Application of Intuitionistic Fuzzy Multisets in Appointment Process

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ABSTRACT
In this paper, a precise note on intuitionistic fuzzy multisets is given and the concept is applied to appointment process. This process was carried out assuming three sets of 10-man committees screened five candidates vying for positions in an organization independently to obtain intuitionistic fuzzy multi-data. The obtained data are compared with the organization requirements of appointments via a new distance measure.

Keywords
Fuzzy multisets, intuitionistic fuzzy sets, intuitionistic fuzzy multisets, intuitionistic fuzzy sets, appointment process, distance measures.

1. INTRODUCTION
The invention of fuzzy sets by Zadeh [22] as the generalization of crisp sets proffers solution to uncertainties in decision making, artificial intelligent, engineering and computer programming. Zadeh is single-handedly responsible for the early development of fuzzy set which he defined as the collection of object with graded membership. The concept of characteristic function in crisp set was replaced by membership function which take elements from a universe X to form image in a closed interval [0, 1]. Yager [21] generalized the concept of fuzzy sets to form a set called fuzzy multiset. By extending fuzzy set in such a way that it contains two functions called the membership function and non-membership function with hesitation zone, Atanassov [1] proposed the idea of intuitionistic fuzzy set which is more viable in tackling uncertainties as seen in [2], [7]. Shinoj and Sunil [18] introduced intuitionistic fuzzy multisets from the combination of the concepts of fuzzy multisets and intuitionistic fuzzy sets. However, intuitionistic fuzzy multiset is more or less the generalization of intuitionistic fuzzy set or the extension of fuzzy multiset [13]. Some researchers have studied the concept of intuitionistic fuzzy multisets and its applications in medical diagnosis, binomial distribution, pattern recognition, robotics science, etc. as seen in [3], [4], [5], [6], [8], [9], [10], [11], [14], [15], [16], [17], [19], [20].

Owning to the fact that, the idea of intuitionistic fuzzy multisets is more accurate in handling imprecision and uncertainties, we seek to apply its concept to appointment procedure or process as an improvement of the work in [7] using a new distance measure in [12].

2. INTUITIONISTIC FUZZY MULTISETS
Definition 2.1[1]. Let X be nonempty set. An intuitionistic fuzzy set (IFS) A in X is an object having the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$, where the functions $\mu_A(x), \nu_A(x): X \longrightarrow [0,1]$ define the degree of membership and degree of non-membership of the element $x \in X$ to the set A. For every $x \in X, 0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

Furthermore, $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ is the intuitionistic fuzzy set index or hesitation margin and is the degree of indeterminacy concerning the membership of x in A, then $\pi_A(x) \in [0,1]$. Whenever $\pi_A(x) = 0$, an IFS reduces automatically to fuzzy set.

Definition 2.2 [5]. Let X be nonempty set. An IFMS A drawn from X is given as $A = \{\langle \mu_A^1(x), \ldots, \mu_A^n(x), \ldots, \nu_A^1(x), \ldots, \nu_A^n(x), \ldots\rangle : x \in X\}$ where the functions $\mu_A^i(x), \nu_A^i(x): X \rightarrow [0,1]$ define the belongingness degrees and the non-belongingness degrees of A in X such that $0 \leq \mu_A^i(x) + \nu_A^i(x) \leq 1$ for $i = 1, \ldots$. If the sequence of the membership functions and non-membership (belongingness functions and non-belongingness functions) have only n-terms (i.e. finite), n is called the dimension of A. Consequently $A = \{\langle \mu_A^1(x), \ldots, \mu_A^n(x), \nu_A^1(x), \ldots, \nu_A^n(x)\rangle : x \in X\}$ for $i = 1, \ldots, n$. when no ambiguity arises, we write $A = \{\langle \mu_A^i(x), \nu_A^i(x)\rangle : x \in X\}$.

For each IFMS A in X, $\pi_A^i(x) = 1 - \mu_A^i(x) - \nu_A^i(x)$ is the intuitionistic fuzzy multisets index or hesitation margin of x in A. The hesitation margin $\pi_A^i(x)$ for each $i = 1, \ldots, n$ is the degree of non-determinacy of $x \in X$, to the set A and $\pi_A^i(x) \in [0,1]$. Similarly, $\pi_A^i(x)$ as in IFS, is the function that expresses lack of knowledge of whether $x \in A$ or $x \notin A$. Then, $\mu_A^i(x) + \nu_A^i(x) + \pi_A^i(x) = 1$ for each $i = 1, \ldots, n$.

We henceforth denote the set of all intuitionistic fuzzy multisets over X as IFMS(X). Also, we denote an intuitionistic fuzzy multiset A as $A = \left(\mu_A^i(x), \nu_A^i(x)\right)$ for simplicity.

Definition 2.3. Let $\{A_j\}_{j \in J}$ be an arbitrary family of IFMSs in X, where $A = \left(\mu_{A_j}^i(x), \nu_{A_j}^i(x)\right) \in IFMS(X)$ for each $j \in J$, we define

$\bigcap_{j \in J} A = \left(\wedge \mu_{A_j}^i(x), \vee \nu_{A_j}^i(x)\right)$ and

$\bigcup_{j \in J} A = \left(\vee \mu_{A_j}^i(x), \wedge \nu_{A_j}^i(x)\right) \forall x \in X.$

Definition 2.4[13]. Let X be nonempty. If $A \in IFMS(X)$, then;

(i) $\Box A = \left\{\langle x, \mu_A^i(x), 1 - \mu_A^i(x)\rangle : x \in X\right\}$

(ii)) $\Diamond A = \left\{\langle x, 1 - \nu_A^i(x), \nu_A^i(x)\rangle : x \in X\right\}$.

Definition 2.5. For any two IFMSs A and B drawn from X, the following operations hold.

(i)Inclusion:$A \subseteq B \Longrightarrow \mu_A^i(x) \le \mu_B^i(x)$ and $\nu_A^i(x) \ge \nu_B^i(x)\ \forall\ x \in X$.

(ii)Equality:$A = B \Longrightarrow \mu_A^i(x) = \mu_B^i(x)$ and $\nu_A^i(x) = \nu_B^i(x)\ \forall\ x \in X$.

(iii) Complement: $A^c = \nu_A^i(x), \mu_A^i(x)\ \forall\ x \in X$.

(iv)Union:$A \cup B = \mu_A^i(x) \vee \mu_B^i(x), \nu_A^i(x) \wedge \nu_B^i(x)\ \forall\ x \in X$.

(v)Intersection:$A \cap B = \mu_A^i(x) \wedge \mu_B^i(x),\ \nu_A^i(x) \vee \nu_B^i(x)\ \ \forall\ x \in X$.

(vi)Addition:$A \oplus B = \mu_A^i(x) + \mu_B^i(x) - \mu_A^i(x)\mu_B^i(x),\ \nu_A^i(x)\ \nu_B^i(x)\ \forall\ x \in X$.

(vii)Multiplication: $A \otimes B = \mu_A^i(x)\mu_B^i(x),\ \ \nu_A^i(x) + \nu_B^i(x) - \nu_A^i(x)\ \nu_B^i(x)\ \forall\ x \in X$.

(viii)Difference:$A - B = \mu_A^i(x) \wedge \nu_B^i(x), \nu_A^i(x) \vee \mu_B^i(x)\ \forall\ x \in X$.

Definition 2.6. Let $A \in IFMS(X)$, we define the support of A and the cross point of A as follow.

(i)Supp(A)= $\{x \in X:\ \mu_A^i(x) > 0,\ \nu_A^i(x) < 1\}, \forall\ x \in X$.

(ii) The crossover point of A is $\{x \in X:\ \mu_A^i(x) = \frac{1}{2},\ \nu_A^i(x) = \frac{1}{2}\}\ \forall\ x \in X$.

Definition 2.7. Let $A, B \in IFMS(X)$, the new distance measure between A and B is given as

$$d(A,B) = \frac{1}{2n}\sum_{i=1}^{n}\Big[\,|\mu_A^i(x_i) - \mu_B^i(x_i)| + \big|\,|\mu_A^i(x_i) - \nu_A^i(x_i)| - |\mu_B^i(x_i) - \nu_B^i(x_i)|\,\big| + \big|\,|\mu_A^i(x_i) - \pi_A^i(x_i)| - |\mu_B^i(x_i) - \pi_B^i(x_i)|\,\big|\Big]\ \forall\ x \in X.$$

3. INTUITIONISTIC FUZZY MULTISETS AND APPOINTMENT PROCESS

Suppose a new organization wants to appoint members into its management board or an old organization intends to reshuffle it cabinet, the challenge is how to avoid fitting round peg in a flat hole when the to be managers seem to be relatively qualified. In [7], this situation was handled by using intuitionistic fuzzy sets approach. It is assumed that a 10-member committee scored or screened the candidates using intuitionistic fuzzy values.

Since it will be difficult to ascertain how impartial the 10-member committee is, in this method we assume the candidate are screened by three set of 10-member committees such that each of the committees are independent. By compiling the results from each of the three committees, intuitionistic fuzzy multi-data is obtained.

Let $C = \{c_1, c_2, c_3, c_4, c_5\}$ be the set of candidates to be appointed, let Q ={honesty, team spirit, hardworking, transparency, academic fitness, experience, accountability, probity} be the set of qualifications expected from the candidates, and $P = \{p_1, p_2, p_3, p_4, p_5\}$ be the set of positions to be occupied. Assumed after the committees screened the candidates, the following results were obtained.

Table 1. Candidates and Qualifications

	honesty	team spirit	hard working	transparency	academic fitness	experience	accountability	probity
c_1	(0.50,0.40) (0.40,0.35) (0.60,0.45)	(0.80,0.20) (0.90,0.15) (0.70,0.25)	(0.40,0.30) (0.50,0.40) (0.30,0.20)	(0.40,0.50) (0.50,0.40) (0.30,0.60)	(0.60,0.20) (0.50,0.15) (0.70,0.25)	(0.7,0.10) (0.6,0.05) (0.8,0.15)	(0.75,0.10) (0.80,0.15) (0.85,0.05)	(0.65,0.15) (0.70,0.25) (0.75,0.20)
c_2	(0.80,0.20) (0.75,0.25) (0.85,0.15)	(0.60,0.30) (0.50,0.35) (0.70,0.25)	(0.60,0.10) (0.50,0.15) (0.70,0.05)	(0.30,0.50) (0.50,0.40) (0.10,0.60)	(0.80,0.10) (0.75,0.15) (0.85,0.05)	(0.80,0.15) (0.75,0.10) (0.85,0.05)	(0.90,0.00) (0.85,0.00) (0.95,0.00)	(0.80,0.15) (0.85,0.05) (0.75,0.10)
c_3	(0.60,0.30) (0.50,0.40) (0.70,0.20)	(0.70,0.30) (0.60,0.40) (0.80,0.20)	(0.70,0.20) (0.60,0.30) (0.80,0.10)	(0.20,0.50) (0.10,0.60) (0.30,0.70)	(0.85,0.15) (0.90,0.10) (0.95,0.05)	(0.75,0.15) (0.85,0.20) (0.80,0.25)	(0.95,0.15) (0.90,0.10) (0.85,0.05)	(0.65,0.15) (0.70,0.25) (0.75,0.20)
c_4	(0.70,0.10) (0.60,0.10) (0.80,0.10)	(0.80,0.10) (0.75,0.05) (0.85,0.15)	(0.90,0.00) (0.85,0.00) (0.95,0.00)	(0.50,0.40) (0.60,0.35) (0.40,0.45)	(0.55,0.35) (0.45,0.30) (0.50,0.25)	(0.75,0.10) (0.80,0.15) (0.85,0.05)	(0.85,0.00) (0.90,0.00) (0.95,0.00)	(0.65,0.15) (0.70,0.05) (0.75,0.10)
c_5	(0.80,0.10) (0.70,0.05) (0.90,0.15)	(0.50,0.40) (0.45,0.35) (0.55,0.45)	(0.80,0.05) (0.90,0.15) (0.70,0.10)	(0.80,0.20) (0.85,0.15) (0.75,0.25)	(0.60,0.30) (0.50,0.25) (0.70,0.35)	(0.50,0.30) (0.45,0.35) (0.55,0.25)	(0.60,0.25) (0.65,0.35) (0.55,0.30)	(0.60,0.20) (0.50,0.15) (0.70,0.25)

The data in the Table above can be represented by a matrix given as, $\begin{bmatrix} \mu^1 & \nu^1 & \pi^1 \\ \mu^2 & \nu^2 & \pi^2 \\ \mu^3 & \nu^3 & \pi^3 \end{bmatrix}$, where the first row represents an IFMS data for the first committee, second row represents an IFMS data for the second committee, and the third row represents an IFMS data for the third committee, where $\pi^i = 1 - \mu^i - \nu^i$ for $i = 1,2,3$.

Applying the method in [6], the intuitionistic fuzzy multi-data become intuitionistic fuzzy data as shown in the Table below.

Table 2. Candidates and Qualifications

	honesty	team spirit	hard working	transparency	academic fitness	experience	accountability	probity
c_1	(0.5,0.4)	(0.8,0.2)	(0.4,0.3)	(0.4,0.5)	(0.6,0.2)	(0.7,0.1)	(0.8,0.1)	(0.7,0.2)
c_2	(0.8,0.2)	(0.6,0.3)	(0.6,0.1)	(0.3,0.5)	(0.8,0.1)	(0.8,0.1)	(0.9,0.0)	(0.8,0.1)
c_3	(0.6,0.3)	(0.7,0.3)	(0.7,0.2)	(0.2,0.6)	(0.9,0.1)	(0.8,0.2)	(0.9,0.1)	(0.7,0.2)
c_4	(0.7,0.1)	(0.8,0.1)	(0.9,0.0)	(0.5,0.4)	(0.5,0.3)	(0.8,0.1)	(0.9,0.0)	(0.7,0.1)
c_5	(0.8,0.1)	(0.5,0.4)	(0.8,0.1)	(0.8,0.2)	(0.6,0.3)	(0.5,0.3)	(0.6,0.3)	(0.6,0.2)

The Table below is assumed to be the organization requirements for appointment into the five positions or offices.

Table 3. Positions and Qualifications

	honesty	team spirit	hard working	transparency	academic fitness	experience	accountability	probity
p_1	(0.6,0.3)	(0.5,0.4)	(0.7,0.2)	(0.6,0.3)	(0.5,0.3)	(0.5,0.4)	(0.6,0.2)	(0.7,0.3)
p_2	(0.7,0.2)	(0.8,0.1)	(0.7,0.1)	(0.6,0.3)	(0.8,0.1)	(0.8,0.0)	(0.7,0.1)	(0.9,0.0)
p_3	(0.8,0.1)	(0.7,0.2)	(0.8,0.1)	(0.7,0.1)	(0.8,0.1)	(0.8,0.1)	(0.6,0.2)	(0.8,0.2)
p_4	(0.6,0.3)	(0.5,0.3)	(0.6,0.2)	(0.4,0.4)	(0.6,0.1)	(0.5,0.4)	(0.5,0.3)	(0.6,0.3)
p_5	(0.4,0.5)	(0.5,0.2)	(0.5,0.3)	(0.6,0.4)	(0.6,0.3)	(0.5,0.3)	(0.6,0.3)	(0.6,0.1)

Using Def. 2.7, the distance between the positions and the candidates with respect to each of the qualifications is calculated and the Table below is obtained.

Table 4. Candidates and Positions

	p_1	p_2	p_3	p_4	p_5
c_1	0.0438	0.0406	0.0516	0.0406	0.0359
c_2	0.0219	0.0297	0.0313	0.0484	0.0547
c_3	0.0469	0.0344	0.0375	0.0484	0.0578
c_4	0.0438	0.0344	0.0375	0.0531	0.0578
c_5	0.0250	0.0500	0.0320	0.0359	0.0313

From the Table above, the following decision are made on shortest distance bases between the candidates c_i and the positions p_i for i = 1,2,3,4,5. Candidate c_1 can assist c_5 in p_5. Candidate c_2 is fit for p_1. Candidates c_3 is fit for p_2 and c_4 can assist c_5 in p_3 and also fit for p_2. Candidate c_5 is fit for p_5 an

4. CONCLUSION

Intuitionistic fuzzy multiset is a veritable tool in decision making. This method curbed the leeway for possible manipulations because of the intuitionistic fuzzy multi-nature of the exercise. If incorporated into appointment procedure, it will enhance high rate of efficiency and performance in organizations. Intuitionistic fuzzy multisets is recommended for application in pattern recognition, medical imaging etc.

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