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Square Difference 3-Equitable Labeling of Some Graphs

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ABSTRACT

A square difference 3-equitable labeling of a graph G with vertex set V is a bijection f from V to $\{1, 2, \ldots, |V|\}$ such that if each edge uv is assigned the label -1 if $|[f(u)]^2 - [f(v)]^2| \equiv -1(mod\,4)$, the label 0 if $|[f(u)]^2 - [f(v)]^2| \equiv 0(mod\,4)$ and the label 1 if $|[f(u)]^2 - [f(v)]^2| \equiv 1(mod\,4)$, then the number of edges labeled with i and the number of edges labeled with j differ by atmost 1 for $-1 \leq i, j \leq 1$. If a graph has a square difference 3-equitable labeling, then it is called square difference 3-equitable graph. In this paper, we investigate the square difference 3-equitable labeling behaviour of middle graph of paths, fan graphs, (P_{2n}, S_1), mK_3, triangular snake graphs and friendship graphs. ■

AMS subject classification: 05C78
Key words: Square difference 3-equitable labeling, square difference 3-equitable graphs

1. INTRODUCTION

DEFINITION 1. *Let $G = (V, E)$ be a graph. A mapping $f : V(G) \to \{-1, 0, 1\}$ is called ternary vertex labeling of G and $f(v)$ is called the label of the vertex v of G under f.*

For an edge $e = uv$, the induced edge labeling is given by $f^* : E(G) \to \{-1, 0, 1\}$. Let $v_f(-1)$, $v_f(0)$, $v_f(1)$be the number of vertices of G having labels -1, 0, 1 respectively under f and $e_f(-1)$, $e_f(0)$, $e_f(1)$ be the number of edges having labels -1, 0, 1 respectively under f^*.

DEFINITION 2. *A ternary vertex labeling of a graph G is called a* **3-equitable labeling** *if $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$ for all $-1 \leq i, j \leq 1$. A graph G is 3-equitable if it admits 3-equitable labeling.*

DEFINITION 3. *A square difference 3-equitable labeling of a graph G with vertex set $V(G)$ is a bijection $f : V(G) \to \{1, 2, 3, ..., |V|\}$ such that the induced edge labeling $f^* : E(G) \to \{-1, 0, 1\}$ is defined by*

$$f^*(e = uv) = \begin{cases} -1 & if\ |[f(u)]^2 - [f(v)]^2| \equiv -1(mod\,4) \\ 0 & if\ |[f(u)]^2 - [f(v)]^2| \equiv 0(mod\,4) \\ 1 & if\ |[f(u)]^2 - [f(v)]^2| \equiv 1(mod\,4) \end{cases}$$

and $|e_f(i) - e_f(j)| \leq 1$ for all $-1 \leq i, j \leq 1$. A graph which admits square difference 3-equitable labeling is called square difference 3-equitable graph.

EXAMPLE 1. *Consider the following graph G.*

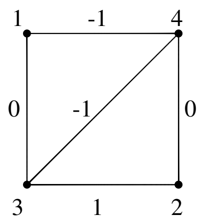

Fig 1. A square difference 3-equitable graph

We see that $e_f(-1) = e_f(0) = 2$ and $e_f(1) = 1$.
Thus $|e_f(i) - e_f(j)| \leq 1$ for all $-1 \leq i, j \leq 1$ and hence G is square difference 3-equitable.

DEFINITION 4. *The middle graph of G, denoted by $M(G)$, is $V(G) \bigcup E(G)$ such that two vertices x,y in the vertex set of $M(G)$ are adjacent in $M(G)$ in case one of the following holds.*
(i) x,y are in $E(G)$ and x,y are adjacent in G.
(ii) x is in $V(G)$, y is in $E(G)$ and x,y are incident in G.

2. MAIN RESULTS

THEOREM 1. *The middle graph $M(P_n)$ of path P_n admits square difference 3-equitable labeling.*

PROOF. Let $V(M(P_n)) = \{u_i, v_j | 1 \leq i \leq n, 1 \leq j \leq n - 1\}$ and
$E(M(P_n)) = \{v_j v_{j+1}, u_i v_i, v_i u_{i+1} | 1 \leq i \leq n - 1, 1 \leq j \leq n - 2\}$.
Define

$$f(u_i) = 2i - 1,\ 1 \leq i \leq n$$
$$\text{and } f(v_j) = 2j,\ 1 \leq j \leq n - 1.$$

Then

$$f^*(v_j v_{j+1}) = 0,\ 1 \le j \le n-2$$
$$f^*(u_i v_i) = -1,\ 1 \le i \le n-1$$
$$\text{and } f^*(v_i u_{i+1}) = 1,\ 1 \le i \le n-1.$$

Hence $|e_f(i) - e_f(j)| \le 1$ for all $-1 \le i, j \le 1$ and therefore $M(P_n)$ is a square difference 3-equitable graph. □

EXAMPLE 2. *The square difference 3-equitable labeling of $M(P_4)$ is shown below.*

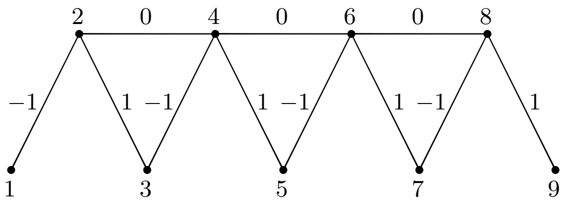

Fig 2. Square difference 3-equitable labeling of $M(P_4)$

THEOREM 2. *The fan graph $F_{2,n}$ admits square difference 3-equitable labeling.*

PROOF. Let the apex vertices of the fan graph $F_{2,n}$ be u_1, u_2 and let the other vertices along the diagonal be $u_3, u_4, ..., u_{n+2}$.
Then $|V(F_{2,n})| = n + 2$ and $|E(F_{2,n})| = 3n - 1$.
Define $f : V(F_{2,n}) \to \{1, 2, ..., n + 2\}$ by

$$f(u_i) = i,\ \ 1 \le i \le n+2.$$

Case(i): n is even
For $1 \le i \le \frac{n}{2}$,

$$f^*(u_1 u_{2i+1}) = 0$$
$$f^*(u_2 u_{2i+2}) = 0$$
$$f^*(u_2 u_{2i+1}) = 1$$
$$f^*(u_1 u_{2i+2}) = -1$$
$$f^*(u_{2i+1} u_{2i+2}) = -1$$

and $1 \le i \le \frac{n-2}{2}$,

$$f^*(u_{2i+2} u_{2i+3}) = 1.$$

Thus $e_f(-1) = e_f(0) = n$ and $e_f(1) = n - 1$.
Case(ii): n is odd
For $1 \le i \le \frac{n-1}{2}$,

$$f^*(u_2 u_{2i+1}) = 0$$
$$f^*(u_{2i+2} u_{2i+3}) = 1$$
$$f^*(u_1 u_{2i+2}) = -1$$
$$f^*(u_{2i+1} u_{2i+2}) = -1$$

and $1 \le i \le \frac{n+1}{2}$,

$$f^*(u_1 u_{2i+1}) = 0$$
$$f^*(u_2 u_{2i+1}) = 1.$$

Thus $e_f(0) = e_f(1) = n$ and $e_f(-1) = n - 1$.
Hence $|e_f(i) - e_f(j)| \le 1$ for all $-1 \le i, j \le 1$ and therefore $F_{2,n}$ is a square difference 3-equitable graph. □

EXAMPLE 3. *The square difference 3-equitable labeling of the fan graph $F_{2,5}$ is shown below.*

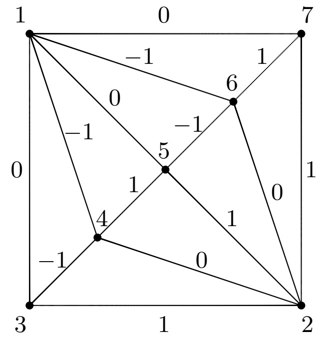

Fig 3. Square difference 3-equitable labeling of $F_{2,5}$

THEOREM 3. *(P_{2n}, S_1) is a square difference 3-equitable graph.*

PROOF. Let $P_{2n} : u_1 u_2 ... u_{2n}$ be the path.
Let $v_1, v_2, ..., v_{2n}$ be the vertices adjacent to $u_1, u_2, ..., u_{2n}$ and $w_1, w_2, ..., w_{2n}$ be the vertices adjacent to $v_1, v_2, ..., v_{2n}$.
Let G be the graph (P_{2n}, S_1).
Define $f : V(G) \to \{1, 2, ..., 6n\}$ by

$$f(u_i) = i,$$
$$f(v_i) = 4n - i + 1$$
$$\text{and } f(w_i) = 6n - i + 1 \text{ for } 1 \le i \le 2n.$$

Then for $1 \le i \le n$,

$$f^*(u_{2i-1} u_{2i}) = -1$$
$$f^*(u_{2i-1} v_{2i-1}) = -1$$
$$f^*(u_{2i} v_{2i}) = 1,$$

for $1 \le i \le n-1$,

$$f^*(u_{2i} u_{2i+1}) = 1$$

and for $1 \le i \le 2n$,

$$f^*(v_i w_i) = 0.$$

Thus $e_f(-1) = e_f(0) = 2n$ and $e_f(1) = 2n - 1$.
Hence $|e_f(i) - e_f(j)| \le 1$ for all $-1 \le i, j \le 1$ and therefore (P_{2n}, S_1) is a square difference 3-equitable graph. □

EXAMPLE 4. *The square difference 3-equitable labeling of (P_6, S_1) is shown below.*

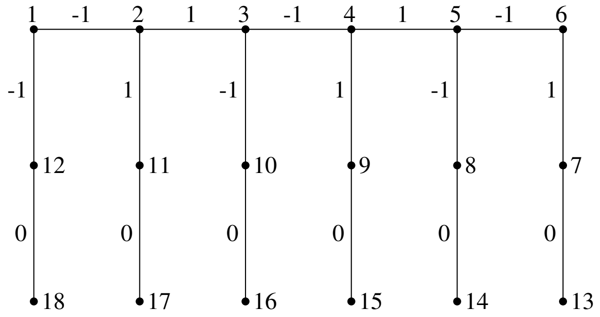

Fig 4. Square difference 3-equitable labeling of (P_6, S_1)

THEOREM 4. *mK_3 is a square difference 3-equitable graph.*

PROOF. Let $G_1, G_2, ..., G_m$ be m copies of K_3. Let u_1, u_2, u_3 be vertices of G_1, u_4, u_5, u_6 be vertices of G_2, ... , and $u_{3m-2}, u_{3m-1}, u_{3m}$ be vertices of G_m.
Define $f : V(mK_3) \to \{1, 2, ..., 3m\}$ by

$$f(u_i) = i,\ 1 \le i \le 3m$$

Case(i): m is even
For $i = 1, 2, ..., \frac{m}{2}$,

$$f^*(u_{6i-5}u_{6i-4}) = -1$$
$$f^*(u_{6i-1}u_{6i}) = -1$$
$$f^*(u_{6i-4}u_{6i-3}) = 1$$
$$f^*(u_{6i-2}u_{6i-1}) = 1$$

and for $i = 1, 2, ..., m$,

$$f^*(u_{3i-2}u_{3i}) = 0.$$

Case(ii): m is odd
For $i = 1, 2, ..., \frac{m+1}{2}$,

$$f^*(u_{6i-5}u_{6i-4}) = -1$$
$$f^*(u_{6i-4}u_{6i-3}) = 1$$

for $i = 1, 2, ..., \frac{m-1}{2}$,

$$f^*(u_{6i-1}u_{6i}) = -1$$
$$f^*(u_{6i-2}u_{6i-1}) = 1$$

and for $i = 1, 2, ..., m$,

$$f^*(u_{3i-2}u_{3i}) = 0.$$

Thus in both cases, $e_f(-1) = e_f(0) = e_f(1) = m$.
Hence $|e_f(i) - e_f(j)| \le 1$ for all $-1 \le i, j \le 1$ and therefore mK_3 is a square difference 3-equitable graph. □

EXAMPLE 5. *The square difference 3-equitable labeling of $7K_3$ is shown below.*

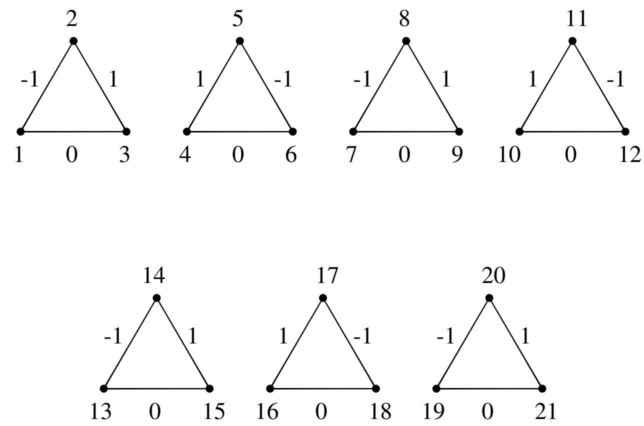

Fig 5. Square difference 3-equitable labeling of $7K_3$

THEOREM 5. *The triangular snake graph is a square difference 3-equitable graph.*

PROOF. Let G be the triangular snake graph.
Let $V(G) = \{u_i, v_j | 1 \le i \le n, 1 \le j \le n-1\}$
and
$E(G) = \{u_iu_{i+1}, u_iv_i, v_iu_{i+1} | 1 \le i \le n-1\}$.
Define

$$f(u_i) = 2i - 1,\ 1 \le i \le n$$
$$\text{and } f(v_j) = 2j,\ 1 \le j \le n-1.$$

Then for $1 \le i \le n-1$,

$$f^*(u_iv_i) = -1$$
$$f^*(u_iu_{i+1}) = 0$$
$$f^*(v_iu_{i+1}) = 1.$$

Hence $|e_f(i) - e_f(j)| \le 1$ for all $-1 \le i, j \le 1$ and therefore G is a square difference 3-equitable graph. □

EXAMPLE 6. *The square difference 3-equitable labeling of a triangular snake graph is shown below.*

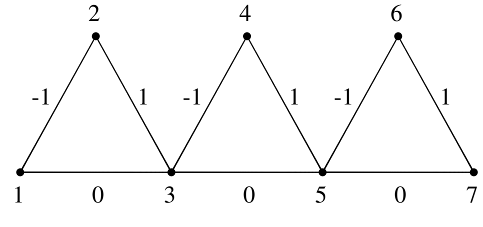

Fig 6. A triangular snake graph and its labeling

THEOREM 6. *The friendship graph F_n admits square difference 3-equitable labeling.*

PROOF. Let $u_1, u_2, u_3, ..., u_{2n}, u_{2n+1}$ be the vertices of the fan F_n with u_1 as the central vertex.

Then $|V(F_n)| = 2n + 1$ and $|E(F_n)| = 3n$.
Define $f : V(F_n) \rightarrow \{1, 2, ..., 2n + 1\}$ by

$$f(u_i) = i, \;\; 1 \leq i \leq 2n + 1.$$

Then for $i = 1, 2, ..., n$,

$$f^*(u_1 u_{2i}) = -1$$
$$f^*(u_1 u_{2i+1}) = 0$$
$$\text{and } f^*(u_{2i} u_{2i+1}) = 1.$$

Thus

$$e_f(-1) = e_f(0) = e_f(1) = n.$$

Hence $|e_f(i) - e_f(j)| \leq 1$ for all $-1 \leq i, j \leq 1$ and therefore F_n is a square difference 3-equitable graph. □

EXAMPLE 7. *The square difference 3-equitable labeling of F_5 is shown below.*

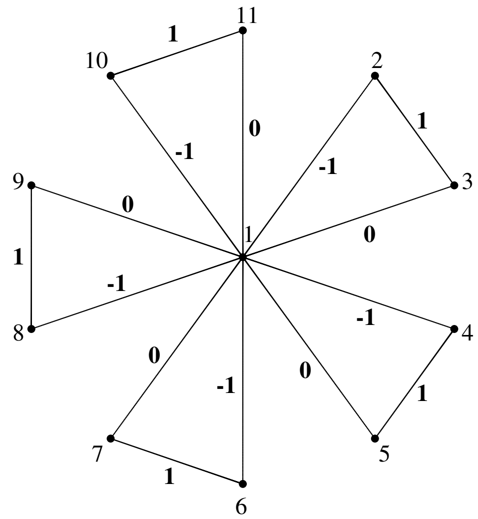

Fig 7. Square difference 3-equitable labeling of F_5

3. CONCLUSION

In [7], it was proved that paths and cycles admit square difference 3-equitable labelings. In the present work, it is proved that middle graph of paths, fan graphs, (P_{2n}, S_1), mK_3, triangular snake graphs and friendship graphs admit square difference 3-equitable labelings. To investigate analogous results for different graphs is an open area of research.

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