Edge Odd Gracefulness of 2-NC$_4$, 3-NC$_4$, 4-NC$_4$

Dr. A. Solairaju
Associate Professor of Mathematics
Jamal Mohamed College, Tiruchirapalli – 620 020.
Tamil Nadu, India.

A.Sasikala and C. Vimala
Assistant Professors (SG),
Department of Mathematics,
Periyar Maniammai University, Vallam
Thanjavur – Post.. Tamil Nadu, India.

ABSTRACT
A (p, q) connected graph is edge-odd graceful graph if there exists an injective map f: E(G) → {1, 3, …, 2q-1} so that induced map f_+: V(G) → {0, 1,2, 3, …, (2k-1)}defined by $f_+(x) \equiv \Sigma f(x, y) \pmod{2k}$, where the vertex x is incident with other vertex y and k = max {p, q} makes all the edges distinct.

Key words: Generalised n-squares, Graceful Graphs, Edge-odd graceful labeling, Edge-odd Graceful Graph

1. INTRODUCTION
A.Solairaju and K.Chitra [2009] obtained edge-odd graceful labeling of some graphs related to paths. A. Solairaju et.al. [2009, 2010] that the Cartesian product of path C_3 and circuit C_n for all integer n, is edge-odd graceful.

Section-2: Edge-odd graceful labeling of 2-nC$_4$

Definition 2.1: Graceful Graph: A function f of a graph G is called a graceful labeling with m edges, if f is an injection from the vertex set of G to the set {0, 1, 2, …, m} such that when each edge uv is assigned the label │f(u) – f(v)│and the resulting edge labels are distinct. Then the graph G is graceful.

Definition 2.2: Edge-odd graceful graph: A (p, q) connected graph is edge-odd graceful graph if there exists an injective map f: E(G) → {1, 3, …, 2q-1} so that induced map f_+: V(G) → {0, 1, 2, …,(2k-1)} defined by $f_+(x) \equiv \Sigma f(x, y) \pmod{2k}$, where the vertex x is incident with other vertex y and k = max {p, q} makes all the edges distinct and odd. Hence the graph G is edge- odd graceful.

Definition 2.3:Generalised n-squares: n-C_4 is a collection of n number of C_4.2-nC_4 (n≥1) is a connected graph whose vertex set is $v_1, v_2, v_3, v_4,\ldots, v_{4n}$ whose edge set v_1v_2, v_2v_3, v_3v_4, …., $v_{4n-1}v_{4n}$, $v_{4n}v_{4n-3}$, v_2v_6, v_6v_{10},…, v_{4n-6}, v_{4n-2}, …, v_3v_7, v_7v_{11}, …, $v_{4n-5}v_{4n-1}$.

Theorem 2.1: The square 2-nC$_4$ is is edge -odd graceful.

Proof: The square 2-nC_4 is a connected graph with 4n vertices and 6n-2 edges and the arbitrary labelings for vertices and edges for 2-nC_4 are mentioned below.

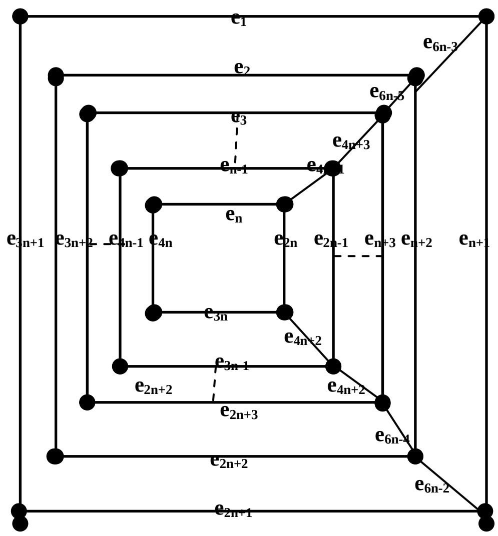


Figure 1: Edge-odd graceful Graph of 2-nC$_4$

To find edge-odd graceful, define f: E(2-nC_4) → {1, 3, …, 2q-1} by
$f(e_i) = 2i-1$, i = 1,2,3,…(n-1),(n+1),(n+2), …, (3n-1), (3n+1),…, (6n-2) Rule(1)

Case i . n is even
$f(e_n) = 6n-1$, $f(e_{3n}) = 2n-1$ Rule (2)

Case ii. n is odd
$f(e_n) = 2n-1$, $f(e_{3n}) = 6n-1$ Rule (3)

Define f_+: V(G) → {0, 1, 2, …, (2k-1)} by
$f_+(v) \equiv \Sigma f(uv) \bmod (2k)$,where this sum run over all edges through v Rule (4)

Thus the induced map f_+ provides the distinct labels for vertices and also the edge labeling is distinct. Hence the square 2-nC_4 is edge odd - graceful.

Example 2.2: The square 2-6C$_4$ is edge-odd graceful.
Proof: The square 2-6C_4 is a connected graph with 24 vertices and 34 edges.
Due to the rules (1), (2) & (4) in theorem 2.1, then 2-6C_4 is edge-odd-graceful.

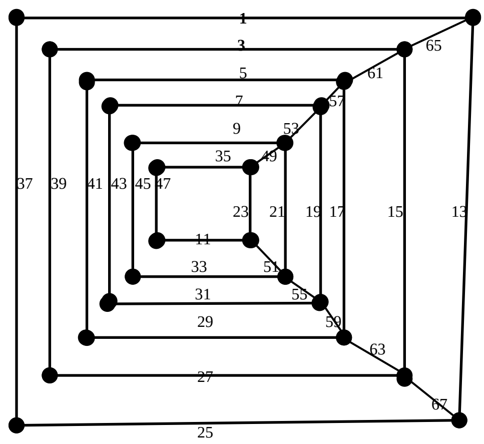


Figure 2-Edge-odd graceful of graph of 2-6C_4

Section-3: Edge-odd graceful labeling of 3-nC4

Definition 3.1:Generalised n-squares: n-C_4 is a collection of n number of C_4.3-nC_4 (n≥1) is a connected graph whose vertex set is $v_1,v_2,v_3,v_4,\ldots\ldots v_{4n}$ whose edge set $v_1v_2,v_2v_3,v_3v_4,\ldots..v_{4n-1}v_{4n},v_{4n}v_{4n-3},v_2v_6,v_6v_{10},\ldots\ldots.v_{4n-6}v_{4n-2},\ldots\ldots.v_3v_7,v_7v_{11},\ldots..v_{4n-5}v_{4n-1},\ldots\ldots\ldots,v_4v_8,v_8v_{12},\ldots\ldots.v_{4n-4}v_4$.

Lemma 3.2: **The square 3-2C_4 is edge -odd graceful**

Proof: The square 3-2C_4 is a connected graph with 8 vertices and 11 edges.
The arbitrary labelings of edge- odd graceful of the required graph is obtained as follows.

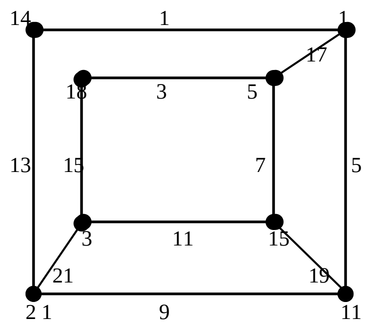


Figure 3: Edge-odd graceful Graph of 3-2C_4

Theorem 3.1: The square 3-nC_4 is edge -odd graceful.

Proof: The square 3-nC_4 is a connected graph with 4n vertices and 7n-3 edges and the arbitrary labelings for vertices and edges for 3-nC_4 are mentioned below.

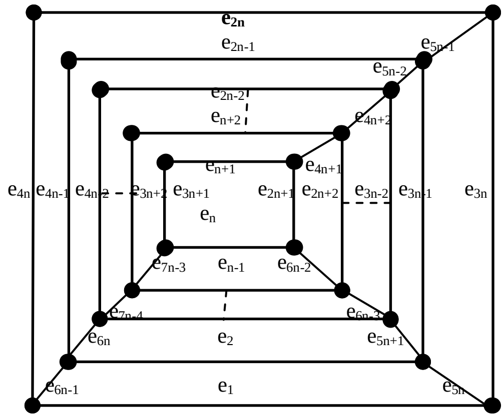


Figure 4: Edge-odd graceful Graph of 3-nC_4

To find edge-odd graceful, define f: E(3-nC_4) → {1, 3, …, 2q-1} by
$f(e_i) = 2i-1$, i = 1,2,3,…(7n-3) Rule(1)
Define f_+: V(G) → {0, 1, 2, …, (2k-1)} by
$f_+(v) \equiv \Sigma f(uv) \bmod (2k)$,where this sum run over all edges through v ……. Rule (2)

Hence the induced map f_+ provides the distinct labels for vertices and also the edge labeling is distinct. Hence the square 3-nC_4 is edge odd - graceful.

Example 3.1: The square 3-5C_4 is edge-odd graceful.

Proof: The square3-5C_4 is a connected graph with 20 vertices and 32 edges

Due to the rules (1) & (2) in theorem 3.1, edge odd-graceful labelings of the required graph is obtained as follows

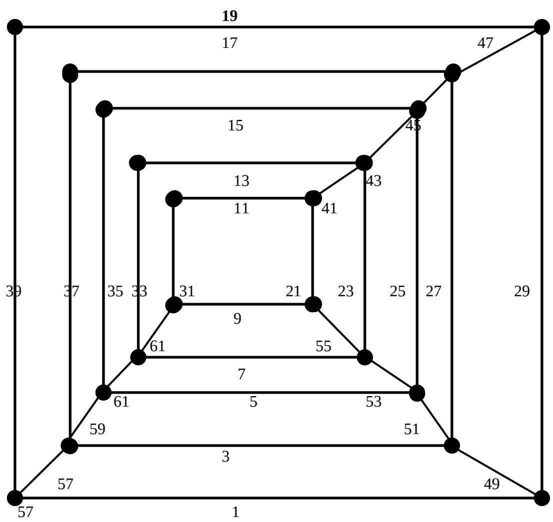


Figure 5: Edge-odd graceful Graph of 3-5C_4

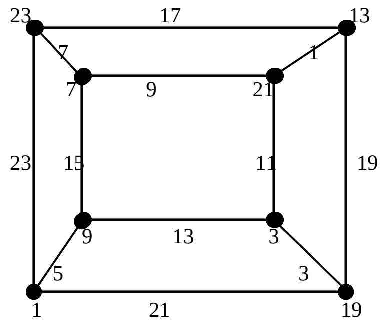
Section-4: Edge-odd graceful labeling of 4-nC4.

Definition 4.1:Generalised n-squares: n-C_4 is a collection of n number of C_4.4-nC_4 (n≥1) is a connected graph whose vertex set is v_1, v_2, v_3, v_4,… , v_{4n} whose edge set v_1v_2, v_2v_3, v_3v_4, …, $v_{4n-1}v_{4n}$, $v_{4n}v_{4n-3}$, v_2v_6, v_6v_{10},…,$v_{4n-6}v_{4n-2}$, … , v_3v_7,

v_7v_{11}, … , $v_{4n-5}v_{4n-1}$, … , v_4v_8, v_8v_{12}, … , $v_{4n-4}v_{4n}$, …, v_1v_5, v_5v_9, … , $v_{4n-7}v_{4n-3}$.

Lemma 4.2: **The square 4-2C_4 is edge -odd graceful**

Proof: The square 4-2C_4 is a connected graph with 8 vertices and 12 edges.
The arbitrary labelings of edge- odd graceful of the required graph is obtained as follows.



F**igure 7: Edge-odd graceful Graph of 4-2C_4**

Theorem 4.1: The square 4-nC_4 is edge -odd graceful.

Proof: The square 4-nC_4 is a connected graph with 4n vertices and 8n-4 edges and the arbitrary labelings for vertices and edges for 4-nC_4 are mentioned below

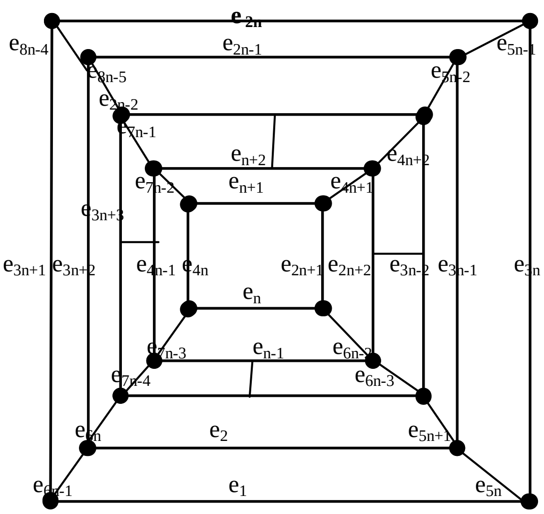


Figure 7: Edge-odd graceful Graph of 4-nC_4

Define f: E(4-nC_4) → {1, 3, …, 2q-1} by
$f(e_i) = 2i-1$, i = 1,2,3,…(8n-4) Rule(1)
Define f_+: V(G) → {0, 1, 2, …, (2k-1)} by
$f_+(v) \equiv \Sigma f(uv) \bmod (2k)$,where this sum run over all edges through v ……. Rule (2)

Thus the induced map f_+ provides the distinct labels for vertices and also the edge labeling is distinct. Hence the square 4-nC_4 is edge odd - graceful.

REFERENCES

1. A.Solairaju and K.Chitra Edge-odd graceful labeling of some graphs " Electronics
2. Notes in Discrete Mathematics Volume 33,April 2009, Pages 15 – 20
3. A.Solairaju, A.Sasikala, C.Vimala Edge-odd Gracefulness of a spanning tree of Cartesian product of P_2 and C_n ,Pacific-Asian Journal of Mathematics, Vol .3, No.1-2. (Jan-Dec.2009) pp:39-42.
4. A.Solairaju, A.Sasikala, C.Vimala Edge-odd Gracefulness of strong product of C_3 and C_n, communicated to serials publications, New Dehli.