What is a minimum of Unpredictable Workload Pattern over all Elastic Scaling in Cloud Computing?

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ABSTRACT
Measurability is a concept in elastic scaling that is based on two assumptions: (1) every cloud service provider is cautious, i.e., does not exclude any cloud consumer's Unpredictable Workload resource pooling pattern choice from consideration, and (2) every cloud service provider respects the cloud consumer's Unpredictable Workload resource pooling pattern preferences, i.e., deems one cloud consumer's Unpredictable Workload resource pooling pattern choice to be infinitely more likely than another whenever it premises the cloud consumer to prefer the one to the other. In this paper we provide a new approach for measurability, by assuming that cloud service providers have asymmetric Unpredictable Workload resource pooling pattern about the cloud consumer's Unpredictable Workload utilities. We show that, if the uncertainty of each cloud service provider about the cloud consumer's Unpredictable Workload utilities vanishes gradually in some regular manner, then the Unpredictable Workload resource pooling pattern choices it can measurably make under common conjecture in measurability are all actually measureable in the original elastic scaling with no uncertainty about the cloud consumer's utilities.

Keywords
Cloud service provider, cloud consumer, Unpredictable Workload, asymmetric, resource pooling pattern, utilities, elastic scaling, behavioral, measurably

1. INTRODUCTION
Elastic scaling deals with the ways the cloud service providers may reason about its cloud consumers before making a decision. More precisely, in elastic scaling cloud service providers base its Unpredictable Workload resource pooling pattern choices on the conjectures about the cloud consumers' behavior, which in turn depend on its conjectures about the cloud consumers' conjectures about other cloud consumers' behavior, and so on [1] [7] [9] [21]. A major goal of elastic scaling in this work is to study such conjecture hierarchies, to impose reasonable conditions on these, and to investigate its resource pooling pattern behavioral implications.

A central idea in elastic scaling is common conjecture in measurability, stating that a cloud service provider premises that its cloud consumers choose measurably, and so on. In our view, one of its most natural refinements is the concept of measurability. Measurability is based on the following two conditions: The first states that cloud service providers are cautious [2] [8] [10] [22], meaning that they do not exclude any cloud consumers' Unpredictable Workload resource pooling pattern choice from consideration. The second condition states that whenever premise that a Unpredictable Workload resource pooling pattern choice a is better than another Unpredictable Workload resource pooling pattern choice b for a cloud consumer, then the probability assign to b must be at most α times the probability assign to a. Under α-measurability there is common conjecture in the event that every cloud service provider is cautious and satisfies the α-actual trembling condition. A Unpredictable Workload resource pooling pattern choice is called actually measureable if it can be chosen under α-measurability for every $\alpha > 0$ [3] [11] [15] [20].

2. RESEARCH CLARIFICATION
The usual interpretation of measurability assumes that cloud consumer makes mistakes, but that deem more costly mistakes much less likely than less costly mistakes. In this paper we offer a rather different approach for measurability. Instead of assuming premise cloud consumer to make mistakes, we rather suppose that have uncertainty about its utility function, while believing that it chooses measurably. We thus consider a elastic scaling with asymmetric Unpredictable Workload resource pooling pattern. Our result states that, if we let uncertainty about the cloud consumer's utility go to zero in some regular manner, then every Unpredictable Workload resource pooling pattern choice that can measurably be made under common conjecture in measurability in the elastic scaling with asymmetric Unpredictable Workload resource pooling pattern, will be actually measureable in the original elastic scaling, in which there is no uncertainty about the cloud consumer's utilities.

In the elastic scaling with asymmetric Unpredictable Workload resource pooling pattern, we impose some regularity conditions on the cloud service providers' conjectures about the cloud consumer's utility functions which can be summarized as follows: First, for every outcome in the elastic scaling, the conjecture that cloud service provider i has about cloud service provider j's utility from this outcome, is always normally distributed with its mean at the "original" utility in the original elastic scaling. As a consequence, cloud service provider i deems any utility function possible for cloud service provider j, and hence every resource pooling pattern choice for cloud service provider j can be optimal for some utility function deemed possible by i. Together with the condition that i premises in j's measurability, this actually makes sure that cloud service provider i deems every Unpredictable Workload resource pooling pattern choice possible for cloud service provider j, thus mimicking the cautiousness condition described above. Secondly, i's conjecture about j's utility function should be independent from its conjecture about j's conjecture hierarchy. This makes intuitive sense since j's conjecture hierarchy is analytic property of this cloud service provider, whereas its utility function is not analytic property [4] [12]

[16] [23]. Therefore there is no obvious reason to expect any correlation between these two characteristics. Thirdly, i's conjecture about j's utilities from different outcomes in the elastic scaling should be independent from each other. Possibly some of these conditions can be relaxed for the proof of our result, but we leave this issue for future research.

The paper is organized as follows: In Section 3 we introduce our elastic scaling model [5] [13] [17] [24] for elastic scaling with asymmetric Unpredictable Workload resource pooling pattern, we formalize the idea of common conjecture in measurability for these elastic scaling, and show that common conjecture in measurability is always possible (Descriptive Study I). In Section 4 we introduce our elastic scaling model for elastic scaling with symmetric Unpredictable Workload resource pooling pattern, and present the concept of measurability for these elastic scaling (Prescriptive Study). In Section 5 we state our result, establishing the connection between common conjecture in measurability in the elastic scaling with asymmetric Unpredictable Workload resource pooling pattern in the presence of small uncertainty about the cloud consumer's utility function, and measurability in the original elastic scaling (Descriptive Study II). In Section 6 we provide some concluding remarks. All proofs are collected in Section 7.

3. DESCRIPTIVE STUDY I

3.1 Elastic Model

Throughout this paper we restrict attention to elastic scaling operations with two sets of cloud service provider. Let $\delta = (C_i, w_i)_{i\epsilon I}$ be a finite, Unpredictable Workload where $I = \{1, 2\}$ is the set of cloud service providers, C_i is the finite set of Unpredictable Workload resource pooling pattern choices of cloud service provider i, w_i is cloud service provider i's utility function. The function w_i assigns to every pair of Unpredictable Workload resource pooling pattern choice $(c_1, c_2) \in C_1 \times C_2$ a utility $w_i(c_1, c_2) \in F$.

In a elastic scaling with asymmetric Unpredictable Workload resource pooling pattern, cloud service providers do not only uncertainty about the cloud consumer's Unpredictable Workload resource pooling pattern choices; they also have uncertainty about the cloud consumer's utility function. Hence a conjecture hierarchy should not only specify what the cloud service provider premises about the cloud consumer's Unpredictable Workload resource pooling pattern choice but also what it premises about the cloud consumer's utility function. Not only this, it should also specify what the cloud service provider premises about the cloud consumer's conjecture about its own Unpredictable Workload resource pooling pattern choice and utility function, and so on. A possible way of modeling such conjecture hierarchies is by means of the following necessary and sufficient condition.

Necessary and sufficient condition 3.1 (*elastic scaling model*). A finite elastic scaling model for δ with asymmetric Unpredictable Workload resource pooling pattern is a tuple $M = (S_i, v_i, K_i)_{i\epsilon I}$ where (1) S_i is the set of Unpredictable Workload types for cloud service provider i. (2) $v_i: S_i \to \theta(C_j \times S_j)$ is the conjecture assignment taking only finitely many different probability distributions on $\theta(C_j \times S_j)$ and (3) k_i is the utility assignment that assigns to every $s_i \in S_i$ a utility function $k_i(s_i): C_1 \times C_2 \to F$. By $\theta(P)$ we denote the set of probability distributions on P. Therefore, in a elastic scaling model, each Unpredictable Workload type s_i has a conjecture about cloud service provider j's resource pooling pattern choice-Unpredictable Workload type combinations. And hence, in particular, it has a conjecture about j's resource pooling pattern choice. But, as cloud service provider j's Unpredictable Workload type also specifies its utility function and its conjecture about i's resource pooling pattern choice, cloud service provider i also has some conjecture about cloud service provider j's utility function, and about cloud service provider j's conjecture about its own resource pooling pattern choice, and so on. In this way one can derive a complete conjecture hierarchy for every given Unpredictable Workload type.

Note that each Unpredictable Workload type s_i can be identified with a pair $\big(k_i(s_i), v_i(s_i)\big)$ where $k_i(s_i)$ is its utility function and $v_i(s_i)$ is its conjecture hierarchy. Since we required the conjecture assignment to take only finitely many different probability distributions, the elastic scaling model contains only finitely many different conjecture hierarchies.

3.2 Limitations on the Elastic Model

Our goal will be to model the situation where the cloud service providers have uncertainty about the cloud consumer's utility function, but where this uncertainty "vanishes in the limit". In order to formalize this we need to impose additional limitations on the elastic scaling-model.

Recall that every Unpredictable Workload type s_i can be identified with a pair $\big(k_i(s_i), v_i(s_i)\big)$, where $k_i(s_i)$ is s_i's utility function and $v_i(s_i)$ is its conjecture hierarchy. Denote by K_i the set of all possible utility functions, and by V_i the set of all conjecture hierarchies in the elastic scaling model $M = (S_i, k_i, v_i)_{i\epsilon I}$. The first condition we impose is that $S_i = K_i \times V_i$, that is, for every possible utility function we can think of, and every conjecture hierarchy in the model, there exists a Unpredictable Workload type in the model with exactly this combination of utility function and conjecture hierarchy. Therefore in a sense we assume that the Unpredictable Workload type is rich enough.

Secondly, we assume that s_i's conjecture about j's utility from (c_1, c_2) is statistically independent from its conjecture j's utility from (c_1', c_2') whenever $(c_1, c_2) \neq (c_1', c_2')$ and that this conjecture is also statistically independent from its conjecture about j's conjecture hierarchy.

Finally we assume that s_i's conjectures about j's utilities from the various outcomes in the elastic scaling are all induced by a unique normal distribution. More formally, s_i's conjecture about j's utility from (c_1, c_2) is given by a normal distribution with its mean at $w_j(c_1, c_2)$ – the "true" utility of cloud service provider j in the original elastic scaling. Therefore, all these conjectures are distributed identically around the mean. By collecting all these conditions we arrive at the following necessary and sufficient condition.

Necessary and sufficient condition 3.2 *(σ–regular elastic scaling model).* Let D be the normal distribution on F with mean 0 and variance $\sigma^2 > 0$. Then a elastic scaling model $M = (S_i, k_i, v_i)_{i\epsilon I}$ is σ–regular if for both cloud service providers i, (1) $S_i = K_i \times V_i$, (2) for every Unpredictable Workload type $s_i \in S_i$, its conjecture about j's utility from (c_1, c_2) is statistically independent from its conjecture about j's utility from (c_1', c_2') whenever $(c_1, c_2) \neq (c_1', c_2')$ and its conjecture about j's utilities is statistically independent from its conjecture about j's conjecture hierarchy, and (3) for every Unpredictable Workload type $s_i \in S_i$, and every resource pooling pattern choice-pair (c_1, c_2), the conjecture of s_i about j's utility from (c_1, c_2) is given by D, upto a shift of the mean to $w_j(c_1, c_2)$.

3.3 σ-Measurability

In this subsection we will define common conjecture in measurability inside a elastic scaling model with asymmetric Unpredictable Workload resource pooling pattern. In addition, if we require the elastic scaling-model to be σ-regular for a given normal distribution with mean 0 and variance σ^2, then we obtain the concept of σ-measurability. We first need some more notations. For given Unpredictable Workload type s_i and Unpredictable Workload resource pooling pattern choice c_i, let $k_i(s_i)(c_i)$ be the expected utility for Unpredictable Workload type s_i from choosing c_i, given its conjecture $v_i(s_i)$ about the cloud consumer's Unpredictable Workload resource pooling pattern choice, and given its utility function $k_i(s_i)$.

Necessary and sufficient condition 3.3 *(Measureable Unpredictable Workload resource pooling choice).* A Unpredictable Workload resource pooling pattern choice c_i is measureable for s_i if $k_i(s_i)(c_i) \geq k_i(s_i)(\acute{c}_i)$ for all $\acute{c}_i \in C_i$.

We will now define common conjecture in measurability. In words it says that a cloud service provider premises that its cloud consumer makes measureable Unpredictable Workload resource pooling pattern choices, and premises that its cloud consumer premises that it makes measureable Unpredictable Workload resource pooling pattern choices, and so on [25].

Formally, for every $\widehat{S_i} \subseteq S_i$, let

$(C_i \times \widehat{S_i})^{quant} = \{(c_i, s_i) \in C_i \times \widehat{S_i} : c_i \text{ is measureable for } s_i\}$.

Necessary and sufficient condition 3.4 *(Common conjecture in Measurability).* For cloud service providers i we define subsets of Unpredictable Workload types $S_i^1, S_i^2, \ldots$ in a recursive way as follows:

$$S_i^1 := \left\{s_i \in S_i : v_i(s_i)\left[\left(C_j \times S_j\right)^{quant}\right] = 1\right\},$$

$$S_i^2 := \left\{s_i \in S_i : v_i(s_i)\left[\left(C_j \times S_j^1\right)^{quan\,t}\right] = 1\right\},$$

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$$.S_i^l := \left\{s_i \in S_i : v_i(s_i)\left[\left(C_j \times S_j^{l-1}\right)^{quant}\right] = 1\right\},$$

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Unpredictable Workload type s_i expresses common conjecture in measurability if $s_i \in \cap_{l \in \mathbb{N}} S_i^l$. A Unpredictable Workload type σ–measureable if it expresses common conjecture in measurability with a σ–regular elastic scaling model.

Necessary and sufficient condition 3.5 *(σ–measureable Unpredictable Workload type).* Let $M = (S_i, v_i, k_i)_{i \in I}$ be a σ–regular elastic scaling model. Every Unpredictable Workload type $s_i \in S_i$ that expresses common conjecture in measurability is called σ–measureable.

Now we show that σ–measureable Unpredictable Workload types always exist.

Proposition 3.1 (σ–measureable Unpredictable Workload types always exist): Consider a finite Unpredictable Workload $\delta = (C_i, w_i)_{i \in I}$, and some $\sigma > 0$. Then there is a σ–regular elastic scaling model $M = (S_i, v_i, k_i)_{i \in I}$ for δ where all Unpredictable Workload types are σ–measureable. The proof can be found in Section 7.

3.4 Limit Measurability

In this subsection we focus on those Unpredictable Workload resource pooling pattern choices, which can measurably be made under common conjecture in measurability when the uncertainty about the cloud consumer's utility vanishes. This will lead to the concept of limit measurability. We first need an additional necessary and sufficient condition.

Necessary and sufficient condition 3.6 *(Constant Unpredictable Workload type and utility assignments).* A Unpredictable Workload sequence of elastic scaling models $((S_i^n, v_i^n, k_i^n)_{i \in I})_{n \in \mathbb{N}}$ has constant Unpredictable Workload type and utility assignments if $S_i^n = S_i^m$ and $k_i^n = k_i^m$ for all n and m, and for cloud service providers i.We are now ready to say the concept of limit measureable Unpredictable Workload resource pooling pattern choice.

Necessary and sufficient condition 3.7 *(Limit measureable resource pooling pattern choice).* Consider a finite Unpredictable Workload $\delta = (C_i, w_i)_{i \in I}$ with cloud service providers. A Unpredictable Workload resource pooling pattern choice c_i is limit measureable if there is a Unpredictable Workload sequence $(\sigma_n)_{n \in \mathbb{N}} \to 0$,and a Unpredictable Workload sequence $(M^n)_{n \in \mathbb{N}}$ of σ_n–regular elastic scaling models with constant Unpredictable Workload type and utility assignments, such that in every M^n there is a σ_n-measureable Unpredictable Workload type s_i^n with utility function w_i, for which Unpredictable Workload resource pooling pattern choice c_i is optimal.

4. PRESCRIPTIVE STUDY

4.1 Elastic Model

Let $\delta = (C_i, w_i)_{i \in I}$ be a finite, Unpredictable Workload with cloud service providers. In a elastic scaling with symmetric Unpredictable Workload resource pooling pattern cloud service providers do not have uncertainty about the cloud consumer's utility function. Therefore a conjecture hierarchy only needs to specify what a cloud service provider premises about the cloud consumer's Unpredictable Workload resource pooling pattern choice, what it premises about the cloud consumer's conjecture about its own Unpredictable Workload resource pooling pattern choice, and so on. Therefore the elastic scaling model will be simpler compared to the case of asymmetric Unpredictable Workload resource pooling pattern.

Necessary and sufficient condition 4.1 *(elastic scaling model).* A elastic scaling model for δ with symmetric Unpredictable Workload resource pooling pattern is a tuple $M = (\Omega_i, \rho_i)_{i \in I}$ where (1) Ω_i is the finite set of Unpredictable Workload types for cloud service provider i, and (2) $\rho_i : \Omega_i \to \theta(C_j \times \Omega_j)$ is the conjecture assignment.

Therefore, in a elastic scaling model, each Unpredictable Workload type τ_i has a conjecture about cloud service provider j's Unpredictable Workload resource pooling pattern choice-Unpredictable Workload type combinations. And hence, in particular, it has a conjecture about j's Unpredictable Workload resource pooling pattern choice. But, as cloud service provider j's Unpredictable Workload type also specifies its conjecture about cloud service provider i's Unpredictable Workload resource pooling pattern choice, cloud service provider i also has some conjecture about cloud service provider j's conjecture about its own Unpredictable Workload resource pooling pattern choice, and so on. In this way one can derive a complete conjecture hierarchy for every given Unpredictable Workload type.

For given Unpredictable Workload type τ_i and Unpredictable Workload resource pooling pattern choice c_i we define $w_i(c_i, \tau_i)$ as the expected utility for Unpredictable Workload type τ_i from choosing c_i given its conjecture $\rho_i(\tau_i)$ about its cloud consumer's Unpredictable Workload resource pooling pattern choice (and *given* its "fixed" utility function w_i). Unpredictable Workload type τ_i is said to *prefer* Unpredictable Workload resource pooling pattern choice c_i to Unpredictable Workload resource pooling pattern choice $\acute{c}_i$ when $w_i(c_i, \tau_i) > w_i(\acute{c}_i, \tau_i)$. We say that a Unpredictable Workload type τ_i *considers possible* some cloud consumer's Unpredictable Workload type τ_i if $\rho_i(\tau_i)(c_j, \tau_j) > 0$ for some $c_j \in C_j$.Now we introduce the key condition in measurability, which is the α–actual trembling condition. Intuitively it says that (1) a cloud service provider should deem possible all cloud consumer's Unpredictable Workload resource pooling pattern choices, and (2) if a cloud service provider premises Unpredictable Workload resource pooling pattern choice a is better than Unpredictable Workload resource pooling pattern choice b for the other cloud service provider, then it should deem Unpredictable Workload resource pooling pattern choice a much more likely than Unpredictable Workload resource pooling pattern choice b.

Necessary and sufficient condition 4.2 *(α-actual trembling condition)*: Let $\alpha > 0$. A Unpredictable Workload type τ_i satisfies the α-actual trembling condition if (1) for each τ_j that τ_i deems possible, $\rho_i(\tau_i)(c_j, \tau_j) > 0$ for all $c_j \in C_j$,and (2) for every τ_j that τ_i deems possible, whenever τ_j *prefers* c_j to $\acute{c}_j$, then $\rho_i(\tau_i)(\acute{c}_j, \tau_j) \leq \alpha \cdot \rho_i(\tau_i)(c_j, \tau_j)$.

Therefore, the first condition says that whenever τ_i deems some Unpredictable Workload type τ_j possible, τ_i also assumes every Unpredictable Workload resource pooling pattern choice is possible for τ_j. Measurability is based on the event that the Unpredictable Workload types should not only satisfy the α-actual trembling condition themselves, but also express common conjecture in the event that Unpredictable Workload types satisfy the α-actual trembling condition.

Necessary and sufficient condition 4.3 *(α-actually measureable Unpredictable Workload type)*. A Unpredictable Workload type τ_i is α-actually measureable if: τ_i satisfies the α-actual trembling condition, τ_i only deems possible cloud consumer's Unpredictable Workload types τ_j which satisfy the α-actual trembling condition, τ_i only deems possible cloud consumer's Unpredictable Workload types τ_j which only deem possible cloud service provider i's Unpredictable Workload types τ_i' which satisfy the α-actual trembling condition, and so on. Actually measureable Unpredictable Workload resource pooling pattern choices are those Unpredictable Workload resource pooling pattern choices, which can measurably be made by α-actually measureable Unpredictable Workload types for all α.

Necessary and sufficient condition 4.4 *(Actually measureable resource pooling pattern choice)*. A Unpredictable Workload resource pooling pattern choice c_i is α-actually measureable if there is a elastic scaling model and a α-actually measureable Unpredictable Workload type τ_i within it for which c_i is optimal. A Unpredictable Workload resource pooling pattern choice c_i is actually measureable if it is α-actually measureable for all $\alpha > 0$.

5. DESCRIPTIVE STUDY II

5.1 Statement of the result

For a Unpredictable Workload we analyzed two contexts, one with asymmetric Unpredictable Workload resource pooling pattern and another with symmetric Unpredictable Workload resource pooling pattern. In the context with asymmetric Unpredictable Workload resource pooling pattern, where cloud service providers have uncertainty about the cloud consumer's utility, we introduced the concept of a limit measureable Unpredictable Workload resource pooling pattern choice. In the context with symmetric Unpredictable Workload resource pooling pattern, where cloud service providers have no uncertainty about the cloud consumer's utility, we discussed the concept of a actually measureable Unpredictable Workload resource pooling pattern choice. In our result we connect these two concepts.

Proposition 5.1 *(Limit Measurability implies Measurability)*: Consider a finite Unpredictable Workload with cloud service providers. Every limit measureable Unpredictable Workload resource pooling pattern choice for the context with asymmetric Unpredictable Workload resource pooling pattern is a actually measureable Unpredictable Workload resource pooling pattern choice for the context with symmetric Unpredictable Workload resource pooling pattern.

5.2 Illustration of the result

By means of an example we provide some intuition for our result. More precisely we show how a measureable Unpredictable Workload type in the context of asymmetric Unpredictable Workload resource pooling pattern can be transformed into an actually measureable Unpredictable Workload type in the context of symmetric Unpredictable Workload resource pooling pattern. Also we show that when σ goes to zero then ϵ goes to zero as well. Let us start with the context of asymmetric Unpredictable Workload resource pooling pattern. Let D be the normal distribution with mean 0 and variance σ^2. From the proof of Proposition 3.1 we know that there exists a regular elastic scaling model $M = (S_i, v_i, k_i)_{i \in I}$ where every Unpredictable Workload type is measureable and all the Unpredictable Workload types have the same conjecture hierarchy. Therefore, Unpredictable Workload types only differ by their utility function. For each of the Unpredictable Workload types s_1 of cloud service provider 1 we denote by ρ_1 the conjecture about cloud service provider 2's Unpredictable Workload resource pooling pattern choice, and for each Unpredictable Workload type s_1 let ρ_1 be the conjecture about cloud service provider 1's Unpredictable Workload resource pooling pattern choice. As we assume that all the Unpredictable Workload types have the same conjecture hierarchy, ρ_1 and ρ_2 are unique.

For both cloud service providers i let O_i be the probability distribution on cloud service provider i's utility functions generated by D. Since the elastic scaling-model is σ-regular every Unpredictable Workload type s_j has the conjecture O_i about i's utility function. Let $K_i(c_i, \rho_i)$ be the set of utility functions for cloud service provider i such that the Unpredictable Workload resource pooling pattern choice c_i is optimal under the conjecture ρ_i about the cloud consumer's Unpredictable Workload resource pooling pattern choice. Since every Unpredictable Workload type s_i expresses common conjecture in measurability, the probability it assigns to a cloud consumer's Unpredictable Workload resource pooling pattern choice c_j is exactly the probability it assigns to the event that j's utility function is in $K_j(c_j, \rho_j)$ which is $O_j\left(K_j(c_j, \rho_j)\right)$.

Since D has full support, it follows that all these probabilities are positive. Now we turn to the context of symmetric Unpredictable Workload resource pooling pattern. We construct a elastic scaling model with a single Unpredictable Workload type τ_1 for cloud service provider 1 and a single Unpredictable Workload type τ_2 for cloud service provider 2. Let the conjecture of τ_1 about the cloud service provider 2's Unpredictable Workload resource pooling pattern choice be given by the ρ_1 constructed above, and similarly for the conjecture of τ_2. Therefore, the conjecture about the cloud consumer's Unpredictable Workload resource pooling pattern choice has not changed by moving from the context with asymmetric Unpredictable Workload resource pooling pattern to the context with symmetric Unpredictable Workload resource pooling pattern.

6. CONCLUDING REMARKS

We premise that measurability is a very natural concept in elastic scaling, but it has not yet received the attention it deserves. In this paper we have established a new approach for measurability from the viewpoint of elastic scaling with asymmetric Unpredictable Workload resource pooling pattern. In elastic scaling with asymmetric Unpredictable Workload resource pooling pattern we define a Unpredictable Workload resource pooling pattern choice as limit measureable if it can measurably be made under common conjecture of measurability when the uncertainty vanishes gradually in some regular way. We show the existence of such Unpredictable Workload resource pooling pattern choices. We then prove that each limit measureable Unpredictable Workload resource pooling pattern choice in the elastic scaling with asymmetric Unpredictable Workload resource pooling pattern is actually measureable for the context with symmetric Unpredictable Workload resource pooling pattern.

7. PROOFS

7.1 Existence of Measureable Unpredictable Workload types

We prove Proposition 3.1, which guarantees the existence of σ-measureable Unpredictable Workload types. Consider a finite Unpredictable Workload $M = (C_i, w_i)_{i\epsilon I}$ and, some $\sigma > 0$. Let D be the normal distribution with mean 0 and variance σ^2. In fact we will construct a σ-regular elastic scaling model where all Unpredictable Workload types of cloud service provider 1 have the same conjecture ρ_2 about cloud service provider 2's Unpredictable Workload resource pooling pattern choice and all Unpredictable Workload types of cloud service provider 2 have the same conjecture ρ_1 about cloud service provider 1's Unpredictable Workload resource pooling pattern choice. We construct ρ_1 and ρ_2 by means of the fixed key of some correspondence.

For every conjecture $\rho_j \,\epsilon\, \theta(C_j)$ and every utility function w_i, we define

$$C_i(\rho_j, w_i) := \{C_i \,\epsilon\, C_i : w_i(c_j, \rho_j) \geq w_i(\acute{c}_i, \rho_j) \text{ for all } \acute{c}_i\}.$$

We also define O_i as the probability distribution on the set of utility functions of cloud service provider i induced by D. For every $\rho_j \,\epsilon\, \theta(C_j)$ we define

$$G_i(\rho_j) := \{\rho_i \,\epsilon\, \theta(C_j) : \rho_i = \int_{w_i \,\epsilon\, K_i} \phi_i(x_i)\; dO_i,$$

where $\phi_i(x_i) \,\epsilon\, \left(C_i(\rho_j, x_i)\right)$ for every $x_i \,\epsilon\, K_i\}$.

Here K_i denotes the set of all possible utility functions for cloud service provider i. Therefore every $\rho_i \,\epsilon\, G_i(\rho_j)$ is obtained by taking for every utility function x_i a randomization over optimal Unpredictable Workload resource pooling pattern choices against ρ_j and then taking the expected randomization with respect to O_i. Now we define a correspondence G from $\theta(C_1) \times \theta(C_2)$ to $\theta(C_1) \times \theta(C_2)$ by

$$G(\rho_1, \rho_2) := G_1(\rho_2) \times G_2(\rho_1).$$

Now we use fixed key position to prove that G has a fixed key. Clearly G is upper hemi-continuous and compact valued. We show that G is convex valued. For this it is sufficient to show that G_1 and G_2 are convex valued. For a given ρ_2, take $\rho_1^{'}$, $\rho_1^{''}$ in (ρ_2). We show that $\psi\rho_1^{'} + (1-\psi)\rho_1^{''}$ is also in $G_1(\rho_2)$. By definition

$$\rho_1^{'} = \int_{x_1} \phi_1^{'}(x_1)\, dO_1 \text{ and } \rho_1^{''} = \int_{x_1} \phi_1^{''}(x_1)\; dO_1$$

where $\phi_1^{'}(x_1)$, $\phi_1^{''}(x_1) \,\epsilon\, \theta\left(C_1(\rho_2, x_1)\right)$ for every x_1. Therefore we have

$$\psi\rho_1^{'} + (1-\psi)\rho_1^{''} = \int_{x_1} (\psi\phi_1^{'}(x_1) + (1-\psi)\phi_1^{''}(x_1))\, dO_1$$

where $\psi\phi_1^{'}(x_1) + (1-\psi)\phi_1^{''}(x_1) \,\epsilon\, \theta\left(C_1(\rho_2, x_1)\right)$ for every x_1. Hence by definition $\psi\rho_1^{'} + (1-\psi)\rho_1^{''} \epsilon\, G_1(\rho_2)$. This implies that G_1 is convex valued. The same applies to G_2 and hence we can conclude that G is convex valued. Now using fixed key position G has a fixed key (ρ_1^*, ρ_1^*).

Since $\rho_1^* \epsilon\, G_1(\rho_2^*)$ it follows that

$$\rho_1^* = \int_{x_1} \phi_1^*(x_1)\; dO_1$$

where $\phi_1^*(x_1) \,\epsilon\, \theta\left(C_1(\rho_2^*, x_1)\right)$ for every x_1. Similarly

$$\rho_2^* = \int_{x_2} \phi_2^*(x_2)\; dO_2$$

where $\phi_2^*(x_2) \,\epsilon\, \theta\left(C_2(\rho_1^*, x_2)\right)$ for every x_2.

We will now construct a elastic scaling model $M = (S_i, v_i, k_i)_{i\epsilon I}$. For both cloud service providers i, define

$$S_i = \{{s_i}^{x_i} : x_i \,\epsilon\, K_i\}.$$

Let the utility assignment k_i be given by

$$k_i({s_i}^{x_i}) = x_i$$

for every ${s_i}^{x_i} \,\epsilon\, S_i$. In order to define the conjecture assignment v_i we first define for every Unpredictable Workload type ${s_i}^{x_i}$ a density function $v_i^{\sim}({s_i}^{x_i})$ on $C_j \times S_j$ as follows: $v_i^{\sim}({s_i}^{x_i})(c_j, {s_j}^{x_j}) := \phi_j^*(x_j)(c_j)$,where $\phi_j^*(x_j)(c_j)$ is the probability that probability distribution $\phi_j^*(x_j)$ assigns to c_j. For every Unpredictable Workload type ${s_i}^{x_i}$ let $v_i({s_i}^{x_i}) \,\epsilon\, \theta(C_j \times S_j)$ be the probability distribution induced by density function $v_i^{\sim}({s_i}^{x_i})(c_j, {s_j}^{x_j})$ and the probability distribution Q_j on K_j. That is, for every set of Unpredictable Workload types $H \subseteq S_j$ given by

$$H := \{{s_j}^{x_j} : x_j \,\epsilon\, G\}$$

We have that

$$v_i({s_i}^{x_i})(\{c_j\} \times H) := \int_{x_j \,\epsilon\, G} v_i^{\sim}({s_i}^{x_i})(c_j, {s_j}^{x_j})\; dO_j$$

It follows that the conjecture of Unpredictable Workload type ${s_i}^{x_i}$ about cloud service provider j's resource pooling pattern choice is given by ρ_j^*. Namely, the probability that Unpredictable Workload type ${s_i}^{x_i}$ assigns to Unpredictable Workload resource pooling pattern choice c_j is equal to

$$\begin{aligned} v_i({s_i}^{x_i})(\{c_j\} \times K_j) &= \int_{x_j\,\epsilon\,K_j} v_i^{\sim}({s_i}^{x_i})(c_j, {s_j}^{x_j})\ dO_j \\ &= \int_{x_j\,\epsilon\,K_j} \phi_j^*(x_j)(c_j)\ dO_j \\ &= \rho_j^*(c_j). \end{aligned}$$

Therefore all Unpredictable Workload types of cloud service provider i have the same conjecture ρ_j^* about cloud service provider j's Unpredictable Workload resource pooling pattern choice. This completes the construction of the elastic scaling model. It follows directly from the construction that the elastic scaling model is σ-regular.We now show that every Unpredictable Workload type in this model expresses common conjecture in measurability. For this it is sufficient to show that every Unpredictable Workload type ${s_i}^{x_i}$ premises in the cloud consumer's measurability. Therefore, we must show for the cloud service providers i and every ${s_i}^{x_i} \epsilon S_i$ that $v_i^{\sim}({s_i}^{x_i})[(C_j \times S_j)^{quant}] = 1$. In order to prove, we show that $v_i^{\sim}({s_i}^{x_i})(c_j, {s_j}^{x_j}) > 0$ only if c_j is measureable for ${s_j}^{x_j}$.

Suppose that $v_i^{\sim}({s_i}^{x_i})(c_j, {s_j}^{x_j}) > 0$. Since $v_i^{\sim}({s_i}^{x_i})(c_j, {s_j}^{x_j}) \coloneqq \phi_j^*(x_j)(c_j)$, it follows that $\phi_j^*(x_j)(c_j) > 0$. As by definition $\phi_j^*(x_j)\,\epsilon\,\theta\left(C_j(\rho_i^*, x_j)\right)$ it follows that $c_j\,\epsilon\,C_j(\rho_i^*, x_j)$. Remember that the conjecture of Unpredictable Workload type ${s_j}^{x_j}$ about cloud service provider i's Unpredictable Workload resource pooling pattern choice is exactly ρ_i^*. Since $c_j\,\epsilon\,C_j(\rho_i^*, x_j)$ it follows that c_j is measureable for Unpredictable Workload type ${s_j}^{x_j}$. Therefore we have shown that $v_i^{\sim}({s_i}^{x_i})(c_j, {s_j}^{x_j}) > 0$ only if c_j is measureable for ${s_j}^{x_j}$. This implies that Unpredictable Workload type ${s_i}^{x_i}$ premises in the cloud consumer's measurability. Since this holds for every Unpredictable Workload type in the model it follows that every Unpredictable Workload type in the elastic scaling model expresses common conjecture in measurability. Therefore every Unpredictable Workload type in the model is σ-measureable because the model is σ-regular. This completes the proof.

7.2 Corollaries

In this subsection we state some technical corollaries, which we need for the proof of the result.

Corollary 7.1. *If P, Q and Rare data valued, independent random variables then* $\Pr(P \geq \max\{Q, R\}) \geq \Pr(P \geq Q) \cdot \Pr(P \geq R)$.

Proof. Let g_Q and g_R be the probability density functions of the random variables Q and R.

Now,

$$\begin{aligned} &\Pr(P \geq \max\{Q, R\}) \\ &= \int_q \int_r \Pr(P \geq \max\{q, r\})\ dg_Q(q)\, dg_R(r) \\ &\geq \int_q \int_r \Pr(P \geq \max\{q, r\}) \cdot \Pr(P \geq \min\{q, r\})\ dg_Q(q)\, dg_R(r) \\ &= \int_q \int_r \Pr(P \geq q) \cdot \Pr(P \geq r)\, dg_Q(q)\, dg_R(r) \\ &= \int_q \Pr(P \geq q)\ dg_Q(q) \cdot \int_r \Pr(P \geq r)\ dg_R(r) \\ &= \Pr(P \geq Q) \cdot \Pr(P \geq R). \end{aligned}$$

Note that the first and third equality follow from the fact that Q and R are independent, and the inequality holds because $\Pr(P \geq \min\{q, r\}) \leq 1$.

Corollary 7.2. *Let P be a random variable with $H(P)$=γ. Then for any number $t > 0$,*

$$(|P - \gamma| \geq t) \leq \frac{\mathrm{Var}(P)}{t^2}$$

Corollary 7.3. *For every $n\,\epsilon\,\mathbb{N}$, let $P_n^1, P_n^2, \ldots P_n^m$ be independent random variables with $H(P_n^i)$=γ^i for all n and i, $\gamma^1 > \gamma^2 > \cdots > \gamma^m$, and $\lim_{n\to\infty} \mathrm{Var}(P_n^i) = 0$ for all i. Then,*

$$\lim_{n\to\infty} \Pr(P_n^1 \geq P_n^2 \geq \cdots \geq P_n^m) = 1.$$

Proof. For a given n,

$$\Pr(P_n^1 \geq P_n^2 \geq \cdots \geq P_n^m) \geq 1 - \Pr(P_n^i \geq P_n^j \text{ for some } i < j).$$

For fixed $i < j$ we have,

$$\begin{aligned} \Pr(P_n^i < P_n^j) &= \Pr(P_n^j - P_n^i > 0) \\ &= \Pr\left((P_n^j - P_n^i) - (\gamma^j - \gamma^i) > \gamma^i - \gamma^j\right) \\ &\leq \Pr\left(\left|(P_n^j - P_n^i) - (\gamma^j - \gamma^i)\right| > \gamma^i - \gamma^j\right) \\ &\leq \frac{\mathrm{Var}(P_n^j - P_n^i)}{(\gamma^i - \gamma^j)^2} \\ &= \frac{\mathrm{Var}(P_n^j) + \mathrm{Var}(P_n^i)}{(\gamma^i - \gamma^j)^2} \end{aligned}$$

The last equality follows from the fact that P_n^j and P_n^i are independent. Now, note that $\lim_{n\to\infty} \mathrm{Var}(P_n^i) = 0$ and $\lim_{n\to\infty} \mathrm{Var}(P_n^j) = 0$, which implies $\lim_{n\to\infty} \mathrm{Var}(P_n^i < P_n^j) = 0$. Then, from above it follows that

$$\lim_{n\to\infty} \Pr(P_n^1 \geq P_n^2 \geq \cdots \geq P_n^m) = 1.$$

Consider a Unpredictable Workload sequence $(D_n)_{n\epsilon\mathbb{N}}$ of normal distributions with mean 0 and variance σ_n^2 such that $\sigma_n \to 0$ as $n \to \infty$. The density function g_n of D_n is given by

$$g_n(p) = \frac{2}{\sigma_n\sqrt{2\pi}}\ \mathrm{e}^{-(p^2/2\sigma_n^2)} \text{ for all } p\,.$$

We show that for large n the right tail of D_n becomes arbitrarily steep everywhere.

Corollary 7.4. *Consider a Unpredictable Workload sequence $(D_n)_{n\epsilon\mathbb{N}}$ of normal distributions with mean 0 and variance σ_n^2, such that $\sigma_n \to 0$ as $n \to \infty$. Let g_n be the density functions of these distributions. Then for all $c > 0$ and $\alpha > 0$ there is $N\,\epsilon\,\mathbb{N}$ such that $g_n(p + c)/g_n(p) \leq \alpha$ for all $n \geq N$ and all $p > 0$.*

Proof. Take $c > 0$ and $\alpha > 0$. Then

$$\frac{g_n(p+c)}{g_n(p)} = \frac{\mathrm{e}^{-((p+c)^2/2\sigma_n^2)}}{\mathrm{e}^{-(p^2/2\sigma_n^2)}} = \mathrm{e}^{-(1/2\sigma_n^2)((p+c)^2-p^2)}$$
$$= \mathrm{e}^{-(1/2\sigma_n^2)(2cp+c^2)} \le \mathrm{e}^{-(c^2/2\sigma_n^2)}$$

Now as $c > 0$ is fixed and $\sigma_n \to 0$ as $n \to \infty$, we can find N large enough such that $\mathrm{e}^{-(c^2/2\sigma_n^2)} \le \alpha$ for $n \ge N$.

Corollary 7.5. *Consider a Unpredictable Workload sequence $(D_n)_{n\in\mathbb{N}}$ of normally distributed random variables such that $H(P_n) = 0$ for all n, and $var(P_n) \to 0$ as $n \to \infty$. Let g_n be the density functions of these random variables. Then, for every $0 < p < q$ it holds that*

$$\lim_{n\to\infty} \frac{\Pr(P_n \ge q)}{\Pr(P_n \ge p)} = 0\,.$$

Proof. Fix $0 < p < q$ and fix a $\alpha > 0$. Then, by corollary 7.4 there is an N such that $g_n(r + (q-p))/g_n(r) \le \alpha$ for all $n \ge N$ and all $r > 0$. Take some $n \ge N$. Then,

$$\Pr(P_n \ge q) = \int_q^\infty g_n(r)\,dr = \int_p^\infty g_n(r + (q-p))\,dr$$
$$\le \alpha \cdot \int_p^\infty g_n(r)\,dr = \alpha \cdot \Pr(P_n \ge p).$$

This implies that

$$\lim_{n\to\infty} \frac{\Pr(P_n \ge q)}{\Pr(P_n \ge p)} = 0\,.$$

7.3 Proof of the result

We finally prove our main proposition, which is Proposition 5.1. We proceed by three steps.

In step 1, we show how a σ-regular elastic scaling model M with asymmetric Unpredictable Workload resource pooling pattern can be transformed into a elastic scaling model M' with symmetric Unpredictable Workload resource pooling pattern. More precisely, we transform every Unpredictable Workload type s_i in M into a Unpredictable Workload type $\tau_i(s_i)$ in M' which has the same conjecture about the cloud consumer's Unpredictable Workload resource pooling pattern choice as s_i. In step 2, we take a Unpredictable Workload resource pooling pattern choice c_i^* that is limit measureable. Therefore we can find a Unpredictable Workload sequence $(D_n)_{n\in\mathbb{N}}$ of normal distributions with mean 0 and variance σ_n^2, with $\sigma_n^2 \to 0$ as $n \to \infty$, and a Unpredictable Workload sequence $(M^n)_{n\in\mathbb{N}}$ of σ_n-regular elastic scaling models with constant Unpredictable Workload type and utility assignments, such that in every M^n there is a σ_n-measureable Unpredictable Workload type s_i^n with utility function w_i for which resource pooling Unpredictable Workload pattern choice c_i^* is optimal. We show that the Unpredictable Workload type s_i^n is transformed into a Unpredictable Workload type $\tau_i(s_i^n)$ which is α_n-actually measureable for some α_n. Since, for all n, c_i^* is measureable for t_i^n and $\tau_i(s_i^n)$ has the same conjecture about the cloud consumer's Unpredictable Workload resource pooling pattern choice and the same utility function as s_i^n, it follows that c_i^* is measureable for $\tau_i(s_i^n)$ for all n. As $\tau_i(s_i^n)$ is α_n-actually measureable for every n, it follows that c_i^* is α_n-actually measureable for all n. In step 3, we prove that $\lim_{n\to\infty} \alpha_n = 0$. Hence, c_i^* is α-actually measureable for every $\alpha > 0$ and therefore actually measureable.

Step 1. Take some $\sigma > 0$. Let $M = (S_i, v_i, k_i)_{i\epsilon I}$ be a σ-regular elastic scaling model for δ with asymmetric Unpredictable Workload resource pooling pattern. Now we transform this elastic scaling model M into a elastic scaling model $M' = (\Omega_i, \rho_i)_{i\epsilon I}$ with symmetric Unpredictable Workload resource pooling pattern. Using the fact that M is σ-regular we can write

$$v_i(s_i) \in \theta(C_j \times k_j \times V_j).$$

Now take $\Omega_i = V_i$ and $\Omega_j = V_j$. Clearly, Ω_i and Ω_j are finite sets as V_i and V_j are finite. For every $s_i \in S_i$ define the Unpredictable Workload type $\tau_i(s_i)$ by

$$\rho_i(\tau_i(s_i)) \coloneqq \max g_{C_j \times V_j} v_i(s_i).$$

Therefore,

$$\rho_i(\tau_i(s_i))(C_j \times V_j) = v_i(s_i)\,(K_j \times \{(C_j \times V_j)\})$$

for all $(C_j \times V_j)$. Hence,

$$\rho_i(\tau_i(s_i)) \in \theta(C_j \times V_j) = \theta(C_j \times \Omega_j)$$

By construction $\tau_i(s_i)$ has the same conjecture about j's Unpredictable Workload resource pooling pattern choice as s_i. This completes the construction of the elastic scaling $M' = (\Omega_i, \rho_i)_{i\epsilon I}$.

Step 2. Take a Unpredictable Workload resource pooling pattern choice c_i^* that is limit measureable. Hence, there exists a Unpredictable Workload sequence $(D_n)_{n\in\mathbb{N}}$ of normal distributions with mean 0 and variance σ_n^2, with $\sigma_n^2 \to 0$ as $n \to \infty$, and a Unpredictable Workload sequence $(M^n)_{n\in\mathbb{N}}$ of σ_n-regular elastic scaling models with constant Unpredictable Workload type and utility assignments, such that in every M^n there is a σ_n-measureable Unpredictable Workload type s_i^n with utility function w_i for which Unpredictable Workload resource pooling pattern choice c_i^* is optimal. Let the constant Unpredictable Workload type in the Unpredictable Workload sequence $(M^n)_{n\in\mathbb{N}}$ of elastic scaling models be S_i and S_j, and the constant utility assignments be $k_i$and k_j.Fix an n. Then, within the elastic scaling model $M^n = (S_i, v_i^n, k_i)_{i\epsilon I}$ there is an σ_n-measureable Unpredictable Workload type $s_i^n \in S_i$ with utility function w_i for which c_i^* is optimal. Since Unpredictable Workload type s_i^n only deems possible j's Unpredictable Workload types which are σ_n-measureable, and only deems possible j's Unpredictable Workload types which only deem possible i's Unpredictable Workload types which are σ_n-measureable and so on. We may assume without loss of generality that all the Unpredictable Workload types in M^n are σ_n-measureable. Let $M'^n = (\Omega_i^n, \rho_i^n)_{i\epsilon I}$ be the corresponding elastic scaling model with symmetric Unpredictable Workload resource pooling pattern, as constructed in step 1. For every $\tau_i \in \Omega_i^n$, we define a number $\alpha_n(\tau_i)$ as follows: Let $\mathrm{Poss}(\tau_i)$ be the set of Unpredictable Workload types in Ω_j that Ω_i deems possible. For a given Unpredictable Workload type $\tau_j \in \mathrm{Poss}(\tau_i)$, suppose that τ_j prefers Unpredictable Workload resource pooling pattern choice c_j^1 to c_j^2, c_j^2 to c_j^3, and so on. Therefore, we obtain an ordering $(c_j^1, c_j^2, c_j^3, \ldots c_j^m)$ of j's Unpredictable Workload resource pooling pattern choices.

Then define

$$\alpha_n(\tau_i, \tau_j) = \max_{t\,\in\{2,3,\ldots,m\}} \frac{\rho_i^n(\tau_i)(c_j^t, \tau_j)}{\rho_i^n(\tau_i)(c_j^{t-1}, \tau_j)}$$

Next we define

$$\alpha_{i,n} = \max_{\tau_i\,\in\,\Omega_i^n,\tau_j\,\in\,\mathrm{Poss}\,(\tau_i)} \alpha_n(\tau_i, \tau_j)$$

Finally let

$$\alpha_n = \max\{\alpha_{i,n}, \alpha_{j,n}\}.$$

Note that by construction every Unpredictable Workload type in M'^{n} satisfies the α_n-actual trembling condition; hence every Unpredictable Workload type in M'^{n} is α_n-actually measureable. In particular $\tau_i(s_i^n)$ is α_n-actually measureable [19] [26].

Step 3. Now we show that $\lim_{n\to\infty} \alpha_n = 0$. It is sufficient to show that

$$\lim_{n\to\infty} \frac{\rho_i^n(\tau_i)(c_j^t, \tau_j)}{\rho_i^n(\tau_i)(c_j^{t-1}, \tau_j)} = 0 \tag{1}$$

for every $\tau_i \in \Omega_i^n$ and every $\tau_j \in \mathrm{Poss}(\tau_i)$ and every t. As before, cloud service provider j's Unpredictable Workload resource pooling pattern choices are ordered $c_j^1, \dots c_j^m$ such that τ_j prefers Unpredictable Workload resource pooling pattern choice c_j^1 to c_j^2, c_j^2 to c_j^3, and so on. We assume, without loss of generality, that all resource pooling pattern preferences are strict. Fix some $\tau_i \in \Omega_i^n$ and $\tau_j \in \mathrm{Poss}(\tau_i)$. Suppose that $\tau_i = \tau_i(s_i)$ for some $s_i \in S_i$ and that $\tau_j = \tau_j(s_j)$ for some $s_j \in S_j$. Let $\phi_j \in \theta(C_i)$ be τ_i's conjecture about i's Unpredictable Workload resource pooling pattern choice [28]. As before, let K_j be the set of utility functions for cloud service provider j. For every $t \in \{1, \dots, m\}$, let $P^t: K_j \to F$ be given by

$$P^t(k_j) := k_j(c_j^t, \phi_j) = \sum_{c_i \in C_i} \phi_j(c_i) \cdot k_j(c_j^t, c_i)$$

for every $k_j \in K_j$. Therefore, $P^t(k_j)$ denotes the expected utility for cloud service provider j induced by Unpredictable Workload resource pooling pattern choice c_j^t, under the conjecture ϕ_j and the utility function k_j. Note that P^t is a random variable, as cloud service provider i holds a probability distribution on K_j, induced by D. The probability distribution of P^t depends on n, and is denoted by $\omega_n^t(P^t)$. Note that P^t has a normal distribution with mean

$$H(P^t) = w_j(c_j^t, \phi_j),$$

and variance

$$Var^n(P^t) = \textstyle\sum_{c_i \in C_i} (\phi_j(c_i))^2 \cdot \sigma_n^2 \tag{2}$$

In particular, it follows that $\lim_{n\to\infty} Var^n(P^t) = 0$, as $\lim_{n\to\infty} \sigma_n^2$. Since, by assumption, τ_j strictly prefers c_j^1 to c_j^2, strictly prefers c_j^2 to c_j^3, and so on, we have that $H(P^1) > H(P^2) > \cdots > H(P^m)$.Let ω^n be the probability distribution of the random set of data value $(P^1, \dots, P^m)$ [6] [14] [18]. Recall that all Unpredictable Workload types in M^n are σ_n-measureable, which implies that all Unpredictable Workload types in M^n express common conjecture in measurability. As such, Unpredictable Workload type $s_i \in S_i$ (which generates τ_i) expresses common conjecture in measurability [27]. In particular, s_i only assigns positive probability to those Unpredictable Workload resource pooling pattern choice-Unpredictable Workload type combinations (c_j, s_j) where c_j is optimal for t_j. Now, as $\tau_i = \tau_i(s_i)$ and $\tau_j = \tau_j(s_j)$, we have that $\rho_i^n(\tau_i)(c_j^t, \tau_j)$ is the probability that c_j^t is optimal for s_j, and that is $\omega^n(P^t \geq P^l \text{for all } l)$. Then,

$$\frac{\rho_i^n(\tau_i)(c_j^t, \tau_j)}{\rho_i^n(\tau_i)(c_j^{t-1}, \tau_j)} = \frac{\omega^n(P^t \geq P^l \text{ for all } l)}{\omega^n(P^{t-1} \geq P^l \text{ for all } l)} \tag{3}$$

Hence, in order to prove (1), we must show that

$$\lim_{n\to\infty} \frac{\omega^n(P^t \geq P^l \text{ for all } l)}{\omega^n(P^{t-1} \geq P^l \text{ for all } l)} = 0$$

for all $t \in \{2, \dots, m\}$,. We distinguish two cases.

Case 1. First we consider the case where $t = 2$. Then we have,

$$\frac{\omega^n(P^t \geq P^l \text{ for all } l)}{\omega^n(P^{t-1} \geq P^l \text{ for all } l)} \leq \frac{\omega^n(P^2 \geq P^1)}{\omega^n(P^1 \geq P^2 \geq P^3 \geq \cdots \geq P^m)}$$

Recall that $H(P^1) > H(P^2) > \cdots > H(P^m)$. But then, by Corollary 7.3, $\omega^n(P^2 \geq P^1) \to 0$ and $\omega^n(P^1 \geq P^2 \geq P^3 \geq \cdots \geq P^m) \to 1$, and hence

$$\frac{\omega^n(P^2 \geq P^1)}{\omega^n(P^1 \geq P^2 \geq P^3 \geq \cdots \geq P^m)} \to 0\,,$$

which implies that

$$\frac{\omega^n(P^t \geq P^l \text{ for all } l)}{\omega^n(P^{t-1} \geq P^l \text{ for all } l)} \to 0\,,$$

as $n \to \infty$.

Case 2. Now we consider the case where $t > 2$.

Let P^{max} be the random variable given by $P^{max} := \max_{j \neq t, t-1} P_j$. We have

$$\frac{\omega^n(P^t \geq P^l \text{ for all } l)}{\omega^n(P^{t-1} \geq P^l \text{ for all } l)}$$

$$= \frac{\omega^n((P^t \geq P^{t-1}) \text{ and} (P^t \geq P^{max}))}{\omega^n((P^{t-1} \geq P^t) \text{ and} (P^{t-1} \geq P^{max}))}$$

$$\leq \frac{\omega^n(P^t \geq P^{max})}{\omega^n((P^{t-1} \geq P^t) \text{ and} (P^{t-1} \geq P^{max}))}$$

$$\leq (\text{by Corollary 7.1}) \ \frac{\omega^n(P^t \geq P^{max})}{\omega^n((P^{t-1} \geq P^t) \cdot \omega^n(P^{t-1} \geq P^{max}))}$$

$$= \frac{\omega^n(P^t \geq P^{max})}{\omega^n(P^{t-1} \geq P^{max})} \cdot \frac{1}{\omega^n(P^{t-1} \geq P^t)}$$

$$= \frac{\omega^n(P^t \geq P^{max})}{\omega^n(P^{t-1} \geq P^{max} - (H(P^{t-1}) - H(P^t)))} \cdot \frac{1}{\omega^n(P^{t-1} \geq P^t)}$$

where the last equality follows from the observation that $P^{t-1} - H(P^{t-1})$ and $P^t - H(P^t)$ have the same distribution.

Now, from Corollary 7.3 it follows that $\omega^n(P^{t-1} \geq P^t) \to 1$ as $n \to \infty$.

We show that

$$\frac{\omega^n(P^t \geq P^{max})}{\omega^n\left(P^{t-1} \geq P^{max} - \left(H(P^{t-1}) - H(P^t)\right)\right)} \to 0$$

as $n \to \infty$.

Let us define $c: H(P^{t-1}) - H(P^t)$. Therefore, we have to show that

$$\frac{\omega^n(P^t \geq P^{max})}{\omega^n(P^{t-1} \geq P^{max} - c)} \to 0 \tag{4}$$

as $n \to \infty$. Note that $\omega^n(P^t \geq P^{max}) \leq \omega^n(P^t \geq P^1)$. We first show that there exists $N \in \mathbb{N}$ such that for all $n \in N$,

$$\omega^n(P^t \geq P^{max} - c) \geq \omega^n(P^t \geq P^1 - c/2) \quad (5)$$

Now,

$$\omega^n(P^t \geq P^{max} - c)$$
$$= \omega^n(P^t \geq P^{max} - c \mid P^{max} = P^1) \cdot \omega^n(P^{max} = P^1)$$
$$+\omega^n(P^t \geq P^{max} - c \mid P^{max} \neq P^1) \cdot \omega^n(P^{max} \neq P^1)$$
$$\geq \omega^n(P^t \geq P^{max} - c|P^{max} = P^1) \cdot \omega^n(P^{max} = P^1)$$
$$= \omega^n(P^t \geq P^1 - c) \cdot \omega^n(P^{max} = P^1)$$

Therefore, to show (5) it is sufficient to show that there exists $N \in \mathbb{N}$ such that for all $n \geq N$,

$$\omega^n(P^t \geq P^1 - c) \cdot \omega^n(P^{max} = P^1) \geq \omega^n(P^t \geq P^1 - c/2) \quad (6)$$

Using Corollary 7.3, $\omega^n(P^{max} = P^1) \to 1$ as $n \to \infty$. We have,

$$\frac{\omega^n(P^t \geq P^1 - c/2)}{\omega^n(P^t \geq P^1 - c)}$$
$$= \frac{\omega^n((P^t \geq P^1) - (H(P^t) - H(P^1)) \geq -c/2 - (H(P^t) - H(P^1)))}{\omega^n((P^t \geq P^1) - (H(P^t) - H(P^1)) \geq -c - (H(P^t) - H(P^1)))}$$

Note that $\omega^n((P^t - P^1) - (H(P^t) - H(P^1)))$ has a normal distribution with mean 0 and where the variance of $\omega^n(P^t - P^1)$ tends to 0 as $n \to \infty$. Moreover, $-c - (H(P^t) - H(P^1)) > 0$ as $H(P^t) - H(P^1) < H(P^t) - H(P^{t-1}) = -c$. Hence, using Corollary 7.5,

$$\frac{\omega^n((P^t \geq P^1) - (H(P^t) - H(P^1)) \geq -c/2 - (H(P^t) - H(P^1)))}{\omega^n((P^t \geq P^1) - (H(P^t) - H(P^1)) \geq -c - (H(P^t) - H(P^1)))}$$
$$\to 0$$

as $n \to \infty$. Then, we have,

$$\frac{\omega^n(P^t \geq P^1 - c/2)}{\omega^n(P^t \geq P^1 - c)} \to 0$$

Therefore, there exists $N \in \mathbb{N}$ such that for all $n \geq N$,

$$\omega^n(P^{max} = P^1) \geq \frac{\omega^n(P^t \geq P^1 - c/2)}{\omega^n(P^t \geq P^1 - c)}$$

This proves (6), which as we have shown, implies (5). Now, by (5) we have

$$\frac{\omega^n(P^t \geq P^{max})}{\omega^n(P^t \geq P^1 - c)}$$
$$\leq \frac{\omega^n(P^t \geq P^1)}{\omega^n(P^t \geq P^1 - c/2)}$$
$$= \frac{\omega^n((P^t \geq P^1) - (H(P^t) - H(P^1)) \geq -(H(P^t) - H(P^1)))}{\omega^n((P^t \geq P^1) - (H(P^t) - H(P^1)) \geq -c/2 - (H(P^t) - H(P^1)))}$$
$$= \frac{\omega^n((P^t \geq P^1) - (H(P^t) - H(P^1)) \geq (H(P^t) - H(P^1)))}{\omega^n((P^t \geq P^1) - (H(P^t) - H(P^1)) \geq (H(P^t) - H(P^1)))}$$
$$\to 0$$

as n goes to infinity. Here the convergence follows from Corollary 7.5 as $(H(P^1) - H(P^t)) \geq -c/2$. Therefore, we have shown (4), which completes case 2. Hence, we have shown that (1) holds for all t. Therefore, $\lim_{n\to\infty} \alpha = 0$ and hence the proof is complete.

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