Edge-odd Gracefulness of Product of P_2 and C_N

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ABSTRACT

A (p, q) connected graph is edge-odd graceful graph if there exists an injective map f: E(G) → {1, 3, …, 2q-1} so that induced map f_+: V(G) → {0, 1,2, 3, …, (2k-1)}defined by $f_+(x) \equiv \Sigma f(x, y) \pmod{2k}$, where the vertex x is incident with other vertex y and k = max {p, q} makes all the edges distinct and odd. In this article, the Edge- odd gracefulness of strong product of P_2 and C_n is obtained.

Key words: , Graceful Graphs, Edge-odd graceful labeling, Edge-odd Graceful Graph

1. INTRODUCTION

A.Solairaju and K.Chitra [2009] obtained edge-odd graceful labeling of some graphs related to paths. A. Solairaju et.al. [2009, 2010] proved that the square 2-nC_4, 3-nC_4, 4-nC_4 are edge -odd graceful.

Section-2: Edge-odd graceful labeling of strong product of $P_2 \boxtimes C_n$

Definition 2.1: Graceful Graph: A function f of a graph G is called a graceful labeling with m edges, if f is an injection from the vertex set of G to the set {0, 1, 2, …, m} such that when each edge uv is assigned the label │f(u) – f(v)│ and the resulting edge labels are distinct. Then the graph G is graceful.

Definition 2.2: Edge-odd graceful graph: A (p, q) connected graph is edge-odd graceful graph if there exists an injective map f: E(G) → {1, 3, …, 2q-1} so that induced map f_+: V(G) → {0, 1, 2, …,(2k-1)} defined by $f_+(x) \equiv \Sigma f(x, y) \pmod{2k}$, where the vertex x is incident with other vertex y and k = max {p, q} makes all the edges distinct and odd. Hence the graph G is edge- odd graceful.

Theorem 2.1: The strong product of $P_2 \boxtimes C_n$ is edge-odd graceful.

Proof: The strong product of the path P_2 and the circuit C_n is given and the arbitrary labelings for vertices and edges for $\mathbf{P_2 \boxtimes C_n}$ are mentioned below.

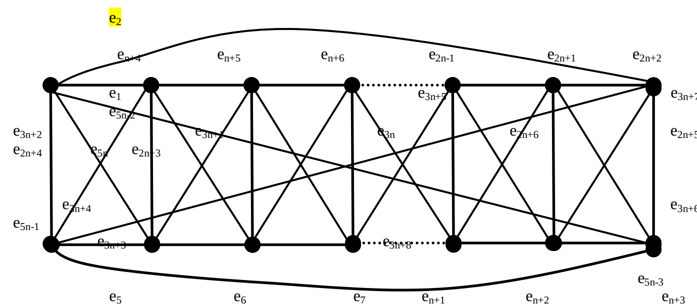

Figure 1: Edge-odd graceful Graph of $P_2 \boxtimes C_n$

To find edge-odd graceful, define by f: E($\mathbf{P_2 \boxtimes C_n}$) → {1, 3, …, 2q-1}

Case i . n ≡ 0 (mod 5)

$f(e_1) = 5$, $f(e_2) = 1$, $f(e_3) = 7$, $f(e_4) = 3$
$f(e_i) = 2i-1$, i = 5,6,7,…5n Rule(1)

Case ii. n ≡2(mod 5)

$f(e_1) = 5$, $f(e_2) = 3$, $f(e_3) = 1$, $f(e_4) = 7$
$f(e_i) = 2i-1$, i = 5,6,7,…,(2n+2), (3n+3), (3n+4), …, 5n
$f(e_{3n+3-i}) = f(e_{2n+2})+2i$, i=1,2,……n Rule (2)

Case iii. n ≡ 4 (mod 5)

$f(e_1) = 5$, $f(e_2) = 7$, $f(e_3) = 1$, $f(e_4) = 3$
$f(e_i) = 2i-1$, i = 5,6,7,…,(2n+2), (3n+3), (3n+4), …, 5n
$f(e_{3n+3-i}) = f(e_{2n+2})+2i$, i=1,2,……n Rule (3)

Case iv. n ≡ 3 (mod 5)

$f(e_1) = 7$, $f(e_2) = 1$, $f(e_3) = 3$, $f(e_4) = 5$
$f(e_i) = 2i-1$, i = 5,6,7,…,(2n+2), (3n+3), (3n+4), .., 5n
$f(e_{3n+3-i}) = f(e_{2n+2})+2i$, i=1,2,……n Rule (4)

For n ≡ 1 (mod 5), the arbitrary labelings for vertices and edges for $P_2 \boxtimes C_n$ are mentioned below

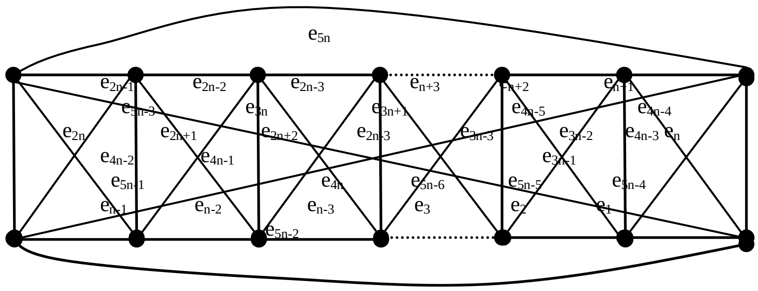

Figure 2: Edge-odd graceful Graph of $P_2 \boxtimes C_n$

Case iv. $n \equiv 1 \pmod 5$

$f(e_i)=2i-1, i=1,2,3,\dots,5n$ Rule (5).
Define f_+: $V(G) \rightarrow \{0, 1, 2, \dots, (2k-1)\}$ by $f_+(v) \equiv \Sigma f(uv) \bmod[2k]$ where this sum run over all edges through v …Rule (6). Hence the induced map f_+ provides the distinct labels for vertices and also the edge labeling is distinct. Hence the strong product graph P_2 and C_n is edge odd - graceful.

Example 2.1: The strong product graph $P_2 \boxtimes C_5$ is edge-odd graceful.

Proof: The strong product graph $P_2 \boxtimes C_5$ is a connected graph with 10 vertices and 25 edges, where $n \equiv 0 \pmod 5$. Due to the rules (1) & (6) in theorem 2.1, edge odd-graceful labelings of the required graph is obtained as follows**.**

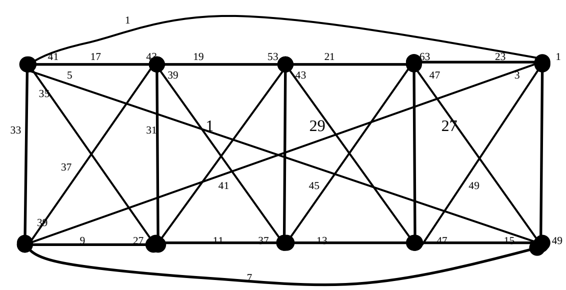

Figure 3: Edge-odd graceful Graph of $P_2 \boxtimes C_n$

Example 2.2: The strong product graph $P_2 \boxtimes C_7$ is edge-odd graceful.

Proof: The strong product graph $P_2 \boxtimes C_7$ is a connected graph with 14 vertices and 35 edges, where

$n \equiv 2 \pmod 5$. Due to the rules (2) & (6) in theorem 2.1, edge odd-graceful labelings of the required graph is obtained as follows.
(Figure 3: Edge-odd graceful Graph of $P_2 \boxtimes C_5$)

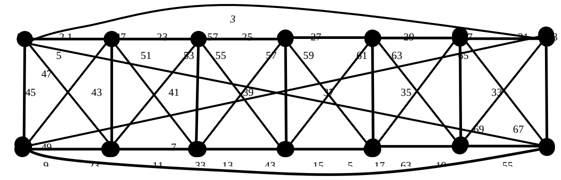

Figure 4: Edge-odd graceful Graph of $P_2 \boxtimes C_7$

Example 2.3: The strong product graph $P_2 \boxtimes C_9$ is edge-odd graceful.

Proof: The strong product graph $P_2 \boxtimes C_9$ is a connected graph with 18 vertices and 45 edges, where
$n \equiv 4 \pmod 5$. Due to the rules (3) & (6) in theorem 2.1, edge odd-graceful labelings of the required graph is obtained as follows.

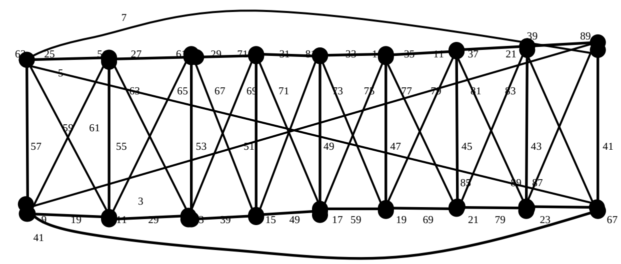

Figure 5: Edge-odd graceful Graph of $P_2 \boxtimes C_9$

Example 2.4: The strong product graph $P_2 \boxtimes C_8$ is edge-odd graceful.
Proof: The strong product graph $P_2 \boxtimes C_8$ is a connected graph with 16 vertices and 40 edges, where $n \equiv 3 \pmod 5$. Due to the rules (4) & (6) in theorem 2.1, edge odd-graceful labelings of the required graph is obtained as follows.

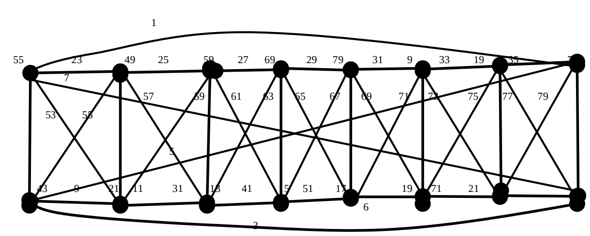

Figure 6: Edge-odd graceful Graph of $P_2 \boxtimes C_8$

Example 2.5: The strong product graph $P_2 \boxtimes C_8$ is edge-odd graceful.

Proof: The strong product graph $P_2 \boxtimes C_6$ is a connected graph with 12 vertices and 30 edges, where $n \equiv 1 \pmod 5$. Due to the rules (5) & (6) in theorem 2.1, edge odd-graceful labelings of the required graph is obtained as follows**.**

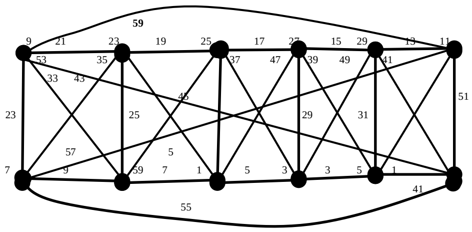

Figure 7: Edge-odd graceful Graph of $P_2 \boxtimes C_6$

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