International Journal of Computer Applications (0975 – 8887)
Volume 10– No.10, November 2010

Edge- Odd Gracefulness of Cartesian product of C_3 and C_N

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ABSTRACT
A (p, q) connected graph is edge-odd graceful graph if there exists an injective map f: E(G) → {1, 3, …, 2q-1} so that induced map f_+: V(G) → {0, 1,2, 3, …, (2k-1)}defined by $f_+(x) \equiv \Sigma f(x, y) \pmod{2k}$, where the vertex x is incident with other vertex y and k = max {p, q} makes all the edges distinct and odd. In this article, the Edge -odd gracefulness of the cartesian product of C_3 and C_n is obtained.

Keywords: Graceful Graphs, Edge-odd graceful labeling, Edge-odd Graceful Graph

1. INTRODUCTION
A.Solairaju and K.Chitra [2009] obtained edge-odd graceful labeling of some graphs related to paths. A. Solairaju et.al. [2009, 2010] that the strong product of path P_3 and circuit C_n for all integer n, is edge-odd graceful.

Section-2: Edge-odd graceful labeling of cartesian product of C_3 and C_n

Definition 2.1: Graceful Graph: A function f of a graph G is called a graceful labeling with m edges, if f is an injection from the vertex set of G to the set {0, 1, 2, …, m} such that when each edge uv is assigned the label │f(u) – f(v)│and the resulting edge labels are distinct. Then the graph G is graceful.

Definition 2.2: Edge-odd graceful graph: A (p, q) connected graph is edge-odd graceful graph if there exists an injective map f: E(G) → {1, 3, …, 2q-1} so that induced map f_+: V(G) → {0, 1, 2, …,(2k-1)} defined by $f_+(x) \equiv \Sigma f(x, y) \pmod{2k}$, where the vertex x is incident with other vertex y and k = max {p, q} makes all the edges distinct and odd. Hence the graph G is edge- odd graceful.

Lemma 2.3: The Cartesian product graph $C_3 \square C_n$ is edge - odd graceful where n = 3, 4, 8.
Proof: The **c**artesian product graph $C_3 \uparrow C_3$ is a connected graph with 9 vertices and 18 edges.

The arbitrary labelings of edge- odd graceful of the required graph is obtained as follows.

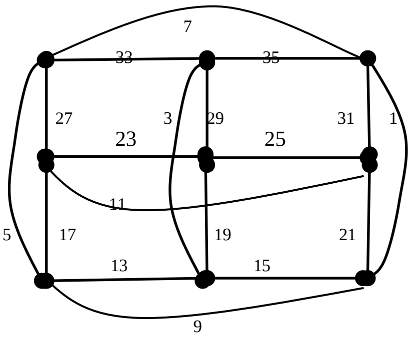

Figure 1: Edge-odd graceful Graph $C_3 \square C_3$

The **c**artesian product graph $C_3 \square C_4$ is a connected graph with 12 vertices and 24 edges. The arbitrary labelings of edge-odd graceful of the required graph is obtained as follows**.**

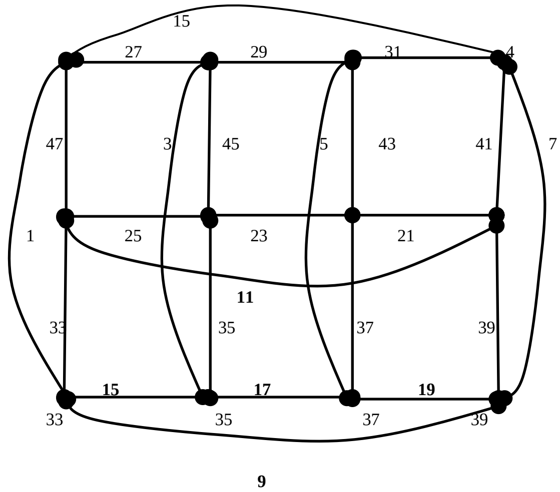

Figure 2: Edge-odd graceful Graph $C_3 \square C_4$

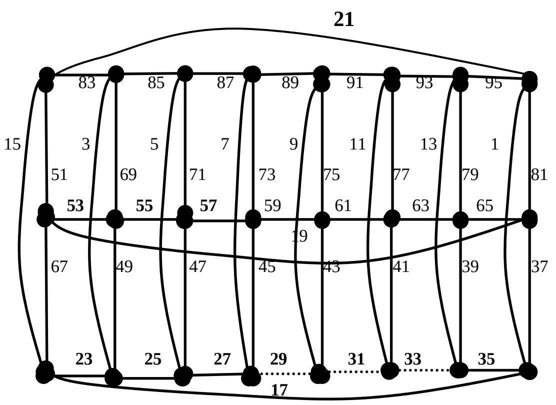

Figure 3: Edge-odd graceful Graph $C_3 \square C_8$

The **c**artesian product graph $C_3 \square C_8$ is a connected graph with 24 vertices and 48 edges. The arbitrary labelings of edge-odd graceful of the required graph is given in figure 3.

Theorem 2.1: The Cartesian product of $C_3 \square C_n$ is edge-odd graceful.

Proof: The Cartesian product of the path C_3 and the circuit C_n is given and the arbitrary labelings for vertices and edges for $C_3 \square C_n$ are mentioned below.

Case (1): n is odd

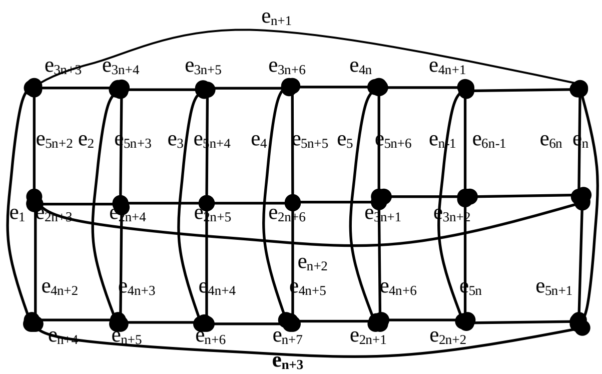

Figure 4: Edge -odd graceful Graph $C_3 \square C_n$, for n is odd

Case (2): n is even

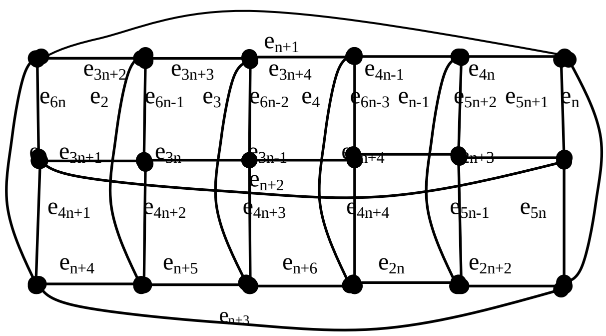

Figure 5: Edge- odd graceful Graph $C_3 \square C_n$, for n is even

To find edge-odd graceful, define f: $E(C_3 \square C_n) \rightarrow \{1, 3, \ldots, 2q-1\}$ by

n is even

Case i: n≡ 0 (mod 4)

$f(e_i) = 2i-1$, $i = 1,2,3,\ldots n,(n+4),(n+5),\ldots 6n$

$f(e_{n+1}) = 2n+3$, $f(e_{n+2}) = 2n+1$, $f(e_{n+3}) = 2n+5$,

$f(e_{5n+1}) = 10n+3$, $f(e_{5n+2}) = 10n+1$ Rule(1)

Case ii: n ≡ 2 (mod 4), n≠8

$f(e_i) = 2i-1$, $i = 1,2,3,\ldots n,(n+4),(n+5),\ldots 6n$

$f(e_{n+1}) = 2n+3$, $f(e_{n+2}) = 2n+1$, $f(e_{n+3}) = 2n+5$ Rule(2)

n is odd

Case iii. n ≡ 1 (mod 4)

$f(e_i) = 2i-1$, $i = 1,2,3, \ldots\ldots 6n$ Rule(3)

Case iv. n ≡ 3 (mod 4)

$f(e_1) = 2n-1$, $f(e_n) = 1$,

$f(e_i) = 2i-1$, $i = 2,3,4,\ldots\ldots,(n-1), (n+4), (n+5), \ldots, 6n$ Rule (4)

$f(e_{n+1}) = 2n+3$, $f(e_{n+2}) = 2n+1$, $f(e_{n+3}) = 2n+5$

Define $f_+: V(G) \rightarrow \{0, 1, 2, \ldots, (2k-1)\}$ by

$f_+(v) \equiv \Sigma f(uv) \bmod (2k)$, where this sum run over all edges through v Rule (5)

Hence the induced map f_+ provides the distinct labels for vertices and also the edge labeling is distinct. Hence the Cartesian product graph $C_3 \square C_n$ is edge- odd graceful.

Example 2.1: The Cartesian product graph $C_3 \square C_{12}$ is edge-odd graceful.

Proof: The **c**artesian product graph $C_3 \uparrow C_{12}$ is a connected graph with 36 vertices and 72 edges, where $n \equiv 0 \pmod 4$. Due to the rules (1) & (5) in theorem 2.1, edge odd-graceful labelings of the required graph is obtained as follows.

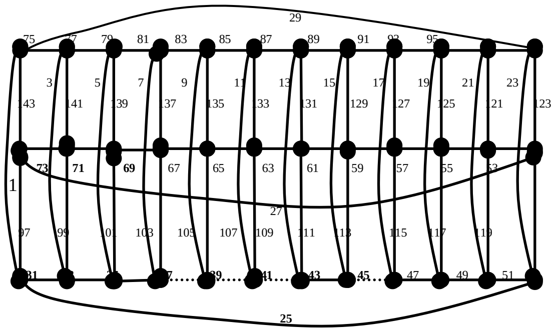

Figure 6: Edge-odd graceful Graph $C_3 \square C_{12}$

Example 2.3: The Cartesian product graph $C_3 \square C_9$ is edge-odd graceful.

Proof: The **c**artesian product graph $C_3 \square C_9$ is a connected graph with 27 vertices and 54 edges, where $n \equiv 1 \pmod 4$. Due to the rules (3) & (5) in theorem 2.1, edge odd-graceful labelings of the required graph is obtained as follows.

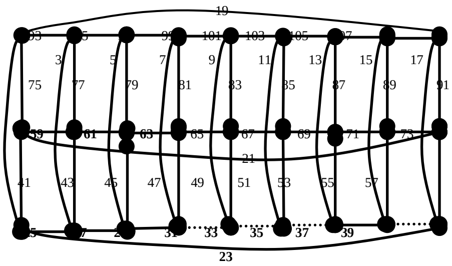

Figure 7: Edge-odd graceful Graph $C_3 \square C_9$

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