Dominating Critical in Intuitionistic Fuzzy Graphs

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ABSTRACT
Let G be an IFG. Then $D \subseteq V$ is said to bae a strong (weak) dominating set if every $v \in V - D$ is strongly (weakly) dominated by some vertex in D. We denote the strong (weak) intuitionistic fuzzy dominating set by sid-set (wid-set). The minimum vertex cardinality over all the sid-set (wid-set) is called the strong (weak) dominating number of an IFG and is denoted by $\gamma_{sid}(G)$ $[\gamma_{wid}(G)]$

In this paper, we introduce the strong (weak) domination in intuitionistic fuzzy graphs and obtain some bounds in IFG.

Keywords
Intuitionistic fuzzy graph, strong (weak) domination, strong (weak) domination number, dominating crictal

1. INTRODUCTION
The first definition of fuzzy graphs was proposed by Kafmann, from the fuzzy relations introduced by Zadeh. Although Rosenfeld introduced another elaborated definition, including fuzzy vertex and fuzzy edges, and several fuzzy analogs of graph theoretic concepts such as paths, cycles, connectedness and etc. The concept of domination in fuzzy graphs was investigated by A. Somasundaram, S. Somasundaram and A. Somasundaram present the concepts of independent domination, total domination, connected domination of fuzzy graphs . C. Natarajan and S.K. Ayyaswamy introduce the strong (weak) domination in fuzzy graph. The first definition of intuitionistic fuzzy graphs was proposed by Atanassov. The concept of domination in intuitionistic fuzzy graphs was investigated by R.parvathi and G.Thamizhendhi. In this paper we introduce a dominating crictial andinvestigate the property of this in intuitionistic fuzzy graph.

2. BASIC DEFINITION
An intuitionistic fuzzy graph (IFG) is of the form G=(V,E) , where $V = \{v_1, v_2, ..., v_n\}$ such that $\mu_1 : V \to [0,1]$, $\gamma_1 : V \to [0,1]$ denote the degree of membership and nonmember ship of the element $v_i \in V$ respectively and $0 \le \mu_1(v_i) + \gamma_1(v_i) \le 1$ for every $v_i \in V, (i = 1,2,...n)$. $E \subseteq V \times V$ where $\mu_2 : V \times V \to [0,1],$ *and* $\gamma_2 : V \times V \to [0,1]$ are such that $\mu_2(v_i, v_j) \le \mu_1(v_i) \wedge \mu_1(v_j)$, $\gamma_2(v_i, v_j) \le \gamma_1(v_i) \vee \gamma_1(v_j)$ and $0 \le \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) \le 1$.

An arc (v_i, v_j) of an IFG G is called an strong arc if

$$\mu_2(v_i, v_j) = \mu_1(v_i) \wedge \mu_1(v_j),$$

$$\gamma_2(v_i, v_j) = \gamma_1(v_i) \wedge \gamma_1(v_j).$$

Let G = (V, E) be an IFG. Then the cardinality of G is defined to be

$$|G| = \left|\sum_{v_i \in V}[\frac{(1+\mu_1(v_i) - \gamma_1(v_i))}{2}]\right| + \left|\sum_{v_i \in V}[\frac{(1+\mu_2(v_i, v_j) - \gamma_1(v_i, v_j))}{2}]\right|$$

Let G = (V, E) be an IFG. The vertex cardinality of G is defined to be $|G| = \left|\sum_{v_i \in V}[\frac{(1+\mu_1(v_i) - \gamma_1(v_i))}{2}]\right|$ for all $v_i \in V, (i = 1,2,...n)$.

Let G = (V, E) be an IFG. An edge cardinality of G is defined to be $|G| = \left|\sum_{v_i \in V}[\frac{(1+\mu_2(v_i, v_j) - \gamma_1(v_i, v_j))}{2}]\right|$ for all $(v_i, v_j) \in V \times V$,

Let G = (V, E) be an IFG. A set $D \subseteq V$ is said to be a dominating set o G if every $v \in V - D$ there exist $u \in D$ such that u dominates v.

An intuitionistic fuzzy dominating D of an IFG, G is called minimal dominating set of G if every node $u \in D$, $D - \{u\}$ is not a dominating set in G.

An intuitionistic fuzzy domination number $\gamma_{if}(G)$ of an IFG, G is the minimum vertex cardinality over all minimal dominating sets in G.

A set $S \subseteq V$ in an IFG, G is said to be an independent if there is no strong between the vertices $v \in S$. An independent set S of IFG ,G is said to be maximal independent set if every node $v \in V - S$ then the set $S \cup \{v\}$ is not an independent set in G. The minimum cardinality among all the maximal independent sets in an IFG, G is called the intuitionistic fuzzy independent number..

Let G be an IFG. The neighbor of a node $v \in V$ is defined by

$$N(v) = \begin{Bmatrix} u \in v : \mu_2(uv) = \mu_1(u) \wedge \mu_1(v) \\ \gamma_2(uv) = \gamma_1(u) \vee \gamma_1(v) \end{Bmatrix}$$

The vertex cardinality of N(v) is the neighborhood degree of v which is denoted by $d_N(v)$.

$$d_N(v) = \sum_{u \in N(v)} \left[\frac{1+\mu_1(u)-\gamma_1(u)}{2} \right]$$

The effective degree of v is an edge cardinality of the strong edge incident v. $d_E(v) = \sum_{u \in V} \left[\frac{1+\mu_2(uv)-\gamma_2(uv)}{2} \right]$ uv is a strong arc .

2.1 Strong Domination

Let u and v be any two vertices in an IFG, G. Then u strongly dominates v (v weakly dominates u) if (i). uv is a strong arc (ii) $d_N(u) \geq d_N(v)$.

Some results in strong domination

- Let D be a minimal strong dominating set of an IFG G. then for each $v \in D$. One of the following holds

1) No vertex in D strongly dominates v
2) There exist $u \in V - D$ such that v is the only vertex in D which strongly dominates u.

- For an IFG, G (V,E) of order P

$$\gamma_{if}(G) \leq \gamma_{sid}(G) \leq p - \Delta_N(G) \leq p - \Delta_E(G)$$

$$\gamma_{if}(G) \leq \gamma_{wid}(G) \leq p - \delta_N(G) \leq p - \delta_E(G)$$

- If D is a minimal strong (weak) dominating set of a connected IFG G, then V-D is weak (strong) dominating set of G.

- For a complete IFG, G. $v, u \in V$ be the vertices having the minimum and maximum vertex cardinality in G respectively.

$$\gamma_{sid}(G) = \left[\frac{1+\mu_1(v)-\gamma_1(v)}{2} \right] \text{and}$$

$$\gamma_{wid}(G) = \left[\frac{1+\mu_1(u)-\gamma_1(u)}{2} \right]$$

3. DOMINATING CRITICAL

3.1 Definition

Let G be an intuitionistic fuzzy graph. We partition the vertices of G into three disjoint sets according to how their removal affects γ(G). Let $V = V^0 \cup V^+ \cup V^-$ for

$V^0 = \{v \in V: \gamma(G) = \gamma(G-v)\}$

$V^+ = \{v \in V: \gamma(G) < \gamma(G-v)\}$

$V^- = \{v \in V: \gamma(G) > \gamma(G-v)\}$

3.2 Definition

Let G be an intuitionistic fuzzy graph. We partition the vertices of G into three disjoint sets according to how their removal affects $\gamma_{sif}(G)$. Let $V = V_{sif}^{\ 0} \cup V_{sif}^{\ +} \cup V_{sif}^{\ -}$ for

$V_{sif}^{\ 0} = \{v \in V: \gamma_{sif}(G) = \gamma_{sif}(G-v)\}$

$V_{sif}^{\ +} = \{v \in V: \gamma_{sif}(G) < \gamma_{sif}(G-v)\}$

$V_{sif}^{\ -} = \{v \in V: \gamma_{sif}(G) > \gamma_{sif}(G-v)\}$

Similarly for weak dominating set.

3.3 Main Results

3.3.1 Theorem

Let G be an intitionistic fuzzy graph. Let v ∈ V have the greatest neighborhood degree in G, that is $\Delta_N(G) = d_N(v)$, then $v \in V^+$.

Proof: Let G be an intuitionistic fuzzy graph. . Let v ∈ V have the greatest neighborhood degree in G, that is $\Delta_N(G) = d_N(v)$. Clearly large number of strong arc incident at v. Therefore v dominates more vertices in G. Hence v belongs to γ_{if}-set of G. The vertex v dominates N(v). If we remove v in G, no vertex dominates N(v) in γ-set. So we add some vertex in D-v, say u. The set {D-{v}} ∪{u} is a dominating set of G-{v}.This implies $|D|_{if} < |(D-\{v\}) \cup \{u\}|_{if}$, since σ(v)< σ(u), therefore v∈D. So we get $\gamma_{if}(G-v) > \gamma_{if}(G)$, $v \in V^+$. Hence proved.

3.3.2 Theorem

Let G be an intuitionistic fuzzy graph. Let $\Delta_N(G) = d_N(v)$, if x ∈ N(v) and x ∉ D then $x \in V^0$. Here D is a γ-set of G.

Proof**:** Let G be a fuzzy graph. Let v∈V such that $\Delta_N(G) = d_N(v)$. We know that v∈D, D is a γ_{if}-set of G. let x ∈ N(v) , v dominates x in G. Therefore we remove x in G, there is no change in dominating set D in G. Hence $\gamma_{if}(G-v) = \gamma_{if}(G)$. Therefore $x \in V^0$. Hence proved.

3.3.3 Theorem

For any intuitionistic fuzzy graph G=(V,σ,μ) if V- ={v} then A*(v)=φ. Here A*(v)={u: u∉D and N(u)∩D={v}}.

Proof**:** Suppose V- ={v}, this implies $\gamma_{if}(G-v) < \gamma_{if}(G)$ for some vertex v ∈ V and D' be the γ-set of , but D' need not be a dominating set of G. Therefore D=D'∪{v} is a G. Hence A*(v) =φ. Hence proved.

3.3.4 THEOREM

If the removal of a vertex v from G increase γ(G), then (i) v is not an isolated vertex and end vertex ,and (ii) There is no dominating set D for G-N[v] having $|D|_f$ vertices which also dominates N(v). For some γ_{if}-set D containing v.

Proof:

a. Given $v \in V^+$, therefore $\gamma_{if}(G-v) > \gamma_{if}(G)$ and v∈γ-set D. Then clearly v is not an isolated and not a end vertex. Suppose v is an isolated or end vertex, then v is dominated by itself, since v∈D. So we get γ(G-v) ≤ γ(G), a contradiction.
b. Assume there exist a dominating set of G-v with ⊡D⊡f vertices. D is a γ-set of G. Then γ(G-v) ≤ γ(G), a contradiction. Hence proved.

3.3.5 Theorem

If there exist at least u_1, u_2∈A*, then $\gamma_{if}(G-v) > \gamma_{if}(G)$,for some γ-set D containing V.

Proof**:** Suppose there exist at least u_1, u_2∈A*, such that u_1 and u_2 are not dominated by removing v, there are no vertex in D dominates u_1 and u_2, hence $\gamma_{if}(G-v) > \gamma_{if}(G)$. Hence proved.

3.3.6 Theorem

If u_i ∈A*(v), and u_i ∉D then $u_i \in V_0$ for some γ_{if}-set D containing v.

Proof**:** Suppose u_i ∈A*(v) and u_i ∉D such that u_i are dominated by v∈D, therefore $\gamma_{if}(G-u_i) = \gamma_{if}(G)$. Hence $u_i \in V^0$

. Hence proved.

3.3.7 Theorem

If G be an intuitionistic fuzzy graph, there exist vertex u∈V, u is a cut node in G, D is a γ_{if}-set of G, then u∈V^-

Proof: Let G be a fuzzy graph. Let vertex u∈V. u is a cut node in G and u∉D. therefore u is dominated by some vertex in D, u is reduce the connectedness of G. This implies we remove strong arc between u and other vertices. The graph is disconnected. The two components are dominated by the vertex in D. Therefore γ_{if}(G- u) = γ_{if}(G). Therefore u∈V^- . Hence proved.

3.3.8 Theorem

Let G be an intuitionistic fuzzy graph. D be the γ_{sif}- set in G. v∈V, Δ_N(G)= d_N(v), then v∈$V_{sif}{}^+$.

Proof: Let G be a fuzzy graph. D be the γ_{sif}- set in G. v∈V, Δ_N(G)= d_N(v),clearly v is in γ_{sif}- set. In G-v the vertex in N(v) is does not dominated by D. Therefore D-v is not a dominating set of G. Therefore some other vertex are strongly dominate the vertex in N(v). Therefore we get γ_{sif}(G- v) > γ_{sif}(G). Hence v∈$V_{sif}{}^+$.

3.3.9 Theorem

Let G be an intuitionistic fuzzy graph. D be the γ_{sif}- set in G. let v∈V, Δ_N(G)= d_N(v).let u∈V-D and N(u)∩D ={v} then u∈$V_{sif}{}^0$.

Proof: Let G be a fuzzy graph. D be the γs- set in G. let v∈V, Δ_N(G)= d_N(v).let u∈V-D and N(u)∩D ={v}.Note that v∈D, v is strongly dominate G-u. Therefore γ_{ifs}(G- v) = γ_{sif}(G). i.e. u∈$V_{sif}{}^0$ .Hence proved.

3.3.10 Theorem

Let G be a complete intuitionistic fuzzy graph. v is the vertex having the minimum membership value in G. Then the cardinality of v is equal γ_{ifs}(G).

Proof: Let G be a complete IFG. the cardinality of v is σ_1, σ_1 is the minimum cardinality of G. G is an complete IFG therefore there is a strong arc between every pair of vertices, $d_N(v_i)$ ≤ d_N(v) for all v_i ∈V. $d_N(v_i)$= P-σ(v_i),since G is complete, here p is a order of G. p-σ(v_i)< p-σ(v),since the cardinality of v is σ_1 . Therefore v strongly dominates V. Hence `γ_{sif}-set D={v}. Hence proved.

Note: In a complete fuzzy graph, D be the γ_{sif}-set of G, then $V_{ifs}{}^+$ = D and D be the γ_{wif}-set of G, then V_{wif}- = D.

3.3.11 Theorem

Let G be an intuitionistic fuzzy graph. D be the γ_{sif}-set of G, then $V_{sif}{}^0$ =V- D.

Proof**:** Let G be a complete fuzzy graph. D be the γ_{sif}-set of G. D contain the vertex v, such that σ(v)=σ_1, σ_1 is the minimum membership value in G. D={v} there is an strong between V and the other vertices in G. Therefore we remove vertex u∈N(v) is does not affect the minimum strong dominating set. Since V-D =N(v). So v is dominating the vertices in G-u. Hence γ_{sif}(G-u)= γ_{sif}(G), i.e. u∈V^0 and V-D = N(V). Therefore V-D=V_{ifs}0 .Hence proved.

3.3.12 Theorem

Let G be a complete intuitionistic fuzzy graph. D be the γ_{sif}-set of G, then $V_{sif}{}^-$ =φ.

Proof: Let G be a complete IFG. D be the γs-set of G. The previous theorem $V_{isf}{}^+$ =D and $V_{isf}{}^0$ =V-D. We know that V = $V_{sif}{}^0$ ∪$V_{sif}{}^+$ ∪$V_{sif}{}^-$, in a complete IFG V = $V_{sif}{}^0$ ∪$V_{sif}{}^+$. Clearly $V_{sif}{}^-$ =φ. Hence proved.

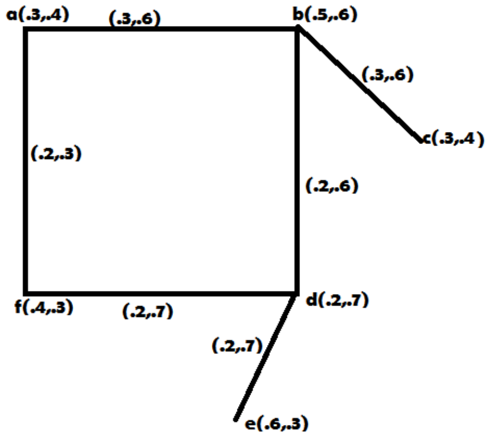
3.3.13 Theorem

Let G be an intuitionistic fuzzy graph. D be the γ_{wif}-set of G, then $V_{wif}{}^0$ =V- D.

3.3.14 Theorem

Let G be an intuitioistic complete fuzzy graph. D be the γ_{wif}-set of G, then $V_{wif}{}^-$ =φ.

Example



For the above intuitionistic fuzzy graphs, Dominating set of G $D=\{b,d\}$ and $\gamma(G)=.65$ $V_{sif}{}^0$={a,c,e,f}, $V_{sif}{}^{--}$={a}, $V_{sif}{}^+$={b,d,e}

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