B_{2,n}W_i$, Coconut Tree and Jellyfish graphs under Fibonacci graceful labeling.

AMS Classification
05C78

Keywords
Comb, Subdivision of Bistar, Jelly fish, Coconut Tree, Fibonacci graceful labeling and Fibonacci graceful graph.

1. INTRODUCTION
In graph labeling vertices or edges or both assigned values subject to the conditions. There are many types of labeling. More number of labeling techniques found their starting point from "graceful labeling" introduced by Rosa (1967)[8] . J.A. Gallian [2] studied a complete survey on graph labeling. David.W. and Anthony. E. Baraaukas [1] have investigated the cycle structure of Fibonacci graceful mapping. A Fibonacci graceful labeling and Super Fibonacci graceful labeling have been introduced by Kathiresan and Amutha [4] in 2006. S.S. Sandhya, C. David Raj, C. Jayasekaran [8] has proved that the crown and combs are harmonic mean graphs. S. Karthikeyan, S. Navaneethakrishnan and R. Sridevi [3] proved subdivision of bistar and are Fibonacci irregular graphs. K. Manimegalai, K. Thirusangu [6] , are proved Comb, Jellyfish, Coconut tree are pair sum graphs . N.Murugesan and R.uma [5] have obtained some Cycle- related graphs under Fibonacci graceful.

2. DEFINITIONS
Definition 2.1. The function $f: V(G) \longrightarrow \{0,1,2 \dots . F_q\}$ (where F_q is the q^{th} Fibonacci number) is said to be Fibonacci graceful if the induced edge labeling $f^*: E(G) \longrightarrow \{F_1, F_2, F_3, \dots F_q\}$ defined by $f^*(uv) = |f(u) - f(v)|$ is bijective.

Definition 2.2. The Comb is graph obtained by joining a single pendent edge to each vertex of a path . It is denoted by P_nAK_1.

Definition 2.3. Let u, v be the central vertices of $B_{2,n}$. Let u_1, u_2 be the vertices joined with u and $v_1, v_2, \dots, v_n$ be the vertices joined with v. Let $w_1, w_2, \dots, w_n$ be the vertices of the subdivision of edges $vv_i (1 \le i \le n)$ respectively and is denoted by $(B_{2,n}\ W_i)\ 1 \le i \le n$.

Definition 2.4 The Jelly fish graph $J(m, n)$ is obtained by joining a 4-cycle whose vertices are v_1, v_2, v_3, v_4 with vertices v_1 and v_3 defined by an edge and appending m pendent edges to v_2 and n pendent edges to v_4.

Definition 2.5 A Coconut Tree $CT(m\ , n)$ is the graph obtained from the path P_m by appending n new pendent edges at an end vertex of P_m .

3. RESULTS
3.1 Theorem
Combs are Fibonacci graceful for $n > 2$.

Proof:

Let $G = (V, E, f)$ be the comb graph P_nAK_1. The order of the comb is $p = 2n$ and the size of the comb is $q = 2n$. Let $V(G) = \{u_1, \dots, u_n, v_1, v_2, \dots, v_n\}$ where $u_1, u_2, \dots, u_n$ are the vertices of the path P_n and and $v_1, v_2, \dots, v_n$ are the pendant vertices attaching by the path P_n. The edge set $E(G) = \{e_i, e_{ij}\}$ where $e_i = (u_i v_i)$ and $e_{ij} = (u_i u_j)$.

Define the function $f: V \longrightarrow \{0,1,2,\dots,F_q\}$ as follows

Case 1: For $n = 3$

$f(u_2) = 0$

$f(v_i) = F_{2i} \qquad for\ i = 1,2$

$f(u_{2i-1}) = F_{2i+1} \qquad for\ i = 1,2$

$f(v_3) = 4$.

Case 2 : For $n \ge 4$

$f(u_n) = 0, f(u_1) = 2, \ f(u_2) = 5,$

$f(v_i) = F_{2i}, \qquad i = 3,4, \dots, n-1$

$f(u_i) = F_{2i+1}, \qquad i = 3,4, \dots, n-1$

$f(v_n) = 1, \ \ f(v_1) = 3\ , \ f(v_2) = 7.$

Then the above defined function f admits Fibonacci graceful labeling. Hence Combs are Fibonacci graceful .

The generalized graph P_nAK_1 is shown in figure 1

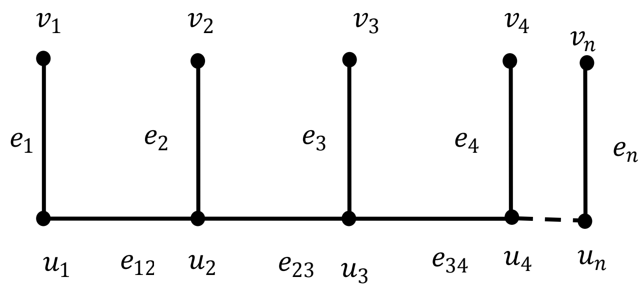

Figure 1. The comb graph P_nAK_1

3.2 Example

The Comb P_5AK_1 is shown in figure 2. In P_5AK_1 , the order is $p = 10$ and the size is $q = 10$.

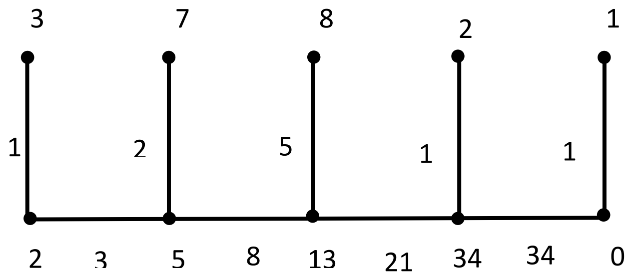

Figure 2. Comb P_5AK_1

3.3 Theorem

The Subdivision of Bistar $B_{2,n}\ w_i, 1 \le i \le n$ is Fibonacci graceful .

Proof:

Let $B_{2,n}\ w_i$,$1 \le i \le n$ be the subdivision of Bistar, By the definition of $B_{2,n}\ w_i$,$1 \le i \le n$ the order and size is $p = 2n + 4$ and $q = 2n + 3$ respectively.The vertex set $V(G) = \{u, v, u_1, u_2, v_1, \dots, v_n, w_1, \dots, w_n\}$. Let u, v be the central vertices of $B_{2,n}$ and u_1, u_2 be the adjacent vertices of u and w_i 's are the vertices which is subdivide the edge $e = (v, v_i)$. Hence $v_i's$ are adjacent to $w_i's$. The edge set $E(G) = \{e_i^*, e_0, e_i, e_{ii}\}$ where $e_1^* = (u, u_i), e_0 = (u, v), e_i = (v, w_i)$ and $e_{ii} = (w_i, v_i)$.

Now let us define the function $f: V(G) \to \{0,1,2, \dots, F_q\}$ as follows

$f(u) = 0,\ \ f(v) = 1$

$f(u_i) = F_{q-(i-1)}, \qquad i = 1,2$

$f(w_i) = F_{(i+1)} + 1, \qquad i = 1,2, \dots, n$

$f(v_i) = w_i + F_{(q-2)-(i-1)},\ i = 1,2, \dots, n$

Then the above defined function f admits Fibonacci graceful labeling.

Hence the graph $B_{2,n}\ W_i$,$1 \le i \le n$ is Fibonacci graceful .

The generalized graph $B_{2,n}\ W_i$ $1 \le i \le n$ is shown in figure 3.

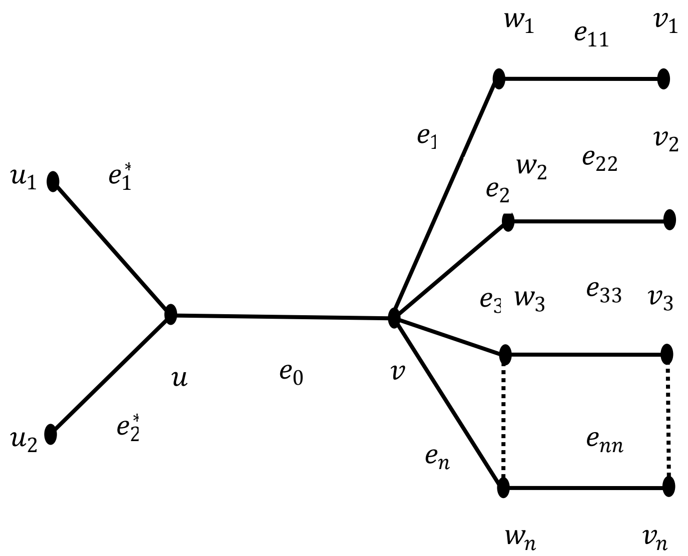

Figure 3. Subdivision of Bistar of $B_{2,n}\ W_i$ $1 \le i \le n$.

3.4 Example

The graph is given in figure 4 is $B_{2,4}\ W_4$,$1 \le i \le n.$ The order and size is $p =$ 12 and $q = 11$ respectively .

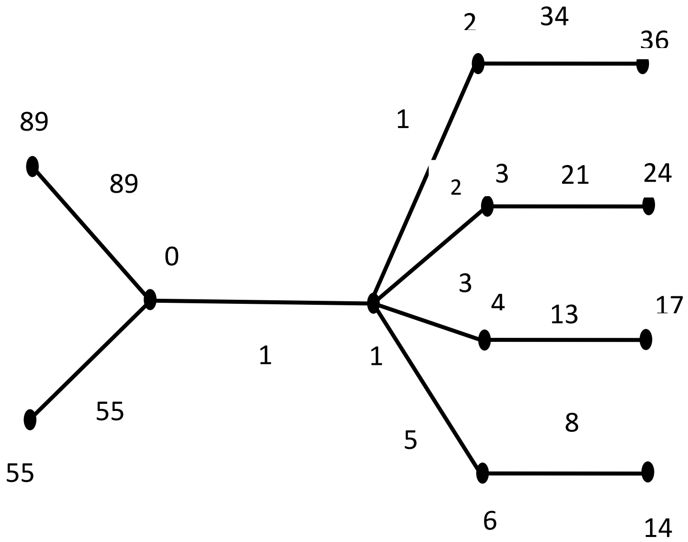

Figure 4. Subdivision of Bistar of $B_{2,4}\ W_4$

3.5 Theorem

The Jelly fish graph $J(m, n)$ is Fibonacci graceful.

Proof

Let $J(m, n)$ be the Jellyfish graph. The order of the $J(m, n)$ is $p = 4 + m + n$ and the size of the $J(m, n)$ is $q = 5 + m + n$. By the definition of $J(m, n)$, the vertex set $V = \{v_1, \dots, v_4, u_1, u_2, \dots, u_n, w_1, \dots, w_n\}$. Let v_1, v_2, v_3, v_4 be the vertices of the cycle C_4. Let $u_1, u_2, \dots, u_m$ be the pendant vertices attaching by v_2 and let $w_1, w_2, \dots, w_n$ be the pendant vertices by v_4 . The edge set $E(G) = \{e_{ij}^*, e_{2i}, e_{1i}, e_3\}$ where $e_{ij}^* = (v_i, v_j)$, $e_{2i} = (v_2, u_i)$, $e_{1i} = (v_1, u_i)$ and $e_{4i} = (v_4, w_i)$.

Now let us define the function $f: V \longrightarrow \{0,1,2,\dots,F_q\}$ as follows

$f(v_2) = 0,\ f(v_1) = 5$

$f(u_i) = F_{q-(i-1)} \qquad i = 1,2,\dots,m$

$f(w_i) = F_{(q-m)-(i-1)}+4 \qquad i = 1,2,\dots,n$

$f(v_i) = i \qquad i = 3,4$

Then the above defined function f admits Fibonacci graceful labeling.

Hence Jelly fish graphs are Fibonacci graceful graphs.

The generalized graph $J(m,n)$ is shown in figure 5.

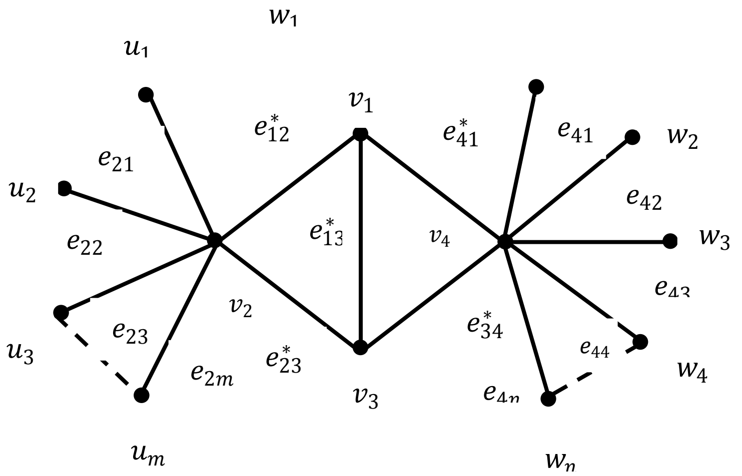

Figure 5. The Jelly fish graph $J(m,n)$

3.6 Example

The Jelly fish graph $J(3,5)$ is shown in figure 6. In (m,n), the order is $p = 12$ and the size is $q = 13$.

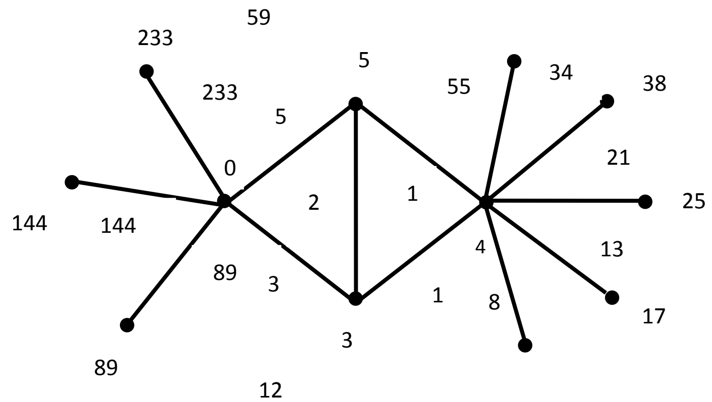

Figure 6. Jelly fish graph $J(3,5)$

3.7 Theorem

The Coconut Trees $CT(m,n)$ are Fibonacci graceful for all n and $m \geq 6$.

Proof :

Let $CT(m,n)$ be the Coconut tree. The order of the $CT(m,n)$ is $p = m+n$ and the size $CT(m,n)$ is $q = (m-1)+n$. By the definition of $CT(m,n)$, the vertex set $V(G) = \{u_1,u_2,\dots,u_m,\ v_1,v_2,\dots,v_n\}$. Let $u_1,u_2,\dots,u_m$ be the vertices of the path P_m and $v_1,v_2,\dots,v_n$ be the pendant vertices attaching by the end vertex of the path P_m. The edge set $E(G) = \{e_{mi}, e_{ij}\}$ where $e_{mi} = (u_m v_i)$ and $e_{ij} = (u_i u_j)$.

Define the function $f: V \longrightarrow \{0,1,2,\dots,F_q\}$ as follows

$f(u_m) = 0$

$f(u_1) = F_{m-2} + 1$

$f(v_i) = F_{m+(i-1)} \qquad i = 1,2,\dots,n$

$f(u_i) = F_i \qquad i = 2,3,\dots,m-1$

Then the above defined function f admits Fibonacci graceful labeling.

Hence Coconut Trees are Fibonacci graceful.

The generalized graph is $CT(m,n)$ is shown in figure 7.

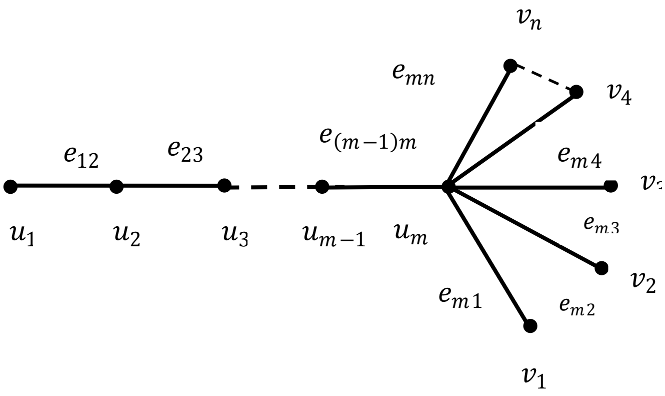

Figure 7.The Coconut Tree $CT(m,n)$

3.8 Example

The Coconut Tree CT (6,5) is shown in Figure 8. In $CT(6,5)$ the order is $p = 11$ and the size is $q = 10$.

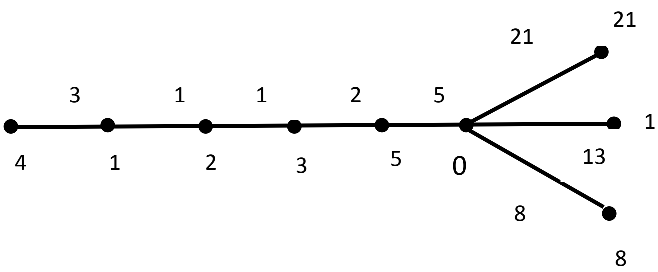

Figure 8. Coconut tree CT (6, 5)

4. CONCLUSION

In this paper, we have shown that Comb, Subdivision of Bistar, Jellyfish and Coconut Trees are Fibonacci graceful. In future the same process will be analyzed for other graphs.

5. ACKNOWLEDGEMENT

The project is supported by UGC under the minor research project scheme 2014-2015.

6. REFERENCES

[1] David.W and Anthony.E.Barakaukas, "Fibonacci graceful graphs".

[2] Joseph A. Gallian, "A Dynamic survey of graph labeling", the electronic journal of combinatorics,(2013).

[3] Kathiresan K. and Amutha S., Fibonacci graceful graphs, Ph.D. Thesis, Madurai Kamaraj University, (October 2006).

[4] S. Karthikeyan, S. Navaneethakrishnan. and R. Sridevi, "Total edge Fibonacci irregular labeling of some star graphs", International journal of Mathematics and Soft computing , Vol 5, No.1 (2015), 73-78.

[5] Murugesan. N and Uma. R, "A conjecture on amalgation of graceful graph star graphs", Int. J. Cont. Math. Science,Vol.7,(2012).

[6] K. Manimekalai and K. Thirusangu, "Pair Sum Labeling of Some Special Graphs", International Journal of Computer Applications (0975-8887) Volume 69, No. 8, May 2013.

[7] S.S Sandhya, C. David Raj and C. Jayasekaran, "Some new results on Harmonic mean Graphs", International Journal of Mathematics Archive-4(5), 2013, 240-245.

[8] Rosa .A, "On certain valuation of vertices of graph",(1967).