A Fuzzy Environment Inventory Model with Partial Backlogging under Learning Effect

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ABSTRACT
In this article we developed an inventory model for non-instantaneous decaying items is considered under crisp and fuzzy environment. In this study we have considered stock dependent demand rate and variable deterioration. It is supposed that shortages are allowed and partially backlogged with exponential backlogging rate. Holding cost follows the learning curve. The deterioration rate, ordering cost, shortage cost and deterioration cost are assumed as a triangular fuzzy numbers. The aim of our study is to defuzzify the total cost function by signed distance method. This model is developed in both crisp and fuzzy surroundings. A numerical experiment is given to demonstrate the developed crisp and fuzzy models. Sensitivity analysis is implemented to examine the effect of parameters. The convexity of the total cost function is shown by graphically.

Keywords
Non-instantaneous-deterioration, Triangular fuzzy numbers, Signed distance, Learning, Partial backlogging

1. INTRODUCTION
Inventory optimization deals with the decision to minimize the total average cost or maximize the total average profit. To do this, the approach is to construct a mathematical model of the real life inventory system by taking into account various assumptions and approximations. In most of the inventory models, it is assumed that various parameters like demand rate and ordering cost, etc. are precisely known. But in reality, the nature of these parameters is uncertain, so it is important to consider them as fuzzy numbers. Considering the fuzzy set theory in inventory model brings authenticity to the model since fuzziness is the closest possible approach to reality. By considering approximations, fuzzy theory helps one to incorporate uncertainties in the model formulation making it closer to reality.

The concept of fuzzy set theory modeling was developed by Zadeh (1965). Jain (1976) deliberated a fuzzy inventory model on decision making in the presence of fuzzy variables. Some operations on fuzzy numbers was defined by Dubois and Prade (1978). A long term inventory policy making through fuzzy decisions was formulated by Kacpryzk and Staniewski (1982). Zimmerman (1983) applied to use fuzzy sets in operational research.

Demand is directly co-related to stock and cost. In a fuzzy environment, data of stock are not clear. So to take care demand & inventory cost, it is important to avoid shortage. There are lots of factor, keeping open eyes on them, we can minimize fuzzy environment. Fuzzy inventory with backorder for fuzzy order quantity was investigated by Yao and Lee(1996). A Single Period Inventory Model with Fuzzy Demand was proposed by Kao and Hsu(2002). An inventory model with total demand and storing cost as triangular fuzzy numbers was developed by Yao and Chiang (2003). They applied the defuzzification by centroid method and signed distance method both. A multi-item, multi-objective inventory model for deteriorating items with stock- and time-dependent demand rate over a finite time horizon in fuzzy stochastic environment was presented by Mahapatra and Maiti (2006). Halim et. al. (2008) deliberated by an EOQ model for perishable items with stochastic demand and partial backlogging. A fuzzy inventory model without shortages by using triangular fuzzy number was presented by De and Rawat(2011). Some recent work in this direction is done by Jaggi et. al. (2012), Saha and Chakrabarti (2012), Dutta and Kumar (2013) and Kumar and Rajput (2015) etc.

Stock maintain has directly related to deterioration specially for perishable items and short expiry period goods. To control these we have to convert larger waiting time to shorter waiting time, by this we can reduce backlogging. For this we have to improve production, logistics and stock in our model. The first model for deteriorating items was formulated by Ghare and Schrader (1963). Chang & Dye (1999) deliberated a decaying inventory model with time varying demand and partial backlogging. An optimal replenishment policy for non-instantaneous deteriorating items with stock-dependent demand was introduced by Wu et. al.(2006).Other inspiring articles related to this research area are Singh & Singh (2008), Sugapriya & Jeyaraman (2008),Dye (2013), Shukla et. al. (2013), Singh & Sharma (2014), Jaggi (2014), Tayal et.al (2015) and Jaggi et. al.(2015), Khurana (2015) etc.

Learning phenomena cannot be ommit. By monitoring demand, shortage, holding cost & backlogged we can improve results as well as performance, shortage cannot be vanish. While developing inventory model we have to improve effective tools and machinery, production procedure, human resource environment. Learning, shortage and backlogging, we have developed a new effective inventory model. The learning phenomena defined by Wright [1936]. Jordan [1958] formulated that how to use the learning curve. An EPQ model under learning effect was developed by Fisk and Ballou [1985]. Balkhi [2003] analyzed an optimal production lot size for deteriorating items with learning effect. An inventory model for imperfect quality items with learning was introduced by Jaber et. al.(2008). Kumar et al. (2013) investigated a Learning effect on an inventory model with two-level storage and partial backlogging. An imperfect quality items with learning under two limited storage capacity was developed by Singh et.al.(2013). Singhal & Singh (2015) proposed an inventory system with multi variate demand under volume flexibility and learning.

In this article a decaying inventory model with variable deterioration and stock dependent demand rate is developed. This model is explained both crisp and fuzzy environment.

Shortages are permitted and partial backlogging. Learning effect is also considered in this model. Therefore, an inventory model with partial-backlogging is considered in fuzzy sense. The signed distance method is applied for defuzzification

2. ASSUMPTIONS AND NOTATIONS:

2.1 Assumptions

The basic assumptions are applied to analyze this inventory model:

1. The demand rate for the product is assumed to be stock dependent which is given by $D(t) = a + mI(t), I(t) > 0$ and $a, I(t) < 0$,where $a > 0,\ 0 < m < 1,$ $a > m$
2. The deteriorating rate $\theta(0 < \theta < 1)$ is time dependent.
3. Shortages are allowed and partial backlogged and backlogging rate is considered to be $B(t) = e^{-\delta(T-t)}, 0 < \delta < 1$.
4. Holding cost is partly constant and partly decreasing in each cycle due to learning effect of employees and is of the form $(h + \frac{h_0}{n^{\beta}}), \beta > 0$
5. t_d is the length of time in which the product has no deterioration.
6. T is the length of the cycle.
7. Lead-time is zero.

2.2 Notations

The basic notations **are** used to develop this inventory model:

1. A is the ordering cost.
2. P is the Purchasing cost.
3. C_1 is the Deterioration Cost.
4. C_2 is the Shortage Cost.
5. C_3 is the Lost Sale Cost.
6. No. of shipments is n.
7. $\tilde{\theta}$ is the fuzzy deterioration rate.
8. $\tilde{A}$ is the fuzzy ordering cost.
9. $\tilde{C}_1$ is the fuzzy deterioration cost.
10. $\tilde{C}_2$ is the fuzzy shortage cost.
11. The Inventory Level at time t is $I_1(t)$ with $t \in [0, t_d]$
12. The Inventory Level at time t is $I_2(t)$ with $t \in [t_d, t_1]$
13. The Inventory Level at time t is $I_3(t)$ with $t \in [t_1, T]$
14. $TC(t_1, T)$ is the total cost of the system for crisp model.
15. $T\tilde{C}(t_1, T)$ is the total cost of the system for fuzzy model.

3. MATHEMATICAL FORMULATION OF THE MODEL

Crisp model:

Let I(t) be the inventory level at time t $(0 \le t \le T)$. During the time interval $(0, t_d)$ the inventory level is depletes only owing to stock-dependent demand rate. The differential equation during the interval $(0, t_d)$ is given by

$$\frac{dI_1(t)}{dt} = -\left[a + mI_1(t)\right], \quad 0 \le t \le t_d \tag{1}$$

Again during the time interval (t_d, t_1) inventory is dropping to zero due to demand rate and deterioration both. The differential equation during the interval (t_d, t_1) is given by

$$\frac{dI_2(t)}{dt} + \theta t I_2(t) = -\left[a + mI_2(t)\right], t_d \le t \le t_1 \tag{2}$$

After that during the time interval (t_1, T) shortage starts and due to partial backlogging some sales are lost. The differential equation during the interval (t_1, T) is given by

$$\frac{dI_3(t)}{dt} = -a\, e^{-\delta(T-t)}, t_1 \le t \le T \tag{3}$$

With boundary conditions

$$I_1(0) = I_{ma.x.}, I_2(t_1) = 0 \;\&\; I_3(t_1) = 0 \tag{4}$$

Solutions of these equations are

$$I_1(t) = e^{-mt}\left[I_{ma.x.} - a\left(t + \frac{mt^2}{2}\right)\right] \tag{5}$$

$$I_2(t) = a(e^{-\left(\frac{\theta t^2}{2} + mt\right)})\left[\left(t_1 - t\right) + \frac{\theta}{6}\left(t_1^3 - t^3\right) + \frac{m}{2}\left(t_1^2 - t^2\right)\right] \tag{6}$$

$$I_3(t) = -a[t - \delta(Tt - \frac{t^2}{2})] + a[t_1 - \delta(Tt_1 - \frac{t_1^2}{2})] \quad (7)$$

Considering the continuity at $t = t_d$ it follows from equation (5) & (6) such that $I_1(t_d) = I_2(t_d)$

$$e^{-mt_d} I_{ma.x.} - e^{-mt_d} a\left(t_d + \frac{mt_d^{\,2}}{2}\right) =$$

$$a(e^{-\left(\frac{\theta t_d^{\,2}}{2}+mt_d\right)})\left[\left(t_1 - t_d\right) + \frac{\theta}{6}\left(t_1^3 - t_d^{\,3}\right) + \frac{m}{2}\left(t_1^2 - t_d^{\,2}\right)\right] \quad (8)$$

Now Maximum inventory level for each cycle is

$$Q_1 = I_{ma.x.} = at_d(1 + \frac{m}{2}t_d) + ae^{mt_d}\left[\left(t_1 - t_d\right) + \frac{\theta}{6}\left(t_1^3 - t_d^{\,3}\right)\right.$$
$$+ \frac{m}{2}\left(t_1^2 - t_d^{\,2}\right) + \frac{\theta}{2}t_d^{\,2}(t_d - t_1) + \frac{\theta m}{4}t_d^{\,2}(t_d^{\,2} - t_1^2)$$
$$\left. + mt_d\left(t_1 - t_d\right) + \frac{\theta m}{6}t_d(t_1^3 - t_d^3)\right] \quad (9)$$

Now Equation (4) becomes

$$I_1(t) = at_d(1 + \frac{m}{2}t_d)e^{-mt} - at(1 + \frac{m}{2}t)e^{-mt} + ae^{m(t_d - t)}$$
$$\left[\left(t_1 - t_d\right) + \frac{\theta}{6}\left(t_1^3 - t_d^{\,3}\right) + \frac{m}{2}\left(t_1^2 - t_d^{\,2}\right) + \frac{\theta}{2}t_d^{\,2}(t_d - t_1)\right.$$
$$\left. + \frac{\theta m}{4}t_d^{\,2}(t_d^{\,2} - t_1^2) + mt_d\left(t_1 - t_d\right) + \frac{\theta m}{6}t_d(t_1^3 - t_d^3)\right] \quad (10)$$

Put $t = T$ in equation (7) we get max. amount of backlogged per cycle as follows

$$Q_2 = -I_3(T) = -a[T - \delta\frac{T^2}{2})] + a[t_1 - \delta(Tt_1 - \frac{t_1^2}{2})] \quad (11)$$

The order quantity is

$$\text{Q}= at_d(1 + \frac{m}{2}t_d) - a[T - \delta\frac{T^2}{2}] + a[t_1 - \delta(Tt_1 - \frac{t_1^2}{2})] +$$
$$ae^{mt_d}\left[\left(t_1 - t_d\right) + \frac{\theta}{6}\left(t_1^3 - t_d^{\,3}\right) + \frac{m}{2}\left(t_1^2 - t_d^{\,2}\right) + \frac{\theta}{2}t_d^{\,2}(t_d - t_1)\right.$$
$$\left. + \frac{\theta m}{4}t_d^{\,2}(t_d^{\,2} - t_1^2) + mt_d\left(t_1 - t_d\right) + \frac{\theta m}{6}t_d(t_1^3 - t_d^3)\right] \quad (12)$$

3.1 The Purchasing cost is given by

$$PC = PQ = P(Q_1 + Q_2)$$

$$\text{PC=P}\left[at_d(1 + \frac{m}{2}t_d) - a[T - \delta\frac{T^2}{2}] + a[t_1 - \delta(Tt_1 - \frac{t_1^2}{2})] + \right.$$
$$ae^{mt_d}\left\{\left(t_1 - t_d\right) + \frac{\theta}{6}\left(t_1^3 - t_d^{\,3}\right) + \frac{m}{2}\left(t_1^2 - t_d^{\,2}\right) + \frac{\theta}{2}t_d^{\,2}(t_d - t_1)\right.$$
$$\left.\left. + \frac{\theta m}{4}t_d^{\,2}(t_d^{\,2} - t_1^2) + mt_d\left(t_1 - t_d\right) + \frac{\theta m}{6}t_d(t_1^3 - t_d^3)\right\}\right] \quad (13)$$

3.2 Ordering Cost is given by

$$\text{OC}= A \quad (14)$$

3.3 Holding cost is given by

$$HC = (h + \frac{h_0}{n^{\alpha_2}})\left[\int_0^{t_d} I_1(t)dt + \int_{t_d}^{t_1} I_2(t)dt\right]$$

$$HC = (h + \frac{h_0}{n^{\alpha_2}})\left[\int_0^{t_d}\left(at_d(1 + \frac{m}{2}t_d)e^{-mt} - at(1 + \frac{m}{2}t)e^{-mt}) + \right.\right.$$
$$(ae^{m(t_d - t)}\left(\left(t_1 - t_d\right) + \frac{\theta}{6}\left(t_1^3 - t_d^{\,3}\right) + \frac{m}{2}\left(t_1^2 - t_d^{\,2}\right) + \frac{\theta}{2}t_d^{\,2}(t_d - t_1)\right.$$
$$\left.\left. + \frac{\theta m}{4}t_d^{\,2}(t_d^{\,2} - t_1^2) + mt_d\left(t_1 - t_d\right) + \frac{\theta m}{6}t_d(t_1^3 - t_d^3)\right)\right)dt +$$
$$\left.\int_{t_d}^{t_1}\left(ae^{-\left(\frac{\theta t^2}{2}+mt\right)}\left(\left(t_1 - t\right) + \frac{\theta}{6}\left(t_1^3 - t^3\right) + \frac{m}{2}\left(t_1^2 - t^2\right)\right)\right)dt\right] \quad (15)$$

3.4 Deterioration Cost is given by

$$DC = I_2(t_d) - \int_{t_d}^{t_1}\left(a + mI_2(t)\right)dt$$

$$DC = I_2(t_d) - \int_{t_d}^{t_1}\left(a + ma\, e^{-\left(\frac{\theta t^2}{2}+mt\right)}\left(\left(t_1 - t\right) + \frac{\theta}{6}\left(t_1^3 - t^3\right)\right.\right.$$
$$\left.\left. + \frac{m}{2}\left(t_1^2 - t^2\right)\right)\right)dt \quad (16)$$

3.5 Shortage cost is given by $SC = -C_2\int_{t_1}^{T} I_3(t)dt$

$$SC = -C_2\int_{t_1}^{T}[-a(t - \delta(Tt - \frac{t^2}{2})) + a(t_1 - \delta(Tt_1 - \frac{t_1^2}{2}))]dt \quad (17)$$

3.6 Lost Sales Cost due to lost sales is given by

$$LS = C_3\int_{t_1}^{T} a(1 - e^{-\delta(T - t)})dt$$

$$LS = C_3\frac{a\delta}{2}(T - t_1)^2 \quad (18)$$

3.7 Therefore, the total relevant inventory cost per unit time is given by (Crisp Model)

$$TC(t_1,T)=\frac{1}{T}[DC+OC+PC+HC+SC+LS] \tag{19}$$

4. OPTIMAL SOLUTION PROCEDURE

The total values of t1 and T which minimize total cost TC can be solved by differentiating equation (21) ,put

$$\frac{\partial TC(t_1,T)}{\partial t_1}=0,\frac{\partial TC(t_1,T)}{\partial T}=0$$

(20)

Provided it satisfies the condition

$$\frac{\partial^2 TC(t_1,T)}{\partial t_1^{\,2}}>0,\frac{\partial^2 TC(t_1,T)}{\partial T^2}>0$$

$$\left[\frac{\partial^2 TC(t_1,T)}{\partial t_1^{\,2}}\right]\left[\frac{\partial^2 TC(t_1,T)}{\partial T^2}\right]-\left[\frac{\partial^2 TC(t_1,T)}{\partial T\partial t_1}\right]^2>0 \tag{21}$$

5. NUMERICAL EXPERIMENT

Let us consider

$A=150,\theta=0.02,P=4,C_1=1.4,a=150,$
$m=0.44,\delta=0.01,C_2=2.5,C_3=2,h=5,$
$h_0=2,n=3,\beta=0.1,t_d=1.2$

Based on these input data, we get

$t_1=2.2718$, T=5.42421 and optimal total cost will be $TC(t_1,T)$ =1494.51

6. SENSITIVITY ANALYSIS

Sensitivity analysis is performed by changing the parameters and considering one parameter at a time, keeping the left over parameters at their original values

Table-(1) Variation in Deterioration cost parameter 'C_1'

C_1	t_1	T	TC
1.41	2.27404	5.44213	1495.21
1.42	2.27458	5.44437	1495.98
1.43	2.27505	5.4466	1496.63
1.44	2.27555	5.44896	1497.31

Table-(2) Variation in Deterioration rate 'θ'

θ	t_1	T	TC
0.03	2.23729	5.29071	1470.43
0.04	2.2062	5.18694	1446.02
0.05	2.17894	5.09124	1425.53
0.06	2.15594	5.04701	1402.01

Table-(3) Variation in shortage cost parameter 'C_2'

C_2	t_1	T	TC
2.6	2.27366	5.40955	1513.2
2.7	2.2709	5.29606	1541.27
2.8	2.26871	5.26503	1555.86
2.9	2.271	5.24598	1575.13

7. OBSERVATIONS

- From Table 1, If we increase the deterioration cost parameter 'C_1'then total cost TC increases.
- From Table 2, If we increase the deterioration rate'θ'then total cost TC decreases.
- From Table 3, If we increase the shortage cost parameter C_2 then the total cost TC increases.

8. FUZZY-MATHEMATICAL MODEL

Let us consider in this inventory model $\theta,A,C_1,and C_2$ as triangular fuzzy numbers, i.e. $\tilde{\theta},\tilde{A},\tilde{C}_1,and\tilde{C}_2$. Let us assume that parameters $\tilde{\theta},\tilde{A},\tilde{C}_1,and\tilde{C}_2$ may change within some limits.

$\tilde{\theta}=(\theta-\Delta_1,\theta,\theta+\Delta_2),$ where $0<\Delta_1<\theta$ and $\Delta_1\Delta_2>0$, (22)

$\tilde{A}=(A-\Delta_3,A,A+\Delta_4)$,where $0<\Delta_3<A$ and $\Delta_3\Delta_4>0$, (23)

$\tilde{C}_1=(C_1-\Delta_5,C_1,C_1+\Delta_6),$ where $0<\Delta_5<C_1$ and $\Delta_5\Delta_6>0$, (24)

$\tilde{C}_2=(C_2-\Delta_7,C_2,C_2+\Delta_8),$ where $0<\Delta_7<C_2$ and $\Delta_7\Delta_8>0$, (25)

By signed distance Method

$$\tilde{\theta}=\theta+\frac{1}{4}(\Delta_2-\Delta_1) \tag{26}$$

$$\tilde{A}=A+\frac{1}{4}(\Delta_4-\Delta_3) \tag{27}$$

$$\tilde{C}_1=C_1+\frac{1}{4}(\Delta_6-\Delta_5) \tag{28}$$

$$\tilde{C}_2=C_2+\frac{1}{4}(\Delta_8-\Delta_7) \tag{29}$$

Now defuzzified total cost per unit time by signed distance method is given by

$$T\tilde{C}(t_1,T)=\frac{1}{4}[T\tilde{C}F_1(t_1,T)+2T\tilde{C}F_2(t_1,T)+T\tilde{C}F_3(t_1,T)] \quad (30)$$

The necessary condition for minimize total cost are

$$\frac{\partial T\tilde{C}(t_1,T)}{\partial t_1}=0, \frac{\partial T\tilde{C}(t_1,T)}{\partial T}=0 \quad (31)$$

Provided it satisfies the condition

$$\frac{\partial^2 T\tilde{C}(t_1,T)}{\partial t_1^2}>0, \frac{\partial^2 T\tilde{C}(t_1,T)}{\partial T^2}>0$$

$$\left[\frac{\partial^2 T\tilde{C}(t_1,T)}{\partial t_1^2}\right]\left[\frac{\partial^2 T\tilde{C}(t_1,T)}{\partial T^2}\right]-\left[\frac{\partial^2 T\tilde{C}(t_1,T)}{\partial T\partial t_1}\right]^2>0 \quad (32)$$

Using the software Mathematica, we get optimum value of t_1^*, T^* and the optimum total cost $T\tilde{C}(t_1,T)$.

9. FUZZY NUMERICAL EXAMPLE

Let us consider

$$\tilde{A}=(130,150,170), a=150, \tilde{C}_1=(1,1.4,1.8), t_d=1.2,$$
$$C_3=2, h_0=2, \delta=0.01, \tilde{C}_2=(2.1,2.5,2.9), h=5,$$
$$\tilde{\theta}=(0.01,0.02,0.03), m=0.44, P=4, n=3, \beta=0.1$$

Based on these input data, we get

$t_1^*=2.1678, T^*=5.20747$ and total cost

$T\tilde{C}(t_1,T)=649.637$.

10. CONCLUSION

This paper presents a crisp and fuzzy inventory model for non-instantaneous decaying items with shortages considering demand rate is time dependent. The demand, deterioration rate, ordering cost and shortage cost are represented by triangular fuzzy numbers. For defuzzification signed distance method is utilized to evaluate the optimal total cost. The proposed model is more practical due to the impreciseness in inventory costs and demand. Considering variables as fuzzy numbers is also more useful business strategy to cope up with ups and downs conditions of the market. To make inventory model more realistic, we also consider stock dependent demand and partial backlogging since all shortages cannot be fully backlogged. At the end numerical example and sensitivity analysis is elaborated. The whole calculation part is done using Mathematica. For the future research we can incorporate some other parameters of inventory control system.

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12. APPENDIX

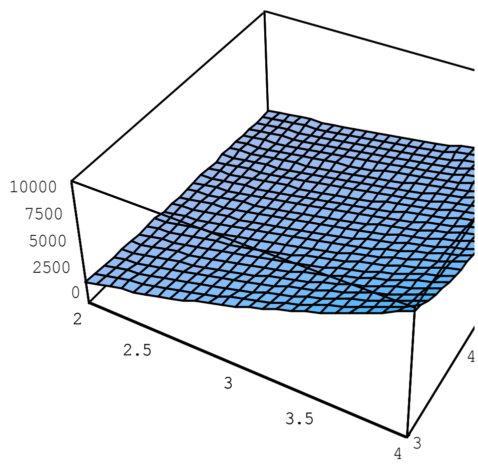

Fig. Convexity of the total cost function (Crisp Model)

Total cost function for fuzzy model from equation no.(30)

Where

$$T\tilde{C}F_1(t_1,T)=\frac{1}{T}\left[(C_1-\Delta_5)\left\{\left(\frac{(\theta-\Delta_1)}{6}\right)(t_1^3-t_d^3)(1+m(t_1+t_d))+(t_d-t_1)\left\{\frac{(\theta-\Delta_1)}{2}t_d^2+m(t_1-t_d)\right.\right.\right.$$

$$\left.\left.+\frac{m(\theta-\Delta_1)}{6}t_1^3\right\}+m(t_1^2-t_d^2)(1-\frac{(\theta-\Delta_1)}{4}t_d^2)-\frac{(\theta-\Delta_1)m}{12}(t_1^4-t_d^4)\right\}+(A-\Delta_3)+\mathrm{P}\left\{ae^{mt_d}\right.$$

$$\left\{(t_1-t_d)+\frac{(\theta-\Delta_1)}{6}\left(t_1^3-t_d{}^3\right)+\frac{m}{2}\left(t_1^2-t_d{}^2\right)+\frac{(\theta-\Delta_1)}{2}t_d{}^2(t_d-t_1)+\frac{(\theta-\Delta_1)m}{4}t_d{}^2(t_d{}^2-t_1^2)+mt_d\,(t_1-t_d)\right.$$

$$\left.\left.+\frac{(\theta-\Delta_1)m}{6}t_d\,(t_1{}^3-t_d^3)\right\}+a\{t_d\,(1+\frac{m}{2}t_d)-\left(T-\delta\frac{T^2}{2}\right))+(t_1-\delta(Tt_1-\frac{t_1^2}{2}))\}\right\}$$

$$+a(h+\frac{h_0}{n^{\alpha_2}})\left\{(t_d+t_d^2(m-\frac{1}{2})\{(t_1-t_d)(1-\frac{(\theta-\Delta_1)}{2}t_d^2+mt_d)+\left(\frac{(\theta-\Delta_1)}{6}\right)(t_1^3-t_d^3)(1+mt_d)\right.$$

$$\left.+\frac{m}{2}(t_1^2-t_d^2)(1-\frac{(\theta-\Delta_1)}{2}t_d^2)\right\}+\frac{t_d^2}{6}(mt_d-3)+(t_1-t_d)(t_1+\frac{(\theta-\Delta_1)}{6}t_1^3+\frac{m}{2}t_1^2)+(\frac{mt_1}{2}+\frac{(\theta-\Delta_1)mt_1{}^3}{12}-\frac{1}{2})$$

$$\left.(t_1^2-t_d^2)-(t_1^3-t_d^3)(\frac{m}{2}+\frac{(\theta-\Delta_1)mt_1{}^2}{12}+\frac{(\theta-\Delta_1)t_1}{6})+\frac{(\theta-\Delta_1)(t_1^4-t_d^4)}{12}+\frac{(\theta-\Delta_1)m(t_1^5-t_d^5)}{60}\right\}+C_3\frac{a\delta}{2}(T-t_1)^2$$

$$-(C_2-\Delta_7)a\left\{\frac{-\delta(T^3-t_1^3)}{6}-\frac{(1-\delta T)(T^2-t_1^2)}{2}+(T-t_1)\left(t_1-\delta(Tt_1-\frac{1}{2}t_1^2\right)\right\}$$

$$TC\tilde{F}_2(t_1,T)=\frac{1}{T}\left[C_1\left\{\left(\frac{\theta}{6}\right)(t_1^3-t_d^3)(1+m(t_1+t_d))+(t_d-t_1)(\frac{\theta}{2}t_d^2+m(t_1-t_d)+\frac{m\theta}{6}t_1^3)\right.\right.$$

$$\left.+m(t_1^2-t_d^2)(1-\frac{\theta}{4}t_d^2)-\frac{\theta m}{12}(t_1^4-t_d^4)\right\}+A+\mathrm{P}\left\{ae^{mt_d}\left\{(t_1-t_d)+\frac{\theta}{6}\left(t_1^3-t_d{}^3\right)+\frac{m}{2}\left(t_1^2-t_d{}^2\right)\right.\right.$$

$$\left.+\frac{\theta}{2}t_d{}^2(t_d-t_1)+\frac{\theta m}{4}t_d{}^2(t_d{}^2-t_1^2)+mt_d\,(t_1-t_d)+\frac{\theta m}{6}t_d\,(t_1{}^3-t_d^3)\right\}+at_d\,(1+\frac{m}{2}t_d)$$

$$\left.-a[T-\delta\frac{T^2}{2})]+a[t_1-\delta(Tt_1-\frac{t_1^2}{2})]\right\}+a(h+\frac{h_0}{n^{\alpha_2}})\left\{(t_d+t_d^2(m-\frac{1}{2})\{(t_1-t_d)(1-\frac{\theta}{2}t_d^2+mt_d)\right.$$

$$\left.+\left(\frac{\theta}{6}\right)(t_1^3-t_d^3)(1+mt_d)+\frac{m}{2}(t_1^2-t_d^2)(1-\frac{\theta}{2}t_d^2)\right\}+\frac{t_d^2}{6}(mt_d-3)+(t_1-t_d)(t_1+\frac{\theta}{6}t_1^3+\frac{m}{2}t_1^2)$$

$$\left.+(t_1^2-t_d^2)(\frac{mt_1}{2}+\frac{\theta mt_1{}^3}{12}-\frac{1}{2})-(t_1^3-t_d^3)(\frac{m}{2}+\frac{\theta mt_1{}^2}{12}+\frac{\theta t_1}{6})+\frac{\theta(t_1^4-t_d^4)}{12}+\frac{\theta m(t_1^5-t_d^5)}{60}\right\}$$

$$+C_3\frac{a\delta}{2}(T-t_1)^2-C_2a\left\{\frac{-\delta(T^3-t_1^3)}{6}-\frac{(1-\delta T)(T^2-t_1^2)}{2}+(T-t_1)\left(t_1-\delta(Tt_1-\frac{1}{2}t_1^2\right)\right\}\Bigg]$$

$$TC\tilde{F}_3(t_1,T)=\frac{1}{T}\Bigg[(C_1+\Delta_6)\left\{\left(\frac{(\theta-\Delta_2)}{6}\right)(t_1^3-t_d^3)(1+m(t_1+t_d))+(t_d-t_1)(\frac{(\theta-\Delta_2)}{2}t_d^2\right.$$

$$\left.+m(t_1-t_d)+\frac{m(\theta-\Delta_2)}{6}t_1^3)+m(t_1^2-t_d^2)(1-\frac{(\theta-\Delta_2)}{4}t_d^2)-\frac{(\theta-\Delta_2)m}{12}(t_1^4-t_d^4)\right\}+(A+\Delta_4)$$

$$+C_3\frac{a\delta}{2}(T-t_1)^2+\mathrm{P}\left\{ae^{mt_d}\left\{\left(t_1-t_d\right)+\frac{(\theta+\Delta_2)}{6}\left(t_1^3-t_d{}^3\right)+\frac{m}{2}\left(t_1^2-t_d{}^2\right)+\frac{(\theta+\Delta_2)}{2}t_d{}^2(t_d-t_1)\right.\right.$$

$$\left.+mt_d\left(t_1-t_d\right)+\frac{(\theta+\Delta_2)m}{4}t_d{}^2(t_d{}^2-t_1^2)+\frac{(\theta+\Delta_2)m}{6}t_d(t_1{}^3-t_d^3)\right\}+a\{(t_1-\delta(Tt_1-\frac{t_1^2}{2}))+$$

$$t_d(1+\frac{m}{2}t_d)-(T-\delta\frac{T^2}{2})\}\Big\}+a(h+\frac{h_0}{n^{\alpha_2}})\left\{(t_d+t_d^2(m-\frac{1}{2})\left\{(1-\frac{(\theta+\Delta_2)}{2}t_d^2+mt_d)(t_1-t_d)\right.\right.$$

$$+\left(\frac{(\theta+\Delta_2)}{6}\right)(t_1^3-t_d^3)(1+mt_d)+\frac{m}{2}(t_1^2-t_d^2)(1-\frac{(\theta+\Delta_2)}{2}t_d^2)\Big\}+(t_1+\frac{(\theta+\Delta_2)}{6}t_1^3+\frac{m}{2}t_1^2)\left(t_1-t_d\right)$$

$$+\frac{t_d^2}{6}(mt_d-3)+(t_1^2-t_d^2)(\frac{mt_1}{2}+\frac{(\theta+\Delta_2)mt_1{}^3}{12}-\frac{1}{2})-(t_1^3-t_d^3)(\frac{m}{2}+(\theta+\Delta_2)(\frac{mt_1{}^2}{12}+\frac{t_1}{6}))+(\theta+\Delta_2)$$

$$(\frac{(t_1^4-t_d^4)}{12}+\frac{m(t_1^5-t_d^5)}{60})\Big\}-C_2a\left\{\frac{-\delta(T^3-t_1^3)}{6}-\frac{(1-\delta T)(T^2-t_1^2)}{2}+(T-t_1)\left(t_1-\delta(Tt_1-\frac{1}{2}t_1^2\right)\right\}\Bigg]$$